Long-Term Growth As A Sequence of Exponential Modes

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Abstract

A world product time series covering two million years is well fit by either a sum of four exponentials, or a constant elasticity of substitution (CES) combination of three exponential growth modes: "hunting," "farming," and "industry." The CES parameters suggest that farming substituted for hunting, while industry complemented farming, making the industrial revolution a smoother transition. Each mode grew world product by a factor of a few hundred, and grew a hundred times faster than its predecessor. This weakly suggests that within the next century a new mode might appear with a doubling time measured in days, not years.

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Introduction

Many have argued that current U.S. stock prices are much higher than can be justified by reasonable profit forecasts based on empirical models derived from the recorded history of the U.S. economy (Shiller, 2000). While it might be theoretically possible for the economy to suddenly switch to a fundamentally new mode with much higher corporate profits, our experience with the U.S. economy offers little evidence that a mode switch of this magnitude has happened anytime in the last century or so (Barro & Sala-i-Martin, 1995), though we may be able to understand smaller changes in stock prices in terms of milder fluctuations in profit trends (Barsky & De Long, 1993).

The arguments against high current U.S. stock prices are thus largely empirical, in contrast with the theoretical nature of the (typically informal) arguments offered in support of investor optimism. These empirical arguments, however, are based primarily on the last century or so of economic experience. Could empirical forecasts based on a longer term history of economic growth offer more support for the rationality of high stock prices? To address this question, this paper will take an empirical approach to modeling very long term data on economic growth, with an eye toward inferring the chances for very large changes in economic growth modes.

In order to empirically model very long term economic history, we need one or more very long term time series, and we need to choose one or more functional forms to use in modeling those time series.

Most attempts to formally model long term human history have focused on population time series, as population estimates have been offered going back many thousands of years, and such time series have been considered to be the most reliable long term series available. Also, for most of human history population seems to have reasonably reflected most fundamental changes, including areas colonized, capital accumulated, and technologies adopted. This seems to be because new productive abilities have mainly resulted in larger populations under Malthusian conditions of near-subsistence per capita consumption.

Nearly constant per capita consumption, however, implies that reasonable estimates of total consumption can be obtained from reasonable estimates of population. We thus have reasonable estimates of world product up until about 1500, when per capita product began to rise. Economic historians have also provided us with reasonable estimates for world product over the past century or so (Maddison, 1995). So all that has stood in the way of a full long term world product time series was to fill in the gap from roughly 1500 to 1850.

Recently economic historian Brad De Long has cleverly used a simple empirical Malthusian relation to fill this gap, and has thus created a world product time series ranging from one million B.C. to today (De Long, 1998). His estimates for before 10,000 B.C., however, are based on forty year old speculations (Deevey, 1960). This paper will therefore replace those old speculations with recent published estimates going back to two million B.C. (Hawks, Hunley, Lee, & Wolpoff, 2000).

This paper will then attempt to empirically model that time series with a few simple functional forms. After considering models of gradually accelerating growth, which have been the most popular in formal models of long term population growth, we will turn to attempting to formalize the more common informal descriptions of history as a sequence of growth modes. And since exponential growth is the most common formal description of short term growth, we will consider modeling long term world product history as a sequence of exponential growth modes.

The simplest sequence of exponentials is a sum of exponentials, and we will find that a sum of four exponential modes fits the data with less than a 2% average error, which is less than the average uncertainty in the estimates. One of these modes, however, appears to be there just to allow a more gradual transition between two other modes than a simple sum of exponentials will allow. So we will then try a constant elasticity of substitution (CES) form for combining exponentials, which can allow a transition to be smooth or sudden by varying a single parameter. We will find that a CES combination of three exponential modes also fits the data with less than a 2% average error.

After describing these models, and discussing their interpretation, we will then use them to consider when we might see a transition to yet another mode of economic growth, and how fast growth might then be. It seems quite unreasonable to expect an infinite series of faster and faster growth modes, since that leads to an infinite world product in finite time. But it may be reasonable to wonder if a world that has so far seen at least three distinct growth modes might see yet one more growth mode.

The Data

Economic historians have reasonable estimates for world product over the last century or so, while other historians have reasonable estimates for world population over the last ten thousand years, and cruder estimates for the two million years before that. Until a few centuries ago, per-capita consumption was typically near subsistence levels, making it easy to infer world product from world population. Thus the obstacle to having a full long term world product time series has been to get reasonable estimates for the intermediate period, from roughly 1500 to 1850.

