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Drift–diffusion in mangled worlds quantum mechanics

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In Everett's many-worlds interpretation, where quantum measurements are seen as decoherence events, inexact decoherence may let large worlds mangle the memories of observers in small worlds, creating a cutoff in observable world measure. I solve a growth–drift–diffusion–absorption model of such a mangled worlds scenario, and show that it reproduces the Born probability rule closely, though not exactly. Thus, inexact decoherence may allow the Born rule to be derived in a many-worlds approach via world counting, using a finite number of worlds and no new fundamental physics.

Keywords: many worlds; decoherence; selection; probability

1. Introduction

Traditional quantum mechanics suffers from many ambiguities regarding quantum measurements. Many-worlds approaches try to resolve these ambiguities by seeing most measurements as decoherence processes produced by standard linear quantum evolution (Everett 1957; DeWitt & Graham 1973). In such processes, local off-diagonal density matrix elements are often naturally and dramatically suppressed due to coupling with a large environment (Dowker & Halliwell 1992).

Unfortunately, the many-worlds approach still suffers from the problem that, when there are a finite number of worlds, the straightforward way to calculate probabilities, i.e. counting the fraction of worlds with a given outcome, does not produce the standard Born probability rule (Kent 1990; Auletta 2000). Some have tried to resolve this by adding new fundamental physics, such as nonlinear dynamics in Weissman (1999), or an infinity of minds per quantum state which diverge via an unknown process in Albert & Loewer (1988). Deutsch (1999) and Wallace (2005) propose new decision theory axioms, saying in essence that we do not care about the number of worlds that see an outcome.

This author has proposed a 'mangled worlds' variation on the many-worlds interpretation (Hanson 2003). This variation attempts to resolve the Born rule problem using only assumptions about the behaviour of standard linear quantum evolution, assumptions that can, in principle, be checked by theoretical analysis of common quantum systems. The basic idea is that decoherence is never exact, and so while decoherence makes off-diagonal terms small relative to a large

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enough world, such terms can be large relative to a small enough world. This may allow larger worlds to mangle much smaller worlds, either by destroying the observers in such worlds or by changing them into observers who remember events from large worlds.[1] This world-mangling process is conjectured to be relatively sudden and thermodynamically irreversible (though mechanically reversible).

While the basic concept has been outlined, many open questions remain to be resolved before this proposal can be considered a serious alternative. These questions include:

(i) Does world-mangling work as conjectured in realistic physical models?
(ii) Are real decoherence rates near the minimum possible, as theory predicts?
(iii) How closely is the Born rule satisfied if mangling goes as conjectured?

This paper addresses only this last question; the other questions remain for now unresolved. In this paper we ask: assuming that the world-mangling process is sudden and practically irreversible, how good an approximation is the Born rule?

This paper addresses this question by finding closed-form solutions of an explicit growth–drift–diffusion–absorption model of world splitting and mangling. This model assumes that there is a range of world sizes (i.e. norms or measures), where worlds quickly become suddenly and irreversibly mangled, that this range is small relative to other relevant ranges, and that this range is located near the world size where half of all measure is in larger worlds.

Given these assumptions, closed-form expressions are given showing exactly how closely the Born rule is followed. Given plausible parameter values, the Born rule would be a very good approximation. Thus, most unmangled worlds would remember having observed frequencies very near that predicted by the Born rule, even though, in fact, Born frequencies do not apply to the vast majority of worlds, and even though such frequencies are not observed in the very largest worlds.

2. Drift–diffusion of all worlds

Within a many-worlds framework, let us start with a single 'world' of unit magnitude (or norm). That is, let us start with a simple quantum state, describing a system interacting with its environment. Next, let this state undergo a 'decoherence event', a finite duration during which this quantum state naturally evolves into a state where the system part is described by a nearly diagonal density matrix. In the many-worlds framework, this new state is considered to be a set of largely autonomous worlds, one for each diagonal system element. Finally, let all these worlds continue to undergo more decoherence events, producing many more worlds describing many more largely autonomous 'worlds'.

More formally, during each decoherence event e, each pre-existing world i splits into a set $J(i)$ of resulting child worlds j, each of which gets some fraction

[1] Saunders (1993) makes the related suggestion that 'In evolutionary terms, any computational capability, if it is to have survival value, will ignore scenarios of small Hilbert-space norm.'

