Numerical results obtained by using Nonlinear Rescaling method on COPS set.

Abstract

In the following we present the numerical results obtained with Nonlinear Rescaling Solver. The NR solver produces generaly speaking infeasible primal and interior point dual sequence. Therefore along with the duality gap we present maximum constraint violation. These two parameters measure the accuracy of the solution. We also present the total number of Newton steps. The numerical results obtained corroborrate the theory of the “hot start” phenomenon. To illustrate this we present the actual performance of the NR multipliers method for some largest problems from the COPS set. The data for COPS set as well as the notations in the tables below one can find in [1].

The NR solver was developed in MATLAB and the COPS set was tested by Ph.D. student Igor Griva.

References

[1] A.S. Bondarenko, D.M. Bortz, J.J. More, *COPS: Large-scale nonlinearly constrained optimization problems,* Mathematics and Computer Science Division, Argonne National Laboratory, Technical Report ANL/MCS-TM-237, October 1999.

Table 1: Largest Small Polygon

	$N_v = 6$	$N_v = 10$	$N_v = 20$	$N_v = 50$	$N_v = 100$
NR solver	Yes	Yes	Yes	Yes	Yes
$f =$	0.6749814429	0.7491373459	0.7768587560	0.7838692737	0.7850565752
constr violation	2.6E-14	8.6E-14	1.0E-14	3.8E-15	4.4E-16
prl-dual gap	4.1E-17	1.3E-16	1.92E-18	5.9E-17	9.4E-18
iterations	36	62	89	519	2109

Table 2: Electrons on a Sphere

	$N_p = 50$	$N_p = 75$	$N_p = 100$	$N_p = 150$	$N_p = 200$
NR solver	Yes	Yes	Yes	Yes	Yes
$f =$	1055.1823147	2454.1369689	4448.3506343	10236.3174669	18438.9368269
constr violation	1.1E-12	1.6E-13	3.9E-12	8.2E-13	5.3E-15
prl-dual gap	4.7E-10	1.2E-10	2.1E-10	2.1E-09	3.3E-11
iterations	54	72	79	272	174

Table 3: Saw Path Tracking

	$d = 2$	$d = 3$	$d = 4$	$d = 5$	$d = 6$
NR solver	Yes	Yes	Yes	Yes	Yes
$f =$	1152.7069161	401.4899555	181.5729928	151.2582875	64.7236938
constr violation	0E+0	6.4E-13	0E+0	1.1E-14	6.2E-13
prl-dual gap	0E+0	8.5E-11	0E+0	9.4E-13	3.8E-13
iterations	1	45	66	81	58

Table 4: Hanging Chain

	$n = 50$	$n = 99$	$n = 100$	$n = 200$	$n = 400$
NR solver	Yes	Yes	Yes	Yes	Yes
$f =$	5.068577962	5.068505564	5.068505062	5.068486411	5.068481694
constr violation	3.4E-10	2.6E-10	4.7E-10	7.8E-10	2.8E-11
prl-dual gap	3.1E-12	2.4E-12	4.2E-12	6.9E-12	2.5E-13
iterations	17	19	18	16	27

Table 5: Two-dimensional Shape Optimization of a Cam

	$n = 10$	$n = 50$	$n = 100$	$n = 200$	$n = 400$
NR solver	Yes	Yes	Yes	Yes	Yes
$f =$	-43.85994780	-214.7608450	-428.4147122	-855.7000466	-1710.275392
constr violation	7.8E-13	2.9E-13	1.7E-15	8.8E-16	2.8E-15
prl-dual gap	1.2E-11	6.7E-10	3.7E-10	6.8E-11	5.8E-10
iterations	143	416	777	1823	5334

Table 6: Isometrization of Alpha-Pinene

	$n_h = 20$	$n_h = 50$	$n_h = 100$	$n_h = 150$	$n_h = 200$
NR solver	Yes	Yes	Yes	Yes	Yes
$f =$	19.87208041	19.87216637	19.87216695	19.87216693	19.87216693
constr violation	2.8E-14	2.3E-13	1.5E-14	1.4E-14	1.4E-14
prl-dual gap	3.2E-10	2.5E-11	4.7E-10	4.5E-10	1.1E-10
iterations	28	25	22	21	20

Table 7: Marine Population Dynamics

	$n_h = 25$	$n_h = 50$	$n_h = 100$	$n_h = 150$	$n_h = 200$
NR solver	Yes	Yes	Yes	Yes	Yes
$f =$	19746526.83	19746529.71	19746529.72	19746529.72	19746529.72
constr violation	4.8E-12	1.1E-11	7.1E-12	8.4E-12	7.9E-12
prl-dual gap	1.6E-10	9.8E-10	3.7E-10	3.3E-10	4.3E-10
iterations	47	59	81	86	66

Table 8: Flow in a Channel

	$n_h = 40$				$n_h = 100$			
	$R = 10$	$R = 100$	$R = 1000$	$R = 10000$	$R = 10$	$R = 100$	$R = 1000$	$R = 10000$
NR solver	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
iterations	4	5	9	12	4	5	9	12
constr viol	4.4E-14	5.8E-13	2.4E-11	7.4E-09	4.0E-14	6.1E-13	2.4E-11	1.1E-09
	$n_h = 200$				$n_h = 400$			
	$R = 10$	$R = 100$	$R = 1000$	$R = 10000$	$R = 10$	$R = 100$	$R = 1000$	$R = 10000$
NR solver	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
iterations	4	5	9	12	4	5	9	12
constr viol	4.5E-14	7.2E-13	1.9E-11	2.1E-10	4.7E-14	7.3E-13	1.8E-11	6.1E-10

