THE GREATEST THING YOU WILL DO ALL WEEK#4

MATH 114 - CALCULUS II - FALL 2020

Professor/TA : ________________________ **Sec:** ________________________

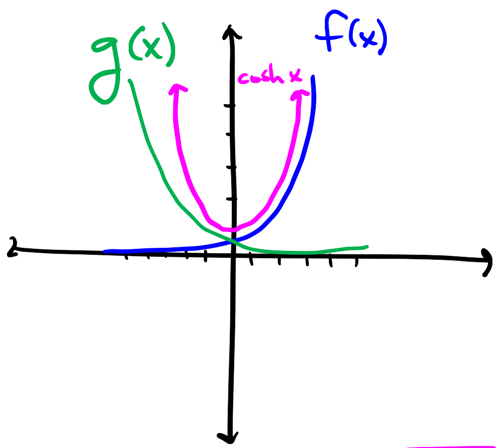
FULL NAME: ________________________ **Partners:** ____________________

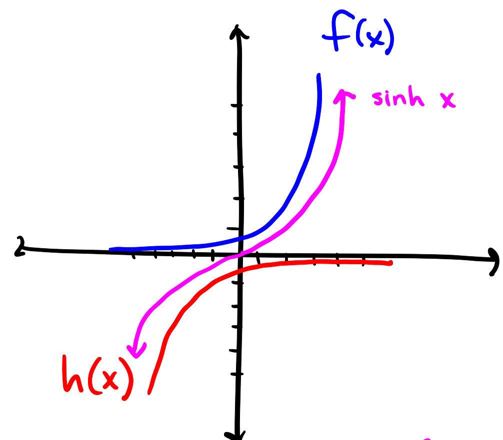
The sagging high voltage cables (hyperbolic functions)

When laying high voltage transition cables a lot of attention needs to be taken care so that trees do not get close to them... or disasters happens: http://www-rohan.sdsu.edu/ rcarrete/teaching/M-151/labs/HangingCable.html
The shape of a hanging cable is a *catenary* that is expressed through hyperbolic functions. The aim of this activity is to learn more about hyperbolic functions and their application to the above high voltage cable problem.
Let us sketch the graphs of the main hyperbolic functions:

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad \text{and} \quad \sinh(x) = \frac{e^x - e^{-x}}{2}.$$

(1) Sketch the functions: $f(x) = \frac{1}{2}e^x$ and $g(x) = \frac{1}{2}e^{-x}$. **(2)** Sketch the functions: $f(x) = \frac{1}{2}e^x$ and $h(x) = -\frac{1}{2}e^{-x}$.





(3) On the same graph as (1) above sketch cosh(x). Also, on same graph as (2) above sketch sinh(x). Comment on the limits when x tends to $-\infty$ and $+\infty$ for cosh(x) and sinh(x). Is this consistent with your graphs?

$$\lim_{x\to-\infty} \cosh x = \infty \qquad \lim_{x\to-\infty} \sinh x = -\infty$$

$$\lim_{x\to\infty} \cosh x = \infty \qquad \lim_{x\to\infty} \sinh x = \infty$$

Are cosh(x) and sinh(x) odd or even functions? Can you prove it mathematically?

cosh (x) ⟷ even
sinh (x) ⟷ odd

$$\text{If } f(-x) = \begin{cases} -f(x), & \text{then odd function} \\ f(x), & \text{then even function} \end{cases}$$

$$\cosh(-x) = \frac{e^{(-x)} + e^{-(-x)}}{2} = \frac{e^x + e^{-x}}{2} = \cosh(x)$$

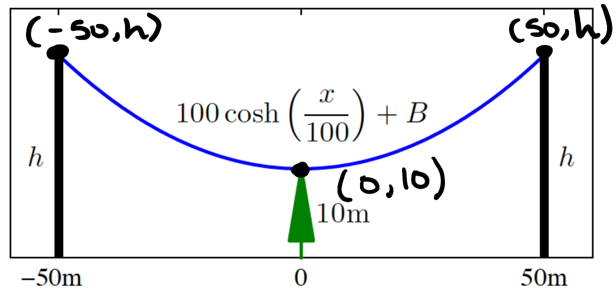
$$\sinh(-x) = \frac{e^{(-x)} - e^{-(-x)}}{2} = \frac{e^{-x} - e^{x}}{2} = -\frac{e^x - e^{-x}}{2} = -\sinh(x)$$

(4) The names of these hyperbolic functions are very similar to that of $\cos(x)$ and $\sin(x)$, which is no accident. Explore their similarities and differences by filling out the following tables.

Function	Derivative
$\sin(x)$	$\cos x$
$\cos(x)$	$-\sin x$
$\sinh(x)$	$\cosh x$
$\cosh(x)$	$\sinh x$

Identities	
$\cos^2(x) + \sin^2(x) =$ 1	— from unit circle
$\cosh^2(x) - \sinh^2(x) =$ 1	— from unit hyperbola

(5) Application: The shape of a hanging cable between two same-height poles satisfies the equation $y = A\cosh(x/C) + B$. Assume that the cable's tension in our example is such that $A = 100$ and $C = 100$ and that the two poles of height h are placed at $x = -50m$ and $x = +50m$ as the graph shows. Then assume that the trees we are dealing with can reach a maximum height of $10m$. What pole height h is needed such that the cable just clears $10m$? Use the following value: $\cosh(0.5) \approx 1.13$ (no calculator needed!).



So when $x = \pm 50$, $y = h$

$$\Rightarrow h = 100 \cosh\left(\frac{50}{100}\right) + B$$

$$h = 100 \cosh(.5) + B$$

$$h = 100(1.13) + B$$

$$h = 113 + B$$

$$h = 113 - 90$$

$$h = 23 \text{ m}$$

When $x = 0$, $y = 10$

$$10 = 100 \cosh(0) + B$$

$$10 = 100\left(\frac{e^0 + e^0}{2}\right) + B$$

$$10 = 100\left(\frac{1+1}{2}\right) + B$$

$$10 = 100 + B$$

$$B = -90$$

Bonus: Completed the work early? Calculate the length of cable for the above problem, using the following formula for cable length.

$$L(x) = \int_a^b \sqrt{1 + (f'(x))^2}\,dx$$

where $f(x)$ is the hyperbolic function provided in the previous example: $f(x) = 100\cosh(x/100) + B$. Use the following value: $\sinh(0.5) \approx 0.52$ (no calculator needed!).

$$f'(x) = \sinh \frac{x}{100}$$

$$L(x) = \int_{-50}^{50} \sqrt{1 + \left(\sinh\left(\frac{x}{100}\right)\right)^2}\,dx = 2\int_0^{50} \underbrace{\sqrt{1 + \left(\sinh\left(\frac{x}{100}\right)\right)^2}\,dx}_{\text{use u-sub and identity}}$$