F_{ji} of the original world's measure or size. That is, if m_i is the measure (or size) of world i, then $m_j = f_{ji} m_i$, where $\sum_{j \in J(i)} F_{ji} = 1$. If $P(i)$ is the parent of world i, $A(i)$ are all its ancestors and r is the root world, then by recursion $m_i = \prod_{j \in A(i) - r} F_{jP(j)}$.

Since we could also write this as the sum $\ln(m_i) = \sum_{j \in A(i) - r} \ln(F_{jP(j)})$, the important parameters for our purposes are the variances of the sets $\{\ln(F_{ji})\}_{j \in J(i)}$. These variances determine how quickly world sizes become different. If we made the very strong assumption that these variances were completely independent of the particular world being split, so that $F_{ji} = F_j$ and $J(i) = J$, then there would only be one relevant parameter: the variance of $\{\ln(F_j)\}_{j \in J}$. Furthermore, after there had been enough decoherence events, the central limit theorem of statistics would assure us that the resulting set of worlds will approach a normal distribution over $\ln(m)$, which is a log-normal distribution over measure m.

While this full independence assumption is very strong, there are many weaker versions of the central limit theorem, versions that require much less than complete independence of the variances of $\{\ln(F_{ji})\}_{j \in J(i)}$. And many real systems satisfying far less than complete independence are observed to display nearly normal distributions. For the purposes of this paper, let us assume enough independence in the local variances to get a nearly log-normal distribution over measure m.

Having done this, we can model the distribution of worlds in terms of log size $x = \ln(m)$ as normal, with some mean $\tilde{x} < 0$ and standard deviation $\sigma > 0$. Since the total measure of all worlds is conserved, the total number of worlds must be $e^{-\tilde{x} - \sigma^2/2} \geq 1$. If there were a constant rate of decoherence events, so that $\tilde{x} = -vt$ and $\sigma^2 = wt$, then $v \geq w/2$, and the distribution of worlds over log size x would be

$$\mu_0(x, t) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\left(v - \frac{w}{2}\right)t - \frac{(x - vt)^2}{2\sigma^2}\right). \tag{2.1}$$

The measure held by these worlds would also be normally distributed over x, with the same standard deviation σ, but with a higher mean of $\hat{x} = \tilde{x} + \sigma^2$. These values $\tilde{x}$ and $\hat{x}$ also indicate the median-sized world and the median measure, respectively.

For example, after N binary decoherence events where the two possible outcomes have relative measure $p > 1/2$ and $1 - p$, there would be 2^N worlds, with median measure and standard deviation given by

$$\hat{x} = N\hat{x}_1 = \frac{N}{2}(\ln(p) + \ln(1 - p)), \tag{2.2}$$

$$\sigma = \sqrt{N}\sigma_1 = \frac{\sqrt{N}}{2}(\ln(p) - \ln(1 - p)), \tag{2.3}$$

with $\tilde{x}_1 = \hat{x}_1 - \sigma_1^2$. A constant event rate r would give $v = -r\tilde{x}_1$ and $w = r\sigma_1^2$.

To suggest some plausible parameter values, consider that the Earth has about 10^{50} atoms, and that since its formation the Earth has emitted about 10^{52} infrared photons, most of which have been absorbed by matter elsewhere in the universe. If this corresponds to $N = 10^{52}$ decoherence events, with each event having only $e^{\sigma_1^2} \approx e^{\hat{x}_1} \approx 10$, then there would be about $10^{10^{52}}$ worlds describing alternative states of the Earth's 10^{50} atoms. For a random pair of such worlds, one world would be about $10^{10^{26}}$ times larger than the other. And most of the measure would be in worlds that are about $10^{10^{52}}$ times larger than a typical world.

The distribution $\mu_0(x, t)$ of equation (2.1) solves the linear growth–drift–diffusion equation

$$\dot{\mu} = v(\nabla\mu + \mu) + (w/2)(\nabla^2\mu - \mu); \tag{2.4}$$

for $t > 0$, given the Dirac delta-function initial condition putting all mass at $x = 0$,

$$\mu(x, 0) = \delta(x). \tag{2.5}$$

3. The mangling of worlds

Consider a total physical system that is decomposed into a local system and an environment, and consider the density matrix describing the system only. 'Decoherence' is the name given to the phenomenon, whereby, in the dynamically relevant basis, interactions between that system and its environment often quickly suppress the magnitude of the system's off-diagonal density matrix terms, relative to its diagonal terms.

