A Network Contraction Algorithm Using an Iterative Learning Approach for Path Finding Problem in Stochastic Time-Varying Networks considering Travel Time Correlations

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Fatemeh Fakhrmoosavi
Doctoral Researcher
Michigan State University
428 S. Shaw Ln., MI 48824, USA
Email: moosavi@msu.edu

Ali Zockaie
Assistant Professor
Michigan State University
428 S. Shaw Ln., MI 48824, USA
Phone: 517-355-8422; Fax: 517-432-1827; Email: zockaiea@egr.msu.edu

Khaled Abdelghany
Associate Professor
Civil and Environmental and Engineering
Bobby B. Lyle School of Engineering,
Southern Methodist University, Dallas, Texas 75275-0340, USA
Tel: 214-768-4309; Fax: (214) 768-2164; Email: khaled@lyle.smu.edu

Hossein Hashemi
Lead Operations Research Scientist
Tiger Analytics Management Consulting
4701 Patrick Henry Drive, Santa Clara, CA 95054
Tel: 817-902-68-30; Email: hossein.hashemi@tigeranalytics.com

Introduction

The path finding problem is a classical operations research problem with numerous applications. In this regard, the deterministic shortest path problem was initially considered with various proposed solution algorithms (*1–8*). The computational burden of the shortest path finding algorithms has a polynomial growth order of the network size, which limits the real-time applications in large-scale networks. Therefore, many studies presented different algorithms to improve the efficiency of shortest path finding algorithms using heuristic approaches (*9*, *10*). A* algorithms are one of the main categories of these heuristic algorithms, in which the scan eligible list order is prioritized based on the probability of a node being in the shortest path (*11–19*). The efficiency of these algorithms is highly dependent on the quality of the travel time estimation from each node to the destination.

The branch pruning approach is another method to limit the search area for the shortest path calculation (*18*, *20–22*) which is quite similar to the A* algorithm with the difference that the low priority nodes in this method are pruned from the search list and will never be scanned. A major limitation of this approach is that it may be terminated without finding the optimal path. Several decomposition-based algorithms are also suggested, in which large-scale networks are decomposed into several small sub-networks (*23–26*). The mentioned heuristic algorithms cannot be applied to stochastic networks due to the non-additivity and non-linearity of link travel times in such networks. Therefore, the reliable path finding problem gained attention to consider uncertainty and dynamic nature of link travel times (*27–33*). However, the computational burden in stochastic networks is much higher relative to the deterministic ones. Overcoming this computational burden requires developing innovative solution approaches, especially for large-scale applications. In this paper, we intend to describe a methodology to improve the computational efficiency of the optimal path-finding algorithms in stochastic dynamic networks, considering heterogeneity of users towards risk and correlation of link travel times.

Methodology Development

There are many algorithms in the literature for optimal paths finding in stochastic networks. However, these algorithms are not computationally efficient, especially for applications in large-scale networks. In the path finding problems, only a part of the entire network is relevant to the optimal path between a certain origin and destination. Thus, this study aims to demonstrate the ability to reduce the network size throughout the iterations of a Monte-Carlo Simulation-based (MCS) approach, recently introduced in the literature.

Each path has a minimum travel time and a maximum travel time. Therefore, comparing the optimistic travel time through any node to the destination, with the pessimistic travel time for the OD pair specifies if the node should be retained in the sub-network or not. The schematic view of this optimistic/pessimistic travel time comparison is illustrated in Figure **1**. One way to find the optimistic and pessimistic travel times is to consider minimum and maximum travel times for all links in the network and calculate the origin-based and destination-based deterministic shortest paths trees from the origin to the destination through any node. The sum of the travel time from the origin of interest to the node, and the travel time from the node to the desired destination while all links are set to the minimum travel time value specifies the optimistic travel time. However, the pessimistic travel time is calculated from the origin to the destination using the deterministic static shortest path algorithm when all links have the maximum value of travel time. These two optimistic and pessimistic travel times are then compared to make a decision about each network node (*34*). However, the probability of all network links being at their maximum or minimum travel time at the same time is low, especially when there are many links in the optimal path. Furthermore, to the best of our knowledge, there is no other approach developed in the literature that can find optimistic/pessimistic bounds that can be implemented efficiently on any general network with different configurations and characteristics. Therefore, a learning approach can be presented to derive efficient optimistic/pessimistic bounds. This learning approach can use the generated information within the early iterations of the simulation-based approach, as previously proposed in the literature (*33*) to solve the path finding problem in stochastic networks.

