Week 3 Recitation Problems

MATH:114, Recitations 309 and 310

Let's sketch the graphs of the hyperbolic functions.

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad \text{and} \quad \sinh(x) = \frac{e^x - e^{-x}}{2}.$$

(1) Sketch the functions: $f(x) = \frac{1}{2}e^x$ and $g(x) = \frac{1}{2}e^{-x}$. **(2)** Sketch the functions: $f(x) = \frac{1}{2}e^x$ and $h(x) = -\frac{1}{2}e^{-x}$.

(3) On the same graph as (1) above sketch $\cosh(x)$. Also, on same graph as (2) above sketch $\sinh(x)$. Comment on the limits when x tends to $-\infty$ and $+\infty$ for $\cosh(x)$ and $\sinh(x)$. Is this consistent with your graphs?

Are $\cosh(x)$ and $\sinh(x)$ odd or even functions? How do you know?

(4) The names of these hyperbolic functions are very similar to that of $\cos(x)$ and $\sin(x)$, which is no accident. Explore their similarities and differences by filling out the following tables.

Function	**Derivative**
$\sin(x)$	
$\cos(x)$	
$\sinh(x)$	
$\cosh(x)$	

Identities
$\cos^2(x) + \sin^2(x) =$
$\cosh^2(x) - \sinh^2(x) =$

(5) Application: The shape of a hanging cable between two same- height poles satisfies the equation $y = A\cosh(x/C) + B$. Assume that the cable's tension in our example is such that $A = 100$ and $C = 100$ and that the two poles of height h are placed at $x = -50m$ and $x = +50m$ as the graph shows. Then assume that the trees we are dealing with can reach a maximum height of $10m$. What pole height h is needed such that the cable just clears $10m$? Use the following value: $\cosh(0.5) \approx 1.13$ (no calculator needed!).

Bonus: Completed the work early? Calculate the length of cable for the above problem, using the following formula for cable length.

$$L(x) = \int_a^b \sqrt{1 + (f'(x))^2}dx$$

where $f(x)$ is the hyperbolic function provided in the previous example: $f(x) = 100\cosh(x/100) + B$. Use the following value: $\sinh(0.5) \approx 0.52$ (no calculator needed!).