Specifically, given two worlds, large 'L' and small 's', corresponding to two diagonal elements of a local system density matrix ρ, after the decoherence process has started, we have

$$|\rho_{\mathrm{Ls}}|^2 \le \epsilon^2(t)|\rho_{\mathrm{LL}}||\rho_{\mathrm{ss}}|, \tag{3.1}$$

where $|\rho_{ab}| \equiv \langle a|\rho_{ab}|b\rangle$, with the coherence parameter $\epsilon(t)$ typically falling at a rapid exponential rate for many doubling times (Dowker & Halliwell 1992). However, in the models that have been solved so far, coherence $\epsilon(t)$ typically eventually asymptotes to a small but non-zero level (Unruh & Zurek 1989; Dowker & Halliwell 1992, 1994; Namiki *et al.* 1997).

As we have seen above, the relative magnitude between any two random worlds increases as $e^{\sigma} = e^{\sqrt{wt}}$, which can reach extremely large magnitudes in plausible scenarios. As discussed elsewhere (Hanson 2003), if $\epsilon(t)$ eventually falls slower than this rate, then eventually the off-diagonal terms ρ_{sL} and ρ_{Ls} may greatly influence the evolution of the smaller diagonal term ρ_{ss}, even though these off-diagonal terms have little influence on and are mainly driven by the larger diagonal term ρ_{LL}. Thus, the evolution of smaller worlds may eventually be driven by larger worlds, plausibly 'mangling' those smaller worlds, i.e. either destroying their observers or turning them into observers who remember outcomes from a large world. Remaining observers would thus only remember the histories of unmangled worlds.

For the purposes of this paper, let us conjecture that large worlds do in fact mangle small enough worlds, and that world mangling is a sudden, global and thermodynamically irreversible process. That is, let us assume that mangling effects on a world go from being detectable to being overwhelming in a relatively short time period, that the rate at which worlds become mangled as a function of their size is similar across the configuration space of worlds, and that while mangling is mechanically reversible, the configurations that produce such reversals are thermodynamically unlikely. These assumptions have *not* yet been established as correct, and this paper will not so establish them. Instead, this paper will simply explore some consequences of these assumptions.

Specifically, let us assume that there is a mangling region in world size, so that a world that has always remained larger than this region remains unmangled, and that any world that becomes smaller than this region becomes suddenly and forevermore mangled. Let us also assume that there have been many decoherence events, and so the mangling region is narrow relative to the standard deviation in log world sizes σ. Finally, let us assume that since it is the measure of some worlds that mangles other worlds, this mangling region remains close to the median measure $\hat{x}$ that would describe the distribution of all worlds in the absence of mangling.

If unmangled worlds evolve locally just as all worlds would in the absence of mangling, and if initially all worlds are unmangled, then the distribution $\mu_1(x, t)$ of unmangled worlds should satisfy equations (2.4) and (2.5), just as the distribution $\mu_0(x, t)$ of all worlds does under the no mangling assumption. To model our assumption of a mangling region narrow compared to σ and remaining close to the median measure $\hat{x}(t) = (w - v)t$, let us impose on the unmangled world distribution $\mu_1(x, t)$ the additional boundary condition

$$\mu(x_b(t), t) = 0, \tag{3.2}$$

for all $t \geq 0$, where $x_b(t) = \hat{x}(t) - \epsilon$, for $\epsilon > 0$. This is an absorbing boundary condition, which says that every world which reaches the point x_b from above immediately falls out of the distribution of unmangled worlds. We will naturally limit our attention to $x \geq x_b(t)$.

4. Solving the drift–diffusion model

We want to solve the set of equations (2.4), (2.5) and (3.2). To achieve this, let us transform from x to a coordinate $y = x - x_b(t)$ that moves along with the absorbing boundary. Let us also factor out the common exponential growth via $\mu(x, t) = \nu(x, t)\mathrm{e}^{(v-(w/2))t}$. Equations (2.4), (2.5) and (3.2) then become

$$\dot{\nu}(y, t) = w\nabla\nu(y, t) + (w/2)\nabla^2\nu(y, t), \tag{4.1}$$

$$\nu(y, 0) = f_0(y) = \delta(y - \epsilon), \tag{4.2}$$

$$\nu(0, t) = 0. \tag{4.3}$$

Equation (4.2) gives two initial conditions, one general and one specific to our problem. Fortunately, Farkas & Fulop (2001) have already solved a closely related set of equations, regarding drift–diffusion between two absorbing barriers. Their solutions can be transformed into general solutions of equations (4.1)–(4.3):