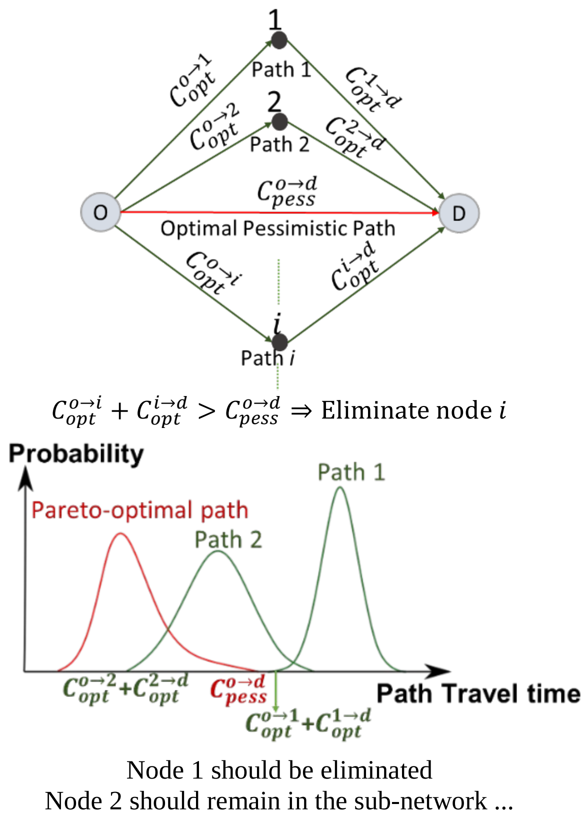

Figure 1 A schematic illustration of network contraction for the reliable path finding problem

Findings

The realizations of link travel times are studied here to find a relation between minimum/maximum labels and free flow travel times. This relation helps to propose realistic optimistic/pessimistic bounds for network contraction. The MCS approach for solving Shortest Path problem with On-Time Arrival Reliability (SPOTAR) problem is used in this study (*33*, *35*). The first stage of the stochastic path finding problem, including solving a deterministic shortest path problem at each iteration, is executed for 100 iterations for a randomly selected destination in the time-dependent Chicago downtown network (destination 84) (*34*). The travel time labels from each node to the destination for different departure time intervals and iterations bring about the insights required for the optimistic/pessimistic bounds. Thus, the minimum realized time-dependent labels of each node over 100 iterations are used to estimate the optimistic travel time from origin through that node to the destination. Similarly, the maximum realized time-dependent labels over 100 iterations of each origin node are used to estimate the pessimistic travel time from that origin to the destination. The maximum and minimum time-dependent labels over 100 realizations of any node located at a certain free flow travel time from the destination, divided by the free flow travel time, are demonstrated in Figure **2**. Each dot in these figures represents a node in the network. The figures reveal that the ratio of optimistic bounds to free flow travel times increases with the growing order of the free flow travel time from each node to the destination, whereas the ratio of pessimistic bounds to their corresponding free flow travel time decreases with growth in the order of free flow travel time from nodes to the destination. Thus, there is a relation between optimistic/pessimistic bounds and free flow travel times. This relation is an intuition for improving the results, especially for OD pairs with large distances, since the maximum bounds are decreasing and the minimum bounds are increasing as the free flow travel time increases.

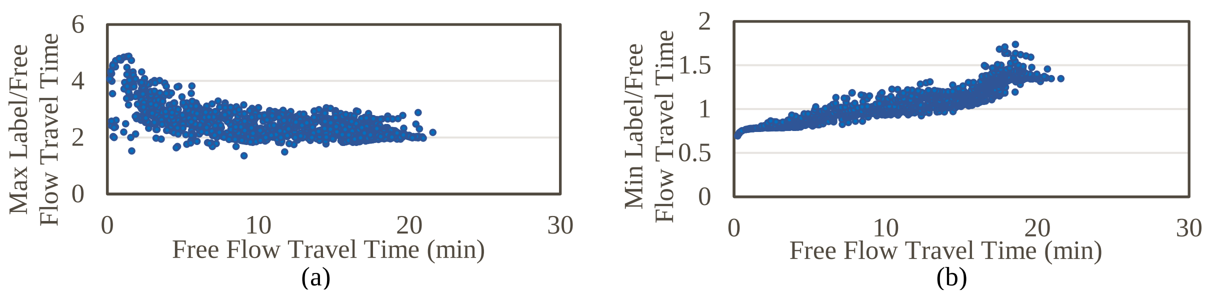

Figure 2 a) Maximum b) Minimum time-dependent label over 100 realizations from each node to destination 84 of the Chicago downtown network, divided by its free flow travel time, versus the free flow travel time from each node to the same destination

Conclusion

Finding optimal paths in a computationally reasonable time is a common requirement of the path finding algorithms. The goal of this study was to show the capability of existing MCS algorithms to use the information of some early iterations and reduce the network size for later iterations. The ratio of maximum and minimum labels to the free flow travel time of all iterations of an MCS approach for solving SPOTAR problem is utilized in this paper to show this capability. Therefore, a learning approach can be proposed to compare the optimistic and pessimistic solutions resulting from the realizations of travel time from the early iterations of MCS approaches. The network can be iteratively contracted till reaching a limit for the optimistic/pessimistic bounds.

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