Week 3 Recitation Problems

MATH:114, Recitations 309 and 310

1. Let

$$f(x) = \frac{1}{2x-1}.$$

Compute the surface area of the solid generated when f is rotated around the x axis where x is between $3/4$ and 4.

Solution: Start by taking the first derivative of f:

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(\frac{1}{2x-1}\right) \\ &= \frac{-1}{(2x-1)^2} \qquad (1) \end{aligned}$$

Then, we can use the surface area formula:

$$\left.\begin{aligned} S &= \int_{-\frac{3}{4}}^{4} 2\pi f(x)\sqrt{1+(f'(x))^2} \\ &= 2\pi \int_{-\frac{3}{4}}^{4} \frac{1}{2x-1}\sqrt{1+\left(\frac{-1}{(2x-1)^2}\right)^2} \\ &= 2\pi \int_{-\frac{3}{4}}^{4} \frac{1}{2x-1} \cdot \int_{-\frac{3}{4}}^{4} \sqrt{1^2+\left(\frac{-1}{(2x-1)^2}\right)^2} \qquad (2) \\ &= 2\pi \int_{-\frac{3}{4}}^{4} \frac{1}{2x-1} \cdot \int_{-\frac{3}{4}}^{4} \sqrt{\left(1+\frac{-1}{(2x-1)^2}\right)^2} \qquad (3) \\ &= 2\pi \int_{-\frac{3}{4}}^{4} \frac{1}{2x-1} \cdot \int_{-\frac{3}{4}}^{4} \left(1+\frac{-1}{(2x-1)^2}\right) \\ &= 2\pi \cdot \ln(2x-1) \cdot \left(x+\frac{1}{2(2x-1)}\right)\Bigg|_{3/4}^{4} \qquad (4)\ (5) \end{aligned}\right\} (6)$$

$$= \frac{221\ln(\pi)}{4}$$

so we have found the surface area of our solid.

(1) bad use of the chain rule!

(2) integrals of products aren't always products of integrals

(3) can't write sums of squares that way

(4) missing u-sub constant on integration of ln(2x-1)

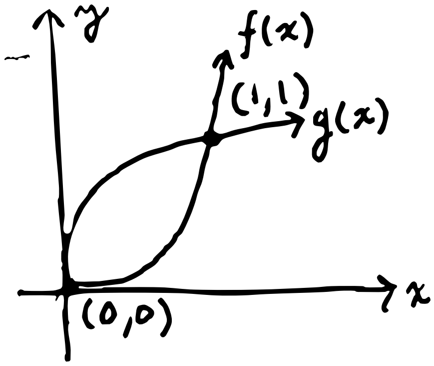
(5) wrong sign on $\frac{1}{2(2x-1)}$ term

(6) no dx anywhere in sight!

2. Plot the functions

$$f(x) = x^3, \; g(x) = \sqrt[3]{x}.$$

Rotate the area between f and g around the x axis to form a solid of rotation. Set up (but do not compute) two integrals to find the volume of the solid.



washer/disk:

$$V = \int_0^1 \pi \left(g(x)^2 - f(x)^2\right) dx$$

$$= \pi \int_0^1 \left(x^{\frac{2}{3}} - x^5\right) dx \quad \left(= \frac{16\pi}{35}\right)$$

shell:

$$V = \int_0^1 2\pi y \left(f^{-1}(y) - g^{-1}(y)\right) dy$$

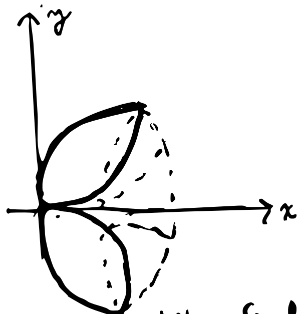
$$= 2\pi \int_0^1 y\left(\sqrt[3]{y} - x^3\right) dy \quad \left(= \frac{16\pi}{35}\right)$$

3. Using f and g from #2, set up (but do not compute) an integral to find the surface area of the solid. Remember that the expression used to find the surface area of a solid is

$$S = \int_a^b 2\pi \cdot h(x) \cdot \sqrt{1 + (h'(x)^2)}\, dx.$$

How does this integral compare to the integral you set up to compute the volume using the *shell* method? Come up with a geometric explanation (a picture counts!).

solid:



two surface areas: outer and inner

$$g'(x) = \frac{1}{3x^{2/3}},$$

$$S_o = \int_0^1 2\pi \cdot g(x) \cdot \sqrt{1 + (g'(x))^2}\, dx$$

$$= 2\pi \int_0^1 \sqrt[3]{x} \cdot \sqrt{1 + \frac{1}{9x^{4/3}}}\, dx \neq$$

$$f'(x) = 3x^2$$

$$S_I = \int_0^1 2\pi \cdot f(x) \cdot \sqrt{1 + (f'(x))^2}\, dx$$

$$= 2\pi \int_0^1 x^3 \sqrt{1 + 9x^4}\, dx$$

When finding surface area, we make the height of each cylinder very small so we can capture the surface area of a tiny width. When finding volume, the width of each cylinder depends on the width of the solid at that moment.

4. Let

$$f(x) = \frac{1}{2}x^2 - \frac{1}{4}\ln(x),$$

and find the length of the curve for $2 \leq x \leq 4$.

$$f'(x) = x - \frac{1}{4x},$$

$$1 + (f'(x))^2 = 1 + \left(x - \frac{1}{4x}\right)^2$$

$$= 1 + x^2 - \frac{1}{2} + \left(\frac{1}{4x}\right)^2$$

$$= x^2 + \frac{1}{2} + \left(\frac{1}{4x}\right)^2$$

$$= \left(x + \frac{1}{4x}\right)^2$$

$$L = \int_2^4 \sqrt{1 + f'(x)^2}\, dx$$

$$= \int_2^4 \sqrt{\left(x + \frac{1}{4x}\right)^2}\, dx$$

$$= \int_2^4 x + \frac{1}{4x}\, dx$$

$$= \frac{1}{2}x^2 + \frac{1}{4}\ln x \Big|_2^4 = 6 + \frac{\ln(2)}{4}$$