$$\nu(y, t) = \frac{4}{\pi}\mathrm{e}^{-wt/4-y}\int_0^\infty g_0(k)\mathrm{e}^{-k^2 wt}\sin(2ky)\mathrm{d}k, \tag{4.4}$$

$$g_0(k) = \int_0^\infty f_0(y)\mathrm{e}^{y}\sin(2ky)\mathrm{d}y. \tag{4.5}$$

If we put back in the exponential growth, we get solutions to our original equations of interest, i.e. equations (2.4), (2.5) and (3.2), except in terms of y instead of x. We find that an initial distribution $\mu_1(y,0)=\delta(y-\epsilon)$ of unmangled worlds evolves into

$$\mu_1(y,t;\epsilon) = \sqrt{\frac{\pi}{8wt}}\mathrm{e}^{\epsilon-y+(v-w)t}\left(\exp\left(-\frac{(y-\epsilon)^2}{2wt}\right)-\exp\left(-\frac{(y+\epsilon)^2}{2wt}\right)\right). \qquad (4.6)$$

For $wt \gg \epsilon^2$, a good approximation to this is

$$\mu_1(y,t;\epsilon) = \frac{\epsilon \mathrm{e}^{\epsilon}}{\sqrt{2\pi}}\frac{\mathrm{e}^{(v-w)t}}{(wt)^{3/2}}\, y \exp\left(-y-\frac{y^2}{2wt}\right). \qquad (4.7)$$

We use this approximation from here on.

This density integrates to give a total unmangled world count

$$W(t;\epsilon) \equiv \int_0^\infty \mu_1(y,t;\epsilon)\mathrm{d}y = \frac{\epsilon \mathrm{e}^{\epsilon}}{2}\mathrm{e}^{(v-w)t}\left[\sqrt{\frac{2}{\pi wt}}-\mathrm{e}^{wt/2}\mathrm{erfc}\left(\sqrt{\frac{wt}{2}}\right)\right]. \qquad (4.8)$$

Note that since density $\mu_1(y,t;\epsilon)$ is proportional to $\mathrm{e}^{-y} \propto m^{-1}$, the vast majority of unmangled worlds have sizes m within a few orders of magnitude of the mangling boundary. However, since count $W(t;\epsilon)$ is proportional to e^{ϵ}, the larger an initial world is, the more descendants it will produce in the long run. Thus, the few largest worlds have a disproportionate influence on the final distribution of worlds.

Note also that if $v > w$, then the total number of unmangled worlds, which grows as $\mathrm{e}(v-w)t$, will increase with time, even though it becomes an exponentially decreasing fraction of the number of all worlds, which grows as $\mathrm{e}^{(v-(w/2))/t}$. To predict our existence in an unmangled world, the mangled worlds approach must predict that $v \geq w$.

5. Born rule accuracy

We have found distributions of world sizes within a mangled worlds framework. Let us now evaluate how well this drift, diffusion, growth and mangling process does at reproducing the Born rule. To do this, we need to express the Born rule in terms of world distributions.

The Born rule says that the probability of observing a particular experimental outcome is proportional to the size (or measure) of that outcome. In a simple many-worlds description of this situation, there would be a single parent world where the experiment started, and each experimental outcome would then correspond to one or more child worlds in which that outcome was recorded. In this context, the Born rule says that the probability that you will find yourself in a world which is descended from a given experimental outcome is proportional to the total size (or measure) of the worlds that see that outcome.

To connect world distributions and probabilities, let us assume:

> *Equal probability assumption.* Consider a person residing in a particular world, and wondering what descendant world he will find himself in at

a particular future date. Such a person should assign an *equal probability* to finding himself in any of the unmangled descendant worlds at that date, and a zero probability to finding himself in a mangled world.

Assumptions about worlds being equally probable seem natural and have a long tradition within the many-worlds approach (e.g. Graham 1973), though they do not appear to be logically necessary.[2] The particular variation used here embodies an observation selection effect, namely that only people in unmangled worlds can use their experimental records to test the Born rule.