Recently, economic historian Brad De Long found a simple linear relation between world population growth rates and per capita product, a relation which seems valid at the low values of per capita product observed from roughly 1850 to 1950 (De Long, 1998). This linear relation has allowed him to created world product estimates for the missing intermediate period, and thus to create a world product time series ranging from one million B.C. to today.

De Long developed several time series based on alternative assumptions, and this paper uses his preferred world product series. The only change made here is to substitute more recent estimates for population prior to 10,000 B.C. De Long continued the use by Kremer (Kremer, 1993) of population estimates from McEvedy and Jones (McEvedy & Jones, 1978) going back to 10 thousand B.C., and of three population estimates from Deevey (Deevey, 1960) for 25 thousand B.C., 300 thousand B.C. and one million B.C.

Deevey's estimates, however, were never published in a peer-reviewed journal, cite no other sources, and are now forty years old. The data set used here excises Deevey's forty year old estimates and instead substitutes two more recent population estimates. Hawks and coauthors (Hawks et al., 2000) estimate a "population bottleneck" of about ten thousand at about two million B.C., and that we "cannot reject exponential growth from" then until ten thousand B.C. Consistent with that conclusion, they also accept a Weiss (Weiss, 1984) population estimate of about half a million between one million B.C. and 500 thousand B.C., which I've coded as at 750 thousand B.C.[1]

The resulting world product time series has 54 data points, is graphed in Figure 1, and is listed in the appendix. In Figure 1, world product is described in units of an equivalent number of humans at a subsistence consumption level.[2] Time is described by the number of years before the somewhat arbitrary date of 2050. This data is also graphed later in Figure 2, which shows world product growth rates versus levels, displaying each step between data points as a horizontal line. This graph looks noisier, because growth rates are noisier, but it has the virtue of avoiding the arbitrary choice of a reference date like 2050.[3]

Choosing a Functional Form

What functional form should we use to model our world product time series? Simple exponential growth is probably the most popular general model for describing positive quantities that grow in time by many orders of magnitude, as both population and world product have. It is, for example, widely used to model the last half century of economic growth (Barro & Sala-i-Martin, 1995) and

[1]These estimates are based on a "multi-regional" model of human evolution (Relethford, 1998). An alternative "out of Africa" model posits a similar population bottleneck at roughly 200 thousand B.C. (Rogers & Jorde, 1995; Ingman, Kaessmann, Paabo, & Gyllensten, 2000). The multi-regional model is based more heavily on non-genetic fossil evidence. Genetic evidence may well estimate the number of our direct ancestors alive on a given date, but non-genetic fossils seem better estimates of the size of the economy or ecological niche occupied by our direct ancestors and their very close relatives on that date. Further footnotes will, however, mention how the alternative model changes this paper's estimates.

[2]Specifically, "subsistence" is $80 1990 "International" dollars, which is the ratio of De Long's preferred world product and population estimates for one million B.C. This per capita product has been assigned to my substitute dates of two million B.C. and 750 thousand B.C., and retained De Long's estimates for all other dates.

[3]In the analysis which follows, we will occasionally compare world product growth since two million B.C. to the earlier exponential growth of maximum brain size. Maximum animal mass and relative brain size have both grown roughly exponentially since the Cambrian explosion about 550 million years ago, when animals first appeared in large numbers in the fossil record. The mass of the currently largest animal doubled roughly every 70 million years, while maximum brain volume relative to body volume (raised to the 2/3 power) has doubled roughly every 50 million years (Russell, 1983). (More details on the details of brain size history are given by Jerison (Jerison, 1991).) Together these estimates suggest that maximum animal brain size has doubled roughly every 34 million years over the last 550 million years (for a total of about 16 doublings). For comparison, the exponential decay rate of Marine genera over this period was a half-life of about 23 million years (Valen, 1973; Newman & Sibani, 1999).

the last few centuries of growth in scientific literature (de Solla Price, 1963). Simple variations on exponential growth, such as linear trends in growth rates, have been particularly popular in making short term population forecasts (Lee, 1990), but seem clearly inadequate for longer term forecasts. Variations on logistic growth have also long been popular in estimating various population trends (Meyer & Ausubel, 1999), but also seem clearly inadequate for modeling very long term population trends (Cohen, 1995).

