exam 2 review

8.2 — integration by parts

the product rule

$$\frac{d}{dx}\left(u(x) \cdot v(x)\right) = u(x) \cdot v'(x) + u'(x) \cdot v(x)$$

integrating the product rule

$$\int \frac{d}{dx}\left(u(x)\cdot v(x)\right) = \int \left(u(x)\cdot v'(x) + u'(x)\cdot v(x)\right) dx$$

integrating the product rule

$$\int \frac{d}{dx}\left(u(x)\cdot v(x)\right) = \int \left(u(x)\cdot v'(x) + u'(x)\cdot v(x)\right) dx$$

$$u(x)\cdot v(x) = \boxed{\int u(x)\cdot v'(x)dx} + \boxed{\int u'(x)\cdot v(x)dx}$$

integrating the product rule

$$\int \frac{d}{dx}\left(u(x)\cdot v(x)\right) = \int \left(u(x)\cdot v'(x) + u'(x)\cdot v(x)\right) dx$$

$$u(x)\cdot v(x) = \boxed{\int u(x)\cdot v'(x)dx} + \boxed{\int u'(x)\cdot v(x)dx}$$

$$\boxed{\int u(x)\cdot v'(x)dx} = u(x)\cdot v(x) - \boxed{\int u'(x)\cdot v(x)dx}$$

2(b), old exam 2

$$\int x^2 \cos x \; dx$$

$$u = x^2,\ dv = \cos(x)dx$$

$$u = x^2,\ dv = \cos(x)dx$$

$$\implies du = 2xdx,\ v = \sin(x)$$

2(b), old exam 2

$$\int u dv = uv - \int v du$$

2(b), old exam 2

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2 \cos(x)dx = x^2 \sin(x) - \int 2x \sin(x)dx$$

2(b), old exam 2

$$u = 2x, \;\; dv = -\sin(x)dx$$

2(b), old exam 2

$$u = 2x, \;\; dv = -\sin(x)dx$$

$$\implies \; du = 2dx, \;\; v = \cos(x)$$

2(b), old exam 2

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2 \cos(x)dx = x^2 \sin(x) - \int 2x \sin(x)dx$$

2(b), old exam 2

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2 \cos(x)dx = x^2 \sin(x) - \int 2x \sin(x)dx$$

2(b), old exam 2

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2 \cos(x)dx = x^2 \sin(x) - \int 2x \sin(x)dx$$

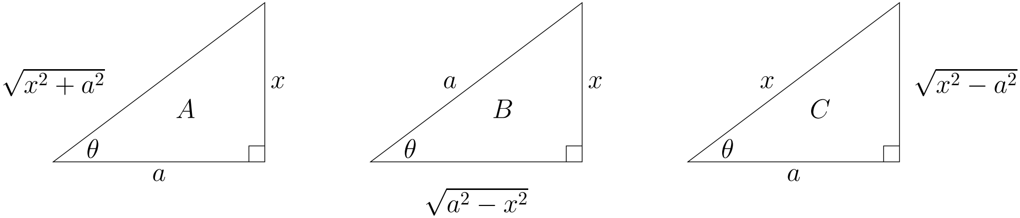
$$= x^2 \sin(x) + 2x \cos(x) - \int 2 \cos(x)dx$$

2(b), old exam 2

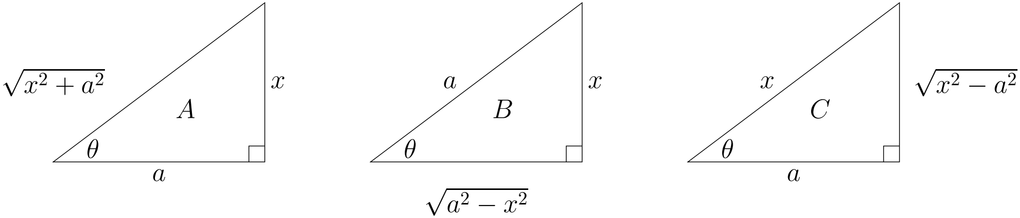
$$\int u\,dv = uv - \int v\,du$$

$$\begin{aligned}\int x^2\cos(x)dx &= x^2\sin(x) - \int 2x\sin(x)dx\\ &= x^2\sin(x) + 2x\cos(x) - \int 2\cos(x)dx\\ &= x^2\sin(x) + 2x\cos(x) - 2\sin(x) + C\end{aligned}$$

