Week 1 Recitation Problems

MATH:114, Recitations 309 and 310

1. Graph (shade) the region bounded by the following curves in the first quadrant:

   (a) $y = f(x) = (x+1)^2 + 1$

   (b) $y = g(x) = 9 - 4x$

   (c) The y-axis.

$f(x) = g(x)$ gives

$9 - 4x = (x+1)^2 + 1$

$= x^2 + 2x + 2,$

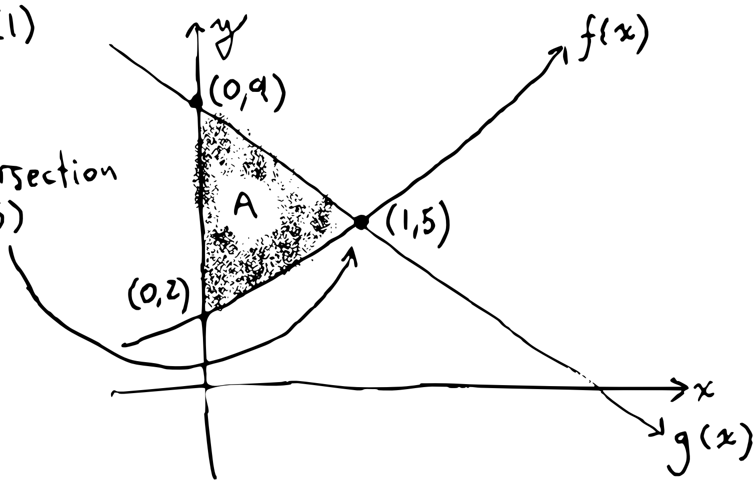
$0 = x^2 + 6x - 7$

with roots @ $\{1, -7\}$

$f(1) = g(1)$

$= 5,$

so intersection @ $(1,5)$



2. Write one (or two) x-integrals that give the exact area of this region. Using your integral(s), compute this area.

$A = \int_0^1 g(x) - f(x)\,dx$

$= \int_0^1 (9-4x) - ((x+1)^2+1)\,dx$

$= \int_0^1 9 - 4x\,dx - \int_0^1 (x+1)^2 + 1\,dx$ expand

$= (9x - 2x^2)\Big|_0^1 - \left(2x + x^2 + \frac{x^3}{3}\right)\Big|_0^1$

$= \left(7x - 3x^2 - \frac{x^3}{3}\right)\Big|_0^1$

$= \left(7 - 3 - \frac{1}{3}\right)$

$= 4 - \frac{1}{3}$

$\boxed{= \frac{11}{3}}$

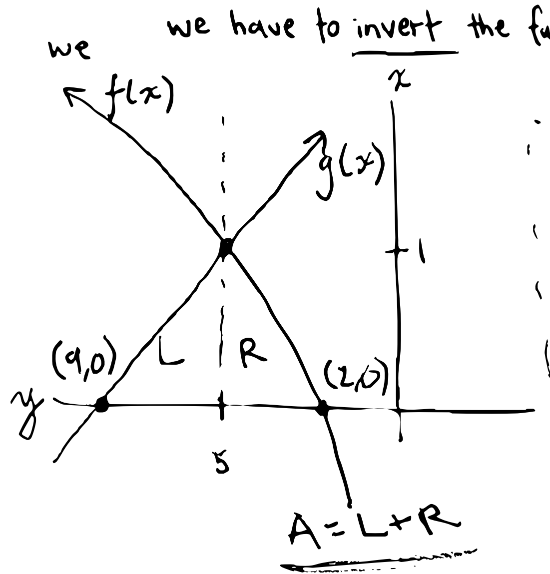
3. Write one (or two) y-integrals that give the exact area of this region. Using your integral(s), compute this area.

we have to invert the functions: $y = (x+1)^2 + 1 \Rightarrow \sqrt{y-1} - 1 = x = f(y)$

