Week 13 Recitation Problems

MATH:114, Recitations 309 and 310

Names: ____

A *power series centered at* c is a series of the form

$$\sum_{n=1}^{\infty} a_n(x-c)^n,$$

where a_n are terms in a sequence. We can also think of this series as the sum of the terms in the sequence S, where

$$\begin{aligned} S &= \left\{a_1(x-c)^1,\ a_2(x-c)^2,\ a_3(x-c)^3, \dots,\ a_n(x-c)^n,\ a_{n+1}(x-c)^{n+1}, \dots\right\} \\ &= \{s_n\}_{n=1}^{\infty} \end{aligned}$$

Now, we want to figure out whether these series *converge* or *diverge*. To do so, we have two important tools: the *ratio test* and the *root test*. The *ratio test* says that

$$\lim_{n\to\infty}\left|\frac{s_{n+1}}{s_n}\right| = L, \text{ if... } \begin{cases} L<1 & \text{the series converges} \\ L>1 & \text{the series diverges} \\ L=1 & \text{we don't know.} \end{cases}$$

....

1. Discuss with your group: why does the *ratio* of the n^{th} and $(n+1)^{\text{th}}$ terms as n gets large help determine convergence? *(Hint: to start, think about last week's geometric series.)*

2. Does the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n(x-2)^n}{n}$$

converge or diverge? Does convergence or divergence depend on the value of x? Use the ratio test to find out.

We can also use another test called the *root test*. This test says that

$$\lim_{n\to\infty} \sqrt[n]{|s_n|} = \lim_{n\to\infty} |s_n|^{1/n} = L, \quad \text{if...} \begin{cases} L < 1 & \text{the series converges} \\ L > 1 & \text{the series diverges} \\ L = 1 & \text{we don't know.} \end{cases}$$

. . . .

3. Discuss with your group: why does the n^{th} root of the terms as n gets large help determine convergence? *(Hint: try playing with the exponents!)*

4. Does the series

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n x^n$$

converge or diverge? Use the root test to find out.

. . . .

5. Taylor series are a type of power series! Write out the Taylor series for the function $f(x) = e^x$ and find the values of x where the Taylor series converges.