Many researchers have had more empirical success explicitly modeling long term population histories with functions that describe accelerating change. The first such efforts used simple hyperbolics (von Foerster, Mora, & Amiot, 1960), and soon became infamous warnings against taking one's model too literally, since they predicted an infinite human population in the early twenty first century.

More recent related efforts (Kremer, 1993; Kapitza, 1996; Johansen & Sornette, 2000) have all included corrections which could describe a recent or upcoming slowdown and other complexities, mostly to allow growth to today be approaching fundamental limits, and hence a final end to accelerating growth.

Some authors have also informally summarized world history as continually accelerating change, but the more common informal summary is of human history as a sequence of specific growth modes. History has, for example, been described as the slow expansion of hunter-gatherers, followed by faster growth following the domestication of plants and animals, followed by even faster growth with commerce, science, and industry (Cipolla, 1967).

Simple exponential growth is a very common model for describing particular historical periods. It is, for example, widely used to model the last half century of world economic growth (Barro & Sala-i-Martin, 1995). Since historical periods tend to be described as exponential growth, formal models of transitions between periods tend to be models of transitions between exponential growth asymptotes. Many have modeled the industrial revolution this way (Marvin Goodfriend, 1995; Hansen & Prescott, 1998; Jones, 1999; Steinmann, Prskawetz, & Feichtinger, 1998; Galor & Weil, 2000; Lucas, 2000). At least one earlier transition, to cities, has also been modeled this way

(Marvin Goodfriend, 1995). And possible current or upcoming transitions, such as due to computers or the internet, have also been modeled this way (Helpman, 1998).

Given all this, it seems natural to formally model the long term history of humanity as a sequence of exponential growth modes. Yet to my knowledge no one has done this. Some authors seem to have come close, by drawing exponential curves suggestively next to data on log-log graphs of population growth (Deevey, 1960; Kates, 1996; Livi-Bacci, 1997). (Some of these authors, however, do not seem to have realized that it could be simple exponentials they were drawing, and instead of "exponential growth followed by a period of approximate stability" (Kates, 1996).)

One author did more explicitly consider and reject a sum of exponentials model for world population history, though without any explicit model fitting or test (Cohen, 1995). (Another author considered a sum of two exponentials, one of which was *decreasing* (Lee, 1988).) In this paper we thus take on the neglected task of more formally modeling long term human history as a sequence of exponential growth modes. In particular, we consider such a model of world product history.

Once we have chosen to model history as a sequence of exponential modes, there still remains the question of how we should mathematically model transitions between exponential growth modes. We will first try a simple sum of exponentials, and then try to replace this sum with a CES (constant elasticity of substitution) form. Such a form allows each transition to be sudden or gradual, depending on a single free parameter per transition. We will end up with a model describing the three classic economic growth modes, which we might call hunting, farming, and industry.

A Sum of Exponentials Model

Apparently, no one has yet tried to formally model long-term growth as a sequence of exponentially growing modes. This paper thus attempts to do so for world product. One very simple model of a sequence of growth modes is a sum of exponentials, which for n modes we will write as $Y(t) = Y_n(t)$, where $Y_0(t) = 0$, and for i from 1 to n,

$$
\begin{aligned}
Y_i(t) &= Y_{i-1}(t) + M_i(t), \\
M_i(t) &= c_i\, 2^{t/\tau_i}.
\end{aligned}
$$

The parameter τ_i is the doubling time of mode i. If data consists of pairs (t_j, Y_j), a min-square-error of logs method searches for model parameters of a function $Y(t)$ that minimize

$$\sum_j (\log(Y(t_j)) - \log(Y_j))^2.$$

Applying this method to my data, the best fit sum of four exponential terms gives an eyeball-pleasing fit, and a (root mean square) average percentage error of 1.8%. This is about the same magnitude as the smallest uncertainty or "indifference range" attributed to the population estimates that this data is based in part on, which is about 2% (McEvedy & Jones, 1978).

Figure 1 compares this model with the data, and indicates that the main errors seem to be due to unmodeled fluctuations with a period of roughly 500-1500 years. Figure 1 also shows the best fit hyperbolic model,[4]

$$Y(t) = c(T - t)^{-\alpha},$$

which has a power of $\alpha = 1.46$, goes to infinity at $T = 2004$, and has a much inferior average percentage error of 9.6%. (Using three terms instead of four in the sum of exponentials model also gives a visibly much worse fit.)