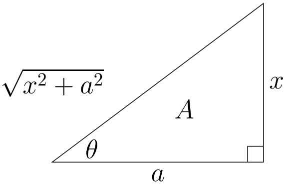
8.3, 8.4 — trig integrals and substitution

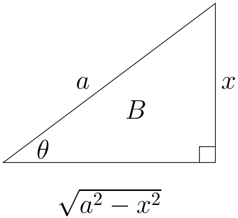


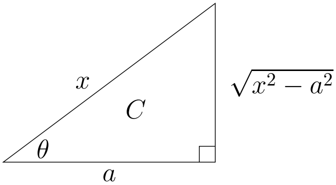
$\sqrt{x^2 + a^2}$
x
A
θ
a
a
x
B
θ
$\sqrt{a^2 - x^2}$
x
$\sqrt{x^2 - a^2}$
C
θ
a



$$a \tan(\theta) = x \qquad a \sin(\theta) = x \qquad a \sec(\theta) = x$$







$$a\tan(\theta) = x \qquad a\sin(\theta) = x \qquad a\sec(\theta) = x$$

$$\sqrt{a^2 + x^2} = a|\sec(\theta)| \qquad \sqrt{a^2 - x^2} = a|\cos(\theta)| \qquad \sqrt{x^2 - a^2} = a|\tan(\theta)|$$

4(b), old exam 2

find a trig substitution for but **do not compute** the integral

$$\int x^3\sqrt{x^2-9}dx$$

matches $\sqrt{x^2 - a^2}$, so

matches $\sqrt{x^2 - a^2}$, so

$$a^2 = 9 \implies a = 3$$

using scenario C, we get

$$x =$$

and

$$\sqrt{x^2 - 3^2} =$$

and

$$dx =$$

using scenario C, we get

$$x = 3\sec(\theta)$$

and

$$\sqrt{x^2 - 3^2} = 3\tan(\theta)$$

and

$$dx = 3\sec(\theta)\tan(\theta)\ d\theta$$

$$\int x^3\sqrt{x^2-9}\,dx =$$

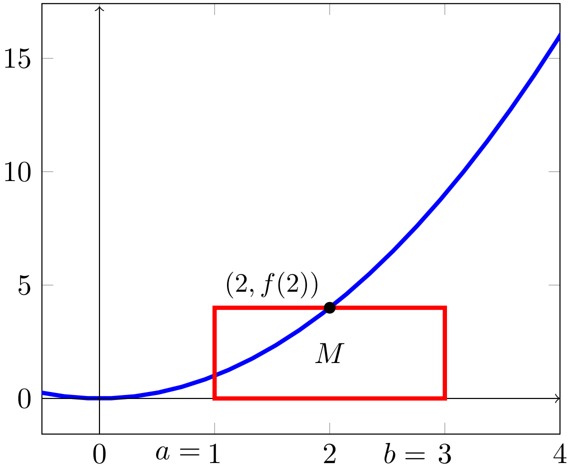
$$\int x^3\sqrt{x^2-9}\ dx = \int (3\sec(\theta))^3 \cdot 3\tan(\theta)\ dx$$

$$\begin{aligned}\int x^3\sqrt{x^2-9}\ dx &= \int (3\sec(\theta))^3 \cdot 3\tan(\theta)\ dx \\ &= \int 3^3\sec^3(\theta)\cdot 3\tan(\theta)\cdot 3\sec(\theta)\tan(\theta)\ d\theta\end{aligned}$$

$$\begin{aligned}\int x^3\sqrt{x^2-9}\ dx &= \int (3\sec(\theta))^3 \cdot 3\tan(\theta)\ dx \\ &= \int 3^3\sec^3(\theta)\cdot 3\tan(\theta)\cdot 3\sec(\theta)\tan(\theta)\ d\theta \\ &= \boxed{3^5\int \sec^4(\theta)\cdot\tan^2(\theta)\ d\theta}\end{aligned}$$

