Week 3 Recitation Problems

MATH:114, Recitations 309 and 310

1. Let

$$f(x) = \frac{1}{2x-1}.$$

Compute the surface area of the solid generated when f is rotated around the x axis where x is between $3/4$ and 4.

Solution: Start by taking the first derivative of f:

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(\frac{1}{2x-1}\right) \\ &= \frac{-1}{(2x-1)^2} \end{aligned}$$

Then, we can use the surface area formula:

$$\begin{aligned} S &= \int_{-\frac{3}{4}}^{4} 2\pi f(x)\sqrt{1+(f'(x))^2} \\ &= 2\pi\int_{-\frac{3}{4}}^{4} \frac{1}{2x-1}\sqrt{1+\left(\frac{-1}{(2x-1)^2}\right)^2} \\ &= 2\pi\int_{-\frac{3}{4}}^{4} \frac{1}{2x-1}\cdot\int_{-\frac{3}{4}}^{4}\sqrt{1^2+\left(\frac{-1}{(2x-1)^2}\right)^2} \\ &= 2\pi\int_{-\frac{3}{4}}^{4} \frac{1}{2x-1}\cdot\int_{-\frac{3}{4}}^{4}\sqrt{\left(1+\frac{-1}{(2x-1)^2}\right)^2} \\ &= 2\pi\int_{-\frac{3}{4}}^{4} \frac{1}{2x-1}\cdot\int_{-\frac{3}{4}}^{4}\left(1+\frac{-1}{(2x-1)^2}\right) \\ &= 2\pi\cdot\ln(2x-1)\cdot\left(x+\frac{1}{2(2x-1)}\right)\Bigg|_{3/4}^{4} \\ &= \frac{221\ln(\pi)}{4} \end{aligned}$$

so we have found the surface area of our solid.

2. Plot the functions

$$f(x) = x^3, \ g(x) = \sqrt[3]{x}.$$

Rotate the area between f and g around the x axis to form a solid of rotation. Set up (but do not compute) two integrals to find the volume of the solid.

3. Using f and g from #2, set up (but do not compute) an integral to find the surface area of the solid. Remember that the expression used to find the surface area of a solid is

$$S = \int_a^b 2\pi \cdot h(x) \cdot \sqrt{1 + (h'(x)^2)}\, dx.$$

How does this integral compare to the integral you set up to compute the volume using the *shell* method? Come up with a geometric explanation (a picture counts!).

4. Let

$$f(x) = \frac{1}{2}x^2 - \frac{1}{4}\ln(x),$$

and find the length of the curve for $2 \le x \le 4$.