Financial Time Series; Returns

In a time series of asset prices, $P_1, P_2, \ldots$, we are generally not as interested in the actual prices, as in their relative changes.

One period simple return from date $t-1$ to date $t$:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

The $k$-period simple compound return, $R_t[k]$, is given by

$$1 + R_t[k] = \frac{P_t}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t-2}} \cdot \cdots \cdot \frac{P_{t-k+1}}{P_{t-k}}$$

$$= \prod_{j=0}^{k-1} (1 + R_{t-j})$$
Annualized Returns and Compound Returns

An annualized return over several years is an “average” return for those years. It is a geometric mean of the returns of the individual years. It provides a basis of comparison.

The length of the period in which a return is realized, or compounded, is important.

A **continuously compounded return** can be computed by letting $k$ increase without bound in an expression of the form

$$\left(1 + \frac{r}{k}\right)^k.$$  

We have

$$\lim_{k \to \infty} \left(1 + \frac{r}{k}\right)^k = e^r.$$  

(can you prove this?)
Log Returns and Excess Returns

Continuous compounding of returns, with the limit result on the last slide, leads to the use of the log return:

\[ r_t = \log(1 + R_t) = \log \left( \frac{P_t}{P_{t-1}} \right) = p_t - p_{t-1}, \]

where \( p_t = \log P_t \).

Often in financial analyses, we compare the returns of a given asset or index with the returns of some standard asset or index, often a “riskless” asset, such as a U.S. Treasury product.

The return of the given asset or index minus the return of the standard is called the “excess return”. It can be negative, of course.
Distribution of Returns

While we expect patterns in the prices, maybe the returns are iid.

This is the first simple statistical model.

What is the distribution? (See Section 1.2.2.)

Tests for normality (see example beginning on page 12).

normalTest in fBasics package.
Using R in This Course

For many of the examples in the text, the R session must include the command

library(fBasics)

It is also useful to store scripts and datasets in a directory set up for this course.

setwd("c:/GMU/GMU-Classes/CSI779/CompFinance/14s")
source("financetools.R")
Getting Real Data; Computing Returns and Log Returns

Using the function `get.stock.price` in the file `financetools.R` sourced in the previous statements and the function `basicStats` included in the `fBasics` package loaded in a previous statement, we can get the adjusted closing prices of American Express from January 1, 1999, to December 31, 2013, compute simple returns, log returns, and simple statistics with the following statements:

```r
AXP_c <- get.stock.price(
    "AXP", start.date=c(1,1,1999), stop.date=c(12,31,2013))
n <- length(AXP_c)
n1 <- n-1
AXP_R <- (AXP_c[-1]-AXP_c[1:n1])/AXP_c[1:n1]
AXP_r <- log(1+AXP_R)
AXP_r_percent <- AXP_r*100
basicStats(AXP_r_percent)
```
Log Returns (in Percentages) of the Amex Data

The output of the `basicStats` function on the previous slide gives the sample mean, standard deviation, skewness, excess kurtosis (sample kurtosis minus 3), minimum, and maximum of the percentage log returns of the adjusted closing prices of American Express (AXP) for the period of 3,772 days ending on December 31, 2013.

For example, the maximum is 18.75. That means on some day in that period, AXP was up 18.75% over the previous day.
Log Returns (in Percentages) of the Amex Data

We now do a simple two-sided t test of the null hypothesis that
the mean of the data-generating process that yielded the ob-
served percentage log returns is 0. We use the regular t.test
function in R.

t.test(AXP_r_percent)

The p-value is 0.39, so we do not reject. Alternatively, we reach
the same conclusion by observing that the 95% confidence in-
terval includes 0.
Log Returns of the Amex Data

There are, of course, many other ways we can look at the log returns.

Are they iid?

Are they normally distributed?

etc., etc., etc.

We can explore these questions using both formal statistical tests and graphical methods.

The following two graphs are instructive.
Log Returns of the Amex Data

![Graph of log returns for the Amex data. The x-axis represents the index, ranging from 0 to 3000, and the y-axis represents the log returns, ranging from -0.2 to 0.2. The graph shows a series of fluctuations around the zero line, indicating the volatility of the log returns over time.]
q-q Plot of Log Returns of the Amex Data Against a Normal Distribution

Normal Q–Q Plot
Log Returns of the Amex Data

What do these graphs suggest?