Table 9: Non-inertial Robot Arm

	$N = 10$	$N = 50$	$N = 100$	$N = 500$
NR solver	Yes	Yes	Yes	No
$f =$	9.27862977005193	9.14574928666614	9.14199465616801	
constr violation	2.1E-14	4.4E-16	8.8E-16	
prl-dual gap	4.6E-13	9.0E-16	6.2E-14	
iterations	142	340	530	time limit

Table 10: Linear Tangent Steering

	$N = 10$	$N = 50$	$N = 100$	$N = 500$
NR solver	Yes	Yes	Yes	Yes
$f =$	0.55757516557550	0.55467279300435	0.55459589851520	0.55457186503115
constr violation	7.1E-15	7.1E-15	3.2E-13	7.1E-15
prl-dual gap	5.8E-13	3.8E-16	9.9E-13	1.4E-14
iterations	68	86	62	106

Table 11: Goddard Rocket

	$N = 10$	$N = 50$	$N = 100$	$N = 500$
NR solver	Yes	No	No	No
$f =$	1062.02845512713			
constr violation	9.0E-13			
prl-dual gap	1.5E-13			
iterations	12110	no conv.	no conv.	no conv

Table 12: Hang Glider

	$N = 10$	$N = 50$	$N = 100$	$N = 500$
NR solver	Yes	Yes	Yes	No
$f =$	1889.25539641168	1285.53192802305	1255.22397059496	
constr violation	1.7E-13	1.7E-13	1.8E-13	
prl-dual gap	8.1E-13	9.6E-13	3.4E-12	
iterations	676	912	1624	time limit

Table 13: Electrons on a Sphere: $N_p = 200$, $n = 600$, $m = 200$

it	gap	constr violat	# of steps
1	1.236553e+00	9.033152e-03	76
2	1.404138e-01	4.842703e-03	75
3	1.455283e-03	2.929818e-07	19
4	8.134942e-06	1.108623e-09	1
5	4.380585e-08	8.178969e-11	1
6	6.347578e-09	1.412204e-12	1
7	3.360881e-11	5.329071e-15	1
Total number of Newton steps			174

Table 14: Saw Path Tracking: $d = 6$, $n = 5$, $m = 196$

it	gap	constr violat	# of steps
1	9.712984e+00	3.026421e-02	18
2	1.906631e-01	1.411460e-02	4
3	2.223766e-03	1.477319e-03	5
4	6.291079e-05	1.080749e-04	7
5	2.419515e-06	2.740194e-06	6
6	1.132301e-07	1.233226e-07	5
7	5.407719e-09	5.832872e-09	3
8	2.582828e-10	2.775808e-10	5
9	1.256300e-11	1.321654e-11	3
10	3.897457e-13	6.290524e-13	2
Total number of Newton steps			58

Table 15: Hanging chain: $n = 400$, $m = 1$

it	gap	constr violat	# of steps
1	2.103840e-05	1.017294e-03	15
2	2.422877e-06	2.115781e-04	4
3	9.679513e-07	7.927070e-05	3
4	2.681858e-08	3.008994e-06	2
5	5.726567e-10	6.425573e-08	1
6	1.197889e-11	1.344103e-09	1
7	2.507625e-13	2.813705e-11	1
Total number of Newton steps			27

Table 16: Izometrization of Alpha-Pinene: $n_p = 200$, $n = 5005$, $m = 5000$

it	gap	constr violat	# of steps
1	3.126107e+00	5.552791e-05	9
2	2.322369e-01	2.539269e-06	6
3	2.508815e-03	3.664246e-08	2
4	6.540839e-06	1.458207e-10	1
5	2.238168e-08	2.559612e-12	1
6	1.157554e-10	1.421085e-14	1
Total number of Newton steps			20

Table 17: Marine Population Dynamics: $n_p = 200$, $n_s = 8$, $n = 8015$, $m = 7392$

it	gap	constr violat	# of steps
1	2.558496e+00	5.505259e-05	61
2	1.936986e-01	4.640585e-06	1
3	1.096573e-02	2.677526e-07	1
4	4.150386e-04	1.048973e-08	1
5	6.354446e-07	1.595930e-11	1
6	4.386419e-10	7.979228e-12	1
Total number of Newton steps			66

Table 18: Linear Tangent Steering: $N = 500$, $n = 2494$, $m = 3997$

it	gap	constr violat	# of steps
1	8.982790e-02	1.116534e-03	84
2	1.815907e-03	2.948428e-05	5
3	9.836249e-04	1.153747e-07	2
4	8.112619e-05	4.774595e-09	1
5	6.697107e-06	3.961302e-10	1
6	5.528534e-07	3.255440e-11	1
7	4.563882e-08	2.688516e-12	1
8	3.767549e-09	2.229328e-13	1
9	3.110165e-10	1.953993e-14	1
10	2.567465e-11	7.105427e-15	1
11	2.119159e-12	5.949371e-15	1
12	1.751849e-13	5.404432e-15	1
13	1.445219e-14	5.130567e-15	1
Total number of Newton steps			106