This equal probability assumption can be consistent with the Born rule if in the long run the number of unmangled worlds corresponding to each measurement outcome is proportional to the size (or measure) of that outcome. That is, if a child world that is a factor F smaller than its parent world produces a number of descendant unmangled worlds $\lambda(F)$ proportional to F, then a person who later finds himself in a random unmangled world would have a chance proportional to F of being in a world that descended from that particular child world. Thus, the probability of a measurement outcome would be proportional to the measure associated with that outcome.

Unfortunately, this $\lambda(F) \propto F$ condition need not hold for all or even most initial worlds. Consider an example of radioactive decay. Within a negligible time period, let an initial world split into one non-decay child world, holding 99% of the initial measure and millions of small decay child worlds, differentiated by the exact time and orientation of the decay. The distributions derived above, such as $\mu_1(y, t; \epsilon)$, typically assign to the vast majority of unmangled worlds sizes between 1% larger and one million times larger than the mangling boundary. Yet if this mangling boundary is sharp, then for every initial world in this size range, *all* of its decay child worlds are mangled, while its single non-decay child world remains unmangled. Thus, for any such initial world, the equal probability assumption requires that a person in such a world assign a *zero* probability to the possibility of radioactive decay. But the Born rule, and well-established observation, suggest that one assign a 1% probability to such decay.

This decay analysis, however, has ignored the disproportionate influence of the few largest worlds. Since we do not know how large a world we are in at the moment, we must integrate over all possible current world sizes when we calculate the future number of unmangled worlds corresponding to each measurement outcome. So that is what we will do in the remainder of this section. Our careful calculation will go through the following steps.

(i) Start with single unmangled world with size $y=\varepsilon$.
(ii) Let this world's descendants evolve (i.e. split and mangle) for a duration t_1.
(iii) Let each resulting world have a child world of relative size F.
(iv) Track the descendants of these children as they evolve for duration t_2.
(v) Count the total descendant unmangled worlds $\lambda(F)$, checking that $\lambda(F) \propto F$.

We will assume that $wt_2 \gg wt_1 \gg \epsilon^2 > 1 > F$.

[2] Some claim that the concept of a world is too ambiguous to allow useful equal calculations (Wallace 2005). Ambiguities in defining the number of worlds containing some outcome seem no worse to me than the ambiguities in defining the number of physical states containing an outcome, i.e. the entropy of that outcome.

Specifically, starting at time $t=0$, let a single unmangled world of size $y=\epsilon>1$ (relative to the mangling region at $y=0$) evolve into a distribution $\mu_1(y;\, t_1;\, \epsilon)$ at time t_1, where $wt_2 \gg \epsilon^2$. This is not intended to be the distribution of all worlds, but rather the distribution of all worlds consistent with the initial conditions of a given experiment to test the Born rule. It is the result of decoherence events both during and before the experiment, events that are not counted in the statistics of the experiment. Given what we know about the experiment, we do not know which of these worlds we are in, and so we must average over these worlds when making experimental predictions.

At time t_1, let each world with value y_1 in $\mu_1(y,\, t_1;\, \epsilon)$ be split into many worlds, one of which is a factor $F<1$ smaller, so that it has the value $y=y_1+\ln(F)$. Let each of these factor F worlds then evolve to produce more worlds over a longer time period $t_2 \gg t_1$. For the Born rule to apply exactly, the number of unmangled worlds descended from factor F worlds at time t_2 should go as F. (It would not contradict observations if the Born rule were violated soon after t_1; the key Born rule data that we want to explain are long-existing historical records of experiments testing the Born rule.)

With help from Mathematica, the final unmangled world count is found to be

$$\lambda(F; t_1, t_2, \epsilon) \equiv \int_0^\infty W(t_2; y)\mu_1(y - \ln(F), t_1; \epsilon)\mathrm{d}y, \tag{5.1}$$

$$\lambda(F; t_1, t_2, \epsilon) = F \,\mathrm{erfc}\left(\frac{-\ln(F)}{\sqrt{2wt_1}}\right)\frac{\epsilon \mathrm{e}^{\epsilon}}{4}\mathrm{e}^{(v-w)(t_1+t_2)}\left[\sqrt{\frac{2}{\pi w t_2}} - \mathrm{e}^{wt_2/2}\mathrm{erfc}\left(\sqrt{\frac{wt_2}{2}}\right)\right]. \tag{5.2}$$