For each term, Table 1 lists a mode description, doubling time[5], the factor by which the doubling

[4]The best fit "log-periodic model," of the form $Y(t) = c(T-t)^{-\alpha}(1+b\cos(\omega\ln(T-t)+\phi))$ (Johansen & Sornette, 2000), has a power of $\alpha = 1.48$, goes to infinity at $T = 2005$, and has an average percentage error of 7.4%.

[5]Under the "out of Africa" theory, the hunting mode began ten times more recently, at 200 thousand B.C. Substituting this estimate for the two multi-regional estimates would reduce the hunting doubling time, increase the hunting doubling time factor, and reduce the farming doubling time factor, all by a factor of ten. All other entries would remain unchanged.

Growth Mode	Doubling Time (years)	Date Began To Dominate	Doubling Time Factor	World Product Factor
Hunting	230,000	2,000,000 B.C.	149?	406
Farming	860	4700 B.C.	267	178
??	58	1730 A.D.	14.9	8.9
Industry	15	1903 A.D.	3.8	> 78

Table 1: Sum of Exponentials Growth Modes

time accelerated between terms, and a "date began to dominate," defined as the d_i that solves

$$M_i(d_i) = M_{i-1}(d_i).$$

(The questionable doubling time factor for the hunting mode comes from comparing it to the previous growth rate of animal brains, as discussed in the footnotes.[6]) Table 1 also gives a world product factor, defined as

$$Y_i(d_{i+1})/Y_{i-1}(d_i),$$

which says how much world product increased due to each term.

CES-Combined Exponentials

While most of the modes in the above model seem to have a familiar interpretation, the second to last mode, the one with a 58 year doubling time, seems more difficult to interpret. We might consider it to represent a commercial revolution, distinct from the later industrial revolution. But visibly, it seems to just be there to allow the model to describe a slower and smoother transition than a simple sum of exponentials will allow between the modes before and after it.

[6]For reference, an additional row of Table 1 describing the earlier brain size growth mode would have entries: 34,000,000, 550,000,000 B.C., ??, and "67,000." The last entry is the total increase of brain size, which isn't directly comparable to the other rows since it is not an increase in world product.

Growth Mode	Doubling Time (year)	Date Began To Dominate	Doubling Time Factor	World Product Factor	Transition CES Power
Hunting	224,000	2,000,000 B.C.	153?	480	?
Farming	909	4860 B.C.	247	190	2.4
Industry	6.3	2020 A.D.	145	> 590	0.094

Table 2: CES-combined Exponentials Growth Modes

To more directly model the possibility of a slow versus fast transitions between modes, we can combine terms not with a sum, but instead with a constant elasticity of substitution (CES) form (Barro & Sala-i-Martin, 1995), as in

$$Y_i(t) = (Y_{i-1}(t)^{a_i} + M_i(t)^{a_i})^{1/a_i}.$$

When the CES power is $a_i = 1$, this is $x + y$, the previous case. When $a_i = 0$, this is proportional to $\sqrt(Y_{i-1}M_i)$, which for exponential terms essentially makes an infinitely slow transition. When a_i is positive infinity, this is $\max(Y_{i-1}, M_i)$, with an infinitely fast transition. Other positive values of a_i allow for other intermediate speeds of transition.

If we now fit such a CES combination of three exponential terms to our data, again using the min square error of logs method, we get an eyeball-pleasing fit, and an average percentage error of 1.7%. This model is described[7] in Table 2, and compared to the data in Figures 1 and 2. This model has the same number of free parameters as the four-term sum of exponentials model, and a modestly smaller total squared error of logs.

Note that the industry mode will not begin to dominate until 2020, has a doubling time which is half of what we have observed over the last half century, and is already responsible for a larger world product factor than either of the previous two modes. The very low transition power allows a new mode to have a strong influence long before it officially dominates, and predicts a mid-transition

[7] An additional row describing the earlier brain size growth mode would have the same entries as before, and the "out of Africa" theory would change the same entries by a factor of ten.

growth rate of about half of the new mode growth rate.

Interpreting the Modes

By describing world product history as either a sum or CES-combination of exponentials, one seems to essentially be saying that among the thousands of large and important changes that the world economy has seen, only a handful are fundamentally big news. Other changes are either "small," at least in the big picture, or such that a change of broadly similar economic magnitude and timing was largely predetermined by the fundamental economic forces at play. Typically, the economy is dominated by one particular mode of economic growth, which produces a roughly constant growth rate.