$y = 9 - 4x \Rightarrow \frac{y-9}{-4} = x = g(y)$

$L = \int_5^9 g(y)\,dy,$

$R = \int_2^5 f(y)\,dy$



$L = \int_5^9 \frac{y-9}{-4}\,dy$

$= \frac{1}{-4}\left(\frac{1}{2}y^2 - 9y\right)\Big|_5^9$

$= 2$

$R = \int_2^5 \sqrt{y-1} - 1\,dy$

u-sub $= \int_2^5 \sqrt{y-1}\,dy - \int_2^5 1\,dy$

$= \int_2^5 \sqrt{u}\,du - \int_2^5 dy$

$= \left(\frac{2}{3}(y-1)^{3/2} - y\right)\Big|_2^5$

$= \frac{5}{3}$

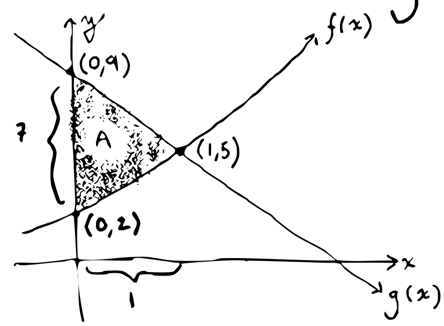
$L + R = 2 + \frac{5}{3}$

$\boxed{= \frac{11}{3}}$

Let's check our work by answering some questions:

1. Are your results for questions 2 and 3 the same?
2. Can you find a way to *approximate* the area between the curves? (Hint: use a bit of geometry!)
3. How does your approximation compare to the values you computed in (2) and (3)?

(2) use a triangle!



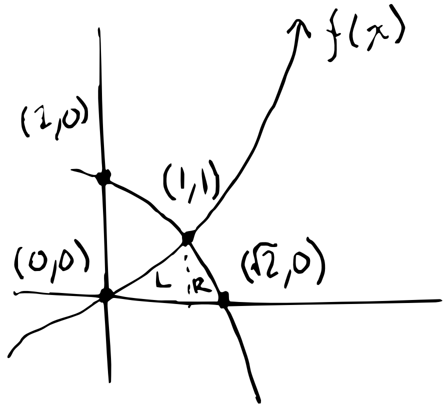
$A \approx \frac{1}{2}(7 \cdot 1)$

$= \frac{7}{2}$,

$\boxed{\frac{7}{2} = 3.5, \quad \frac{11}{3} = 3.\overline{6}}$

4. Repeat the same process with these curves:

   1. $f(x) = x^3$
   2. $g(x) = 2 - x^2$
   3. the x-axis.



$L = \int_0^1 f(x)\,dx$

$= \int_0^1 x^3\,dx$

$= \left(\frac{1}{4}x^4\right)\Big|_0^1$

$= \frac{1}{4}$

(bonus!)

$R = \int_1^{\sqrt{2}} g(x)\,dx$

$= \int_1^{\sqrt{2}} 2 - x^2\,dx$

$= \left(2x - \frac{1}{3}x^3\right)\Big|_1^{\sqrt{2}}$

$= \frac{1}{3} - \frac{2\sqrt{2}}{3} + 2(-1 + \sqrt{2})$

≈ 0.218951

$L + R = \frac{1}{4} + 0.218951$

$\boxed{\approx 0.468951}$

$y = x^3 \Rightarrow \sqrt[3]{y} = x, \quad y = 2 - x^2 \Rightarrow \sqrt{2-y}$

$A = \int_0^1 \sqrt{2-y} - \sqrt[3]{y}\,dy$

$u = 2 - y$

$= \int_0^1 \sqrt{u}\,du - \int_0^1 y^{1/3}\,dy$

$= \left(-\frac{2}{3}(2-y)^{3/2} - \frac{3}{4}y^{4/3}\right)\Big|_0^1$

$= \left(-\frac{2}{3}(1) - \frac{3}{4}(1)\right) - \left(-\frac{2}{3}(2)^{3/2} - \frac{3}{4}(0)\right)$

$= -\frac{2}{3} - \frac{3}{4} + \frac{2}{3}(2)^{3/2}$

$\boxed{\approx 0.46891}$