8.7 — numerical integration

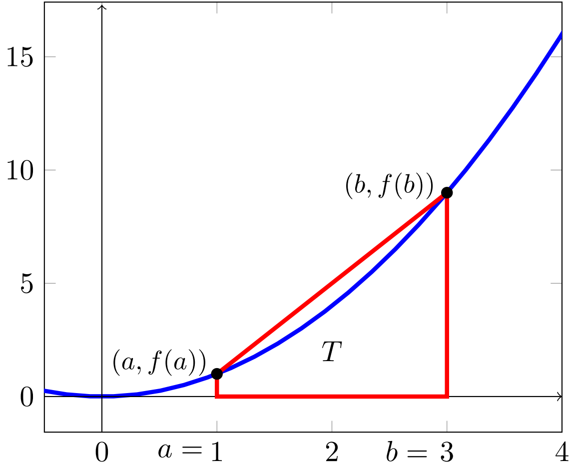
Midpoint rule



$$M = \overbrace{(b-a)\cdot f\left(\frac{b+a}{2}\right)}^{\text{area of a rectangle!}}$$

$$= \Delta x \cdot f(m)$$

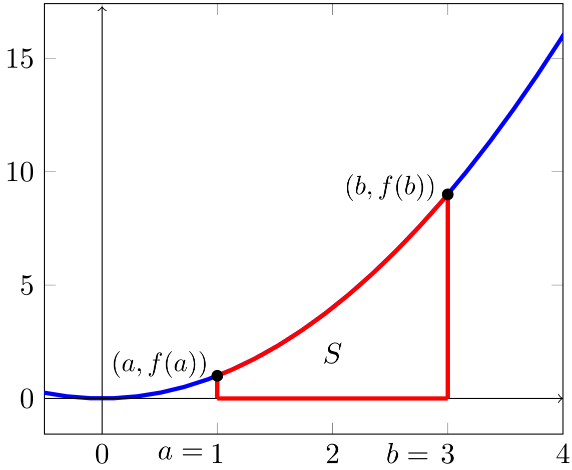
Trapezoid rule



$$T = \overbrace{(b-a) \cdot \frac{f(b)+f(a)}{2}}^{\text{area of a trapezoid!}}$$

$$= \frac{\Delta x}{2} \cdot (f(b) + f(a))$$

Simpson's rule



$$S = \overbrace{\frac{b-a}{3} \cdot \frac{f(a) + 4f\left(\frac{b+a}{2}\right) + f(b)}{2}}^{\text{magic!}}$$

$$= \frac{2M + T}{3}$$

6, old exam 2

If $a = 0$ and $b = 3$, use each rule to estimate the value of

$$\int_0^3 \frac{x}{1 + x + x^2}\, dx$$

$$M =$$

$$M = 3 \cdot \frac{3/2}{1 + 3/2 + (3/2)^2}$$

$$T =$$

$$T = 3 \cdot \frac{0 + \frac{3}{1+3+9}}{2}$$

8.8 — improper integrals

$$\int_a^\infty f(x)\ dx = \lim_{b\to\infty}\left(\int_a^b f(x)\ dx\right)$$

7(b), old exam 2

use the definition of the improper integral to find the value of

$$\int_0^\infty e^{-st}\, dt$$

when $s > 0$ is a constant.

$$\int_0^\infty e^{-st}\ dt = \lim_{b\to\infty} \int_0^b e^{-st}\ dt$$

$aqqQ$

$$\int_0^\infty e^{-st}\ dt = \lim_{b\to\infty} \int_0^b e^{-st}\ dt$$

$$aqqQ = \lim_{b\to\infty} \frac{-1}{s} e^{-st} \Big|_0^b$$

$$\int_0^\infty e^{-st}\ dt = \lim_{b\to\infty}\int_0^b e^{-st}\ dt$$

$$aqqQ = \lim_{b\to\infty} \frac{-1}{s}e^{-st}\Big|_0^b$$

$$= \lim_{b\to\infty}\left(\frac{-1}{s}e^{-sb} - \frac{-1}{s}e^{-s0}\right)$$

$$
\begin{aligned}
\int_0^\infty e^{-st}\ dt &= \lim_{b\to\infty}\int_0^b e^{-st}\ dt \\
aqqQ &= \lim_{b\to\infty}\frac{-1}{s}e^{-st}\Big|_0^b \\
&= \lim_{b\to\infty}\left(\frac{-1}{s}e^{-sb}-\frac{-1}{s}e^{-s0}\right) \\
&= \frac{-1}{s}\cdot 0+\frac{1}{s}
\end{aligned}
$$

$$
\begin{aligned}
\int_0^\infty e^{-st}\ dt &= \lim_{b\to\infty} \int_0^b e^{-st}\ dt \\
aqqQ &= \lim_{b\to\infty} \frac{-1}{s} e^{-st}\Big|_0^b \\
&= \lim_{b\to\infty} \left(\frac{-1}{s} e^{-sb} - \frac{-1}{s} e^{-s0} \right) \\
&= \frac{-1}{s} \cdot 0 + \frac{1}{s} \\
&= \frac{1}{s}
\end{aligned}
$$