While there are often economic processes which grow exponentially at a rate much faster than that of the economy as a whole, such processes almost always slow down as they become limited by the size of the total economy. Very rarely, however, a faster process reforms the economy so fundamentally that overall economic growth rate accelerates to track this new process. The economy might then be thought of as composed of an old sector and a new sector, a new sector which continues to grow at its same speed even when it comes to dominate the economy.

Following this line of interpretation, the simple sum of exponentials model seems to assume that during each transition, the "new" and "old" economies co-exist but do not influence each other. In contrast, the CES-combination form seems to describe interactions between modes. Modes might complement each other ($0 < a < 1$), making for a more gradual transition, or substitute for each other ($a > 1$), making for a sharper transition.

The large transition centered around 5000 B.C. (but surely begin thousands of years earlier) is naturally interpreted as the transition to from "hunting" to "farming" (both broadly construed). Populations relying on food from the domestication of plants and animals seem to have grown exponentially for several millennia before this transition date, nicely fitting a simple interpretation of co-existing new and old economies (Diamond, 1997). In the CES model, the power estimate for the farming transition suggests that farming mostly substituted for hunting, rather than complementing it. Because the data is poor, this suggestion can only be weak. But it does make sense, given that hunting and farming economic modes competed for the use of both land and labor.[8]

The recent “industrial revolution” transition has a CES power describing a much more gradual transition, suggesting less substitution and more complementarity between the old and new economic growth modes. If we interpreted the old and new modes relatively literally as “farming” and “industry”, this makes some sense, in that improved farming technology has freed workers for industry while improved industry technology has often transferred to farming technology.

Such a literal interpretation, however, is argued against by the CES model estimate that we will not “officially” transition to the new mode until 2020, in the sense of having the new contribution to the world economy exceed the old part. For this to make sense, much of our non-farming economy needs to be interpreted as still dominated by the “old” growth mode.

The form of our model suggests that the new growth mode might be found in small quantities well before it has a noticeable effect on total growth rates, and that what distinguishes this new economy inside the old one is that it is growing rapidly, at a rate closely related to what will become the new total economic growth rate. We might thus better understand what distinguishes the current “new” and “old” growth modes by looking for processes which doubled before the transition at a rate similar to the modeled new rates of 6 or 15 years.

Some interesting candidates are found in measures of scientific progress. The number of scientific journals has doubled steadily about every 15 years since about 1750, even though the world product doubling time in 1750 was around a century. (From about 1650 to 1750 both journals and scientific societies doubled roughly every 30 years.) Narrower measures of progress grow slower, while broader measures grow faster. For example, the number of “important” discoveries has doubled every 20 years, while the number of U.S. engineers has doubled every 10 years (de Solla Price, 1963).

[8] Maximum brain size does not translate very directly into world product, but the long roughly steady growth in maximum brain size has been mentioned in this paper because it may be the relevant growth mode prior to the early spread of the human population. Product growth in an economy of human-like creatures seems to have awaited the evolution of a land-based animal with hands, a large enough brain, and perhaps other unknown prerequisites.

This rough coincidence in timing and doubling times suggests, though only weakly, that a new "scientific" evolution and diffusion of knowledge via a subject-specific network of articulate specialists is fundamental to the new industry growth mode. Perhaps the diffusion of seeds and artifacts via a simpler more spatial network of less specialized people is fundamental to the farming mode. If so, the new 15 year doubling time of the sum of exponentials model suggests that published papers are close to the fundamental unit of growth, while the 6 year doubling time of the CES model suggests that a very broad measure of "scientific"-like activity is closer to the fundamental unit of growth.

How can we make sense of the basic idea of the world economy repeatedly switching to new growth modes? That is, if the economy is capable of faster growth modes, what prevented them from happening earlier? Among the many models mentioned before of transitions between historical economic growth modes, perhaps the simplest postulate minimum scale effects, so that a new growth mode will not occur in economies below a certain size, density, or technology level. A stochastic variation on this is to make the number of attempts or chances to discover and transition to a new growth mode be proportional to the current scale of the economy. This variation also predicts that a transition will occur at roughly the time when the economy reaches some predetermined scale.

