Week 5 Recitation Problems

MATH:114, Recitations 309 and 310

Euler's Formula and Hyperbolic Functions

Euler's Formula, written as

$$e^{it} = \cos t + i \sin t,$$

where i is the complex unit $i = \sqrt{-1}$, is an expression which establishes the relationship between continuous complex growth (e^{it}) and trigonometry in the complex plane ($\cos t + i \sin t$).

1. Using an annotated picture or words, describe what Euler's formula tells us. *(Hint: think about the complex unit circle — that is, the circle centered at the origin which intersects the complex (vertical) axis at the points* $(0, i)$ *and* $(0, -i)$ *and the real (horizontal) axis at the points* $(1, 0)$ *and* $(-1, 0)$*. If* t *measures an angle counter-clockwise from the origin, where does each side of the equation end up?)*

2. Verify **Euler's Identity**

$$e^{i\pi} = -1$$

3. (From a previous exam.) The derivative with respect to t of the complex function

$$e^t \cos(2t) + ie^t \sin(2t)$$

is

$$e^t \cos(2t) - 2e^t \sin(2t) + ie^t \sin(2t) + 2ie^t \cos(2t).$$

Show that this derivative is equal to the derivative with respect to t of $e^{(1+2i)t}$.

4. (From a previous exam.) If we define

$$\sinh(t) = \frac{e^t - e^{-t}}{2} \qquad \text{and} \qquad \cosh t = \frac{e^t + e^{-t}}{2},$$

show that

$$(\cosh t)^2 - (\sinh t)^2 = 1$$

for all values of t.

5. Using the definitions of $\cosh$ and $\sinh$ from Problem 4, show that

$$\cosh(2t) = (\cosh t)^2 + (\sinh t)^2$$