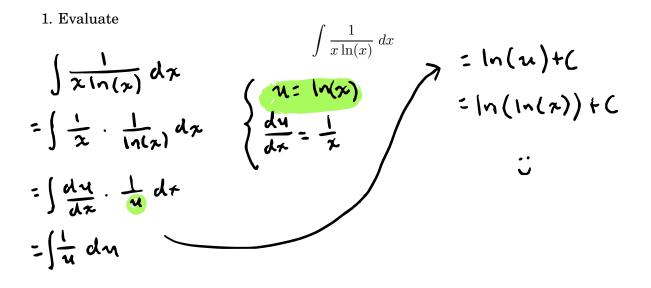
Week 5 Recitation Problems MATH:114, Recitations 309 and 310

Logarithms and Exponential Change



2. Evaluate

$$\int \frac{x}{x^2 + 4} \, dx$$

Use u-sub again!

$$u = \chi^{2} + 4$$
, $\int \frac{\chi}{\chi^{2} + 4} d\chi$
 $\frac{du}{d\chi} = 2\chi$, $\int \frac{1}{2} \frac{du}{d\chi} d\chi$
 $du = 2\chi d\chi$
 $= \frac{1}{2} \int \frac{1}{u} d\chi$
 $= \frac{1}{2} \int \frac{1}{u} d\chi$
 $= \frac{1}{2} \ln(u) + C$
 $= \frac{1}{2} \ln(\chi^{2} + 4) + C$

: : If a function y(t) is increasing or decreasing at an exponential rate, we can say it is **exponen**tially growing or exponentially decreasing, and this rate of change is proportional to its value at a time t. In other words, y(t) is proportional to its own derivative y'(t), so

$$\frac{d}{dt}y(t) = k \cdot y(t). \tag{*}$$

Writing this just in terms of our function y, and treating it like a variable, the following expressions are equivalent:

$$\frac{dy}{dt} = k \cdot y(t) \qquad y'(t) = k \cdot y(t) \qquad y' = k \cdot y$$

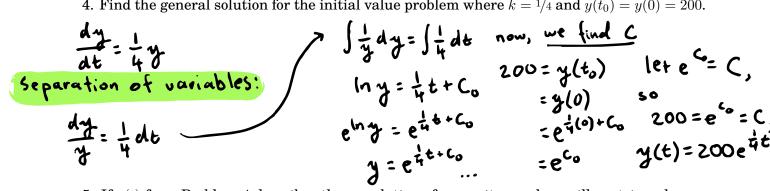
A differential equation is when a function is equated to its own derivative(s), in an expression like the ones above. An **initial value problem** arises when you are given k and the value of y(t) at a "starting value" or initial condition t_0 on its domain — like $t_0 = 0$, so $y(t_0) = y(0) = C$, where C is some constant — and we are tasked with recovering the function y(t). The above types of initial value problems have one specific solution:

$$y(t) = Ce^{kt},$$

where k > 0.

- 3. Check that the function $y(t) = Ce^{kt}$ satisfies the equation in (*).
 - $\frac{d}{dt} y(t) = \frac{d}{dt} C e^{kt}$ so if L = C·k, L is a constant, and = C·k·e^{kt}, $\frac{dy}{dt} = L \cdot e^{kt}$

4. Find the general solution for the initial value problem where k = 1/4 and $y(t_0) = y(0) = 200$.



5. If y(t) from Problem 4 describes the population of mosquitoes, when will we triumph over our pestilent insect overlords and vanquish their population? (In other words, at what time tdoes y(t) = 0?)