Such models could interpret the roughly stair-step shape of Figure 2 somewhat literally. The lip of each step would describe a new mode which would not be realized until world product, moving along the x-axis, reached the base of that step.

Could It Happen Again?

Current U.S. stock prices seem to be much higher than can be justified by the expectation that the U.S. economy will continue much as it has for the last century (Shiller, 2000). If speculators are rational, it seems that they must be assigning a substantial probability to the possibility of a large change in the economy, such as a fall in the risk-premium investors demand, a fall in the real risk of corporate profits, a rise in the share of income taken by corporate profits, or a rise in real economic growth rates. It is this last possibility, of a large rise in economic growth rates, that seems to be offered some support by the empirical analysis of very long term economic growth.

If one takes seriously the idea of long-term economic history as a sequence of exponential growth modes, one might naturally wonder if a world economy that seems to have so far seen at least three dramatic transitions to much faster growth modes will see yet another such transition. One pair of authors, Ausubel and Meyer, did informally "speculate on a fourth [Deevey-like] 'information' pulse starting now that would enable another order-of-magnitude rise" in population. But they took the idea no further (Ausubel & Meyer, 1994).

Are there ways an economy can grow fast that will not be possible until the world economy reaches a threshold in size, density, or the price of some key technology? While it is certainly possible that the economy is approaching fundamental limits to economic growth rates or levels, so that no faster modes are possible, we should also consider the other possibility.

To use our models to estimate how soon the world economy might jump again to a faster growth mode, and how much faster that mode might be, we must make some assumptions about what it is that is similar across past and future modes and mode transitions. The apparent regularities in Figure 2 suggest one approach. They suggest that we treat multiples of world product and growth rates as similar across growth modes. More precisely, if we assume that at each transition new values of the doubling time factor and world product factor are drawn again from the same constant-in-time distributions for these parameters, then previous values of these parameters offer "sample" estimates of future parameter values.

In the sum of four exponentials model, if the current mode were to last through as large a world product factor as one of the previous three growth modes, it would last until one of the sample dates of 1963, 2032, or 2066. If the next doubling time factor were the same as one of the last three values, the next doubling time would be either would be .05, 1, or 4 years.[9]

Table 3 describes sample forecasts based on the CES model (which the author prefers). It

[9] If the next mode were like the brain growth mode, it would start in 2162 and have a doubling time of 0.1 years.

Growth Mode	Doubling Time (DT) Factor	New Doubling Time (days)	World Product (WP) Factor	New Date from WP	New Date from DT
Hunting	153?	14.9?	480	1996	2075
Farming	247	9.3	190	1976	2067
Industry	145	15.8	> 590		

Table 3: Using the CES Model to Forecast The New Mode

includes sample forecasts of the next economic doubling time, and two kinds of estimates of the next transition date.[10] The world product method expects that the world product factor, as previously defined, of the current mode will be similar to that of the previous modes. The doubling time method instead expects to see a similar value for

$$(d_{i+1} - d_i)/\tau_i,$$

which is the number of doubling times during the period when a mode is dominant. Since the "current" mode does not even start to dominate until 2020, the doubling time method naturally gives later date estimates.

The estimates of the new doubling time offer what seems to be a remarkably precise estimate of an amazingly fast growth rate. One is tempted to immediately reject such rapid growth as too preposterous to consider, until one remembers how preposterous it would have seemed to forecast the previous two transitions that did in fact occur.

The robustness of these forecasts can be examined by considering what these types of models would have predicted using only the data up to 1900. From 1900 to 2000, world product increased by a factor of 37. The best fit four term sum of exponentials model forecasts that a factor of 35 increase, while the best fit three term CES model forecasts a factor of 22 increase. The main reason for the CES model failure here is estimating a transition date of about 2060, instead of the 2020 date suggested by the larger dataset. Other parameter estimates were in much closer agreement.

[10] Another row corresponding to a brain growth mode would give sample dates of 2072 and 2120. Using the out of Africa population estimate would change the new doubling time estimates from 14.9?,9.3,15.8 to 1.49?,93,15.8.

2039	2040	2041	2042	2043	2044	2045	2046	2047
6.1%	6.1%	6.6%	8.0%	14%	41%	147%	476%	1023%

Table 4: New Transition Scenario, Instantaneous Annual Growth Rates

For example, the up-to-1900 CES estimate of the new doubling time is 6.4 years, very close to the up-to-2000 estimate of 6.3 years. Timing estimates thus seem less reliable than growth rate estimates.