The key thing to note here is that when wt_1 is large, the Born rule correction,

$$\gamma(F) \equiv \frac{1}{F}\,\frac{\lambda(F; t_1, t_2, \epsilon)}{\lambda(1; t_1, t_2, \epsilon)} = \mathrm{erfc}\left(\frac{-\ln(F)}{\sqrt{2wt_1}}\right), \tag{5.3}$$

changes *very* slowly in the factor F. For example, when $wt_1 = 10^{10}$, it requires a factor of $F = \mathrm{e}^{-10^5} \approx 10^{-43\,000}$ to get the relative number of worlds to be $\gamma(F) \approx 1/3$. The derivative

$$\frac{\partial}{\partial \ln(F)}\left(\frac{\lambda(F)}{\lambda(1)}\right) = -\sqrt{\frac{2}{\pi w t_1}}\exp\left(\frac{-\ln(F)^2}{2wt_1}\right), \tag{5.4}$$

tells a similar story. Thus, this approach is very nearly consistent with the Born rule, while leaving open the possibility of small experimentally detectable deviations from the Born rule.

6. Conclusion

This paper has explored the accuracy of the Born rule under a mangled worlds scenario. In this scenario, inexact decoherence results in larger worlds suddenly and irreversibly mangling any worlds that reach a narrow region in world size. While these assumptions have not yet been established as correct, we have explored their consequences by creating and solving an explicit growth–drift–diffusion–absorption model. Closed-form expressions are given showing that this model reproduces the Born rule closely, but not exactly. This seems to resolve one of the open questions with the mangled worlds attempt to reconcile the many-worlds approach with the Born rule without invoking new fundamental physics or decision theory axioms.

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References

Albert, D. & Loewer, B. 1988 Interpreting the many worlds interpretation. *Synthese* **77**, 195–213. (doi:10.1007/BF00869434)

Auletta, G. 2000 *Foundations and interpretation of quantum mechanics.* Singapore: World Scientific.

Deutsch, D. 1999 Quantum theory of probability and decisions. *Proc. R. Soc. A* **455**, 3129–3197. (doi:10.1098/rspa.1999.0443)

DeWitt, B. & Graham, N. 1973 *The many-worlds interpretation of quantum mechanics.* Princeton, NJ: Princeton University Press.

Dowker, H. & Halliwell, J. 1992 Quantum mechanics of history: the decoherence functional in quantum mechanics. *Phys. Rev. D* **46**, 1580–1609. (doi:10.1103/PhysRevD.46.1580)

Dowker, H. & Halliwell, J. 1994 The decoherence functional in the Caldeira–Leggett model. In *Physical origins of time asymmetry* (ed. J. Halliwell, J. Perez-Mercader & W. Zurek), pp. 234–245. Cambridge: Cambridge University Press.

Everett, H. 1957 'Relative state' formulation of quantum mechanics. *Rev. Mod. Phys.* **29**, 454–462. (doi:10.1103/RevModPhys.29.454)

Farkas, Z. & Fulop, T. 2001 One dimensional drift–diffusion between two absorbing boundaries: application to granular segregation. *J. Phys. A* **34**, 3191–3198. (doi:10.1088/0305-4470/34/15/301)

Graham, N. 1973 The measurement of relative frequency. In *The many-worlds interpretation of quantum mechanics* (ed. B. DeWitt & N. Graham), pp. 229–253. Princeton, NJ: Princeton University Press.

Hanson, R. 2003 When worlds collide: quantum probability from observer selection? *Found. Phys.* **33**, 1129–1150. (doi:10.1023/A:1025642019178)

Kent, A. 1990 Against many-worlds interpretations. *Int. J. Mod. Phys. A* **5**, 1745–1762. (doi:10.1142/S0217751X90000805)

Namiki, M., Pascazio, S. & Nakazato, H. 1997 *Decoherence and quantum measurements.* Singapore: World Scientific.

Saunders, S. 1993 Decoherence, relative states, and evolutionary adaptation. *Found. Phys.* **23**, 1553–1585. (doi:10.1007/BF00732365)

Unruh, W. & Zurek, W. 1989 Reduction of a wave packet in quantum Brownian motion. *Phys. Rev. D* **40**, 1071–1094. (doi:10.1103/PhysRevD.40.1071)

Wallace, D. 2005 Quantum probability from subjective likelihood: improving on Deutsch's proof of the probability rule. Available at http://arxiv.org/abs/quant-ph/0312157.

Weissman, M. 1999 Emergent measure-dependent probabilities from modified quantum dynamics without state-vector reduction. *Found. Phys.* **12**, 407–426. (doi:10.1023/A:1021625209799)