Table 4, Figure 3 and Figure 4 show how a new transition might look, if the new mode had a 15 day doubling time and the same strong complementarity with our current mode that our current mode had with the previous mode, and if the new mode first started to have a noticeable effect on growth rates in 2041. (Transitions starting at any other date would look very similar.)

Table 4 makes it clear that in this scenario, the new mode has an enormous effect on economic growth rates very soon after it has any noticeable effect on growth rates. If this growth mode did not hit limits to slow it down, then by 2047, i.e., within six years of becoming noticeable, the economy would then grow by a larger factor than it had from two million B.C. until 2040.

This is thus not at all a reasonable model for describing the recent small but noticeable increase in U.S. economic growth rates that some now attribute to a "new industrial revolution." Thus if stock prices are now high in anticipation of a transition to a new growth mode, it must be in anticipation of a much smaller change in growth rates, of a change that has not yet detectably changed growth rates, or of a much more gradual transition between growth modes.

In summary, if one takes seriously the model of economic growth as a series of exponential growth modes, and if relative change parameters of a new transition are likely to be similar to such parameters describing old transitions, then it seems hard to escape the conclusion that the world economy could see a very dramatic change within the next century, to a new economic growth mode with a doubling time of roughly two weeks or less.

While it is hard to see in much detail how the world economy could possibly grow that fast, it is at least suggestive that computer hardware costs have in fact been consistently falling for many decades at a doubling time of one to two years (Moravec, 1998; Kurzweil, 1999). Several observers have predicted a transition to a dramatically new economic mode where computers substitute for most human labor, perhaps not long after the raw computational ability of a desktop computer matches the computational ability of a human brain, around 2025 (Moravec, 1998; Kurzweil, 1999). Simple formal models have even been developed suggesting that adding robots to very standard growth models can allow growth rates to naturally increase by a factor of ten, a hundred, or even more (Hanson, 1998). Others have speculated that even faster growth rates are possible with mature nanotechnology.

If the next mode had a "slow" doubling time of two years, and if it lasted through twenty doubling times, longer than any mode seen so far, it would still last only forty years. After that, it is not clear how many more even faster growth modes are possible before hitting fundamental limits. But it is hard to see how such fundamental limits would not be reached within a few decades at most.

Conclusion

Many people have tried to make sense of very long term time series estimating human population. Some have explicitly modeled this history as steadily accelerating growth, while others have more informally described it as a sequence of growth modes. And exponential growth is by far the most common formal model of shorter term growth. Meanwhile, Brad De Long has recently constructed long term world product estimates from recent world product estimates and older population estimates.

This paper has thus attempted the neglected task of more formally describing long term world product growth as a sequence of exponential growth modes. We have found that a time series of world product over the last two million years can be comfortably described as either as a sum of four exponentials, or as a CES-combination of three exponentials. But the CES model seems preferable on theoretical grounds, which can be thought of as describing the three modes of "hunting," "farming," and "industry," broadly construed. (An earlier period of exponential growth in brain sizes may be the relevant previous growth mode.)

In the CES-combination model, there is a sharp transition between hunting and farming, and a smooth transition between farming and industry. These can be interpreted as due to the interaction between hunting and farming being more one of substitution, relative to a more complementary relation between farming and industry. The rough timing and doubling time coincidence between industry and measures of scientific progress weakly suggests that scientific-like creation and diffusion of knowledge might be a key to the current growth mode.

Since there seem to be some rough regularities regarding how much the economy grows between transitions, and how much faster each new growth mode is, this paper has also considered what these regularities suggest about when we might see yet another transition to a much faster mode, if such faster modes are possible. The suggestion is fantastic, namely of a transition to a doubling time of two weeks or less sometime within roughly the next century.

One might think this suggestion too fantastic to consider, were it not for the fact that similar predictions before previous transitions would have seemed similarly fantastic. We should also keep in mind that investors in U.S. stocks seem to be betting that a large change in the nature of the economy is likely soon. They may well be wrong, but this paper shows that the empirical case against such expectations is not as clear as one might expect from empirical analyzes looking at the last century or so. From a purely empirical point of view, *very* large changes are actually to be expected within the next century.

Data Appendix

Year	Product	Year	Product	Year	Product	Year	Product
-2,000,000	10,000	1	231,000,000	1300	401,000,000	1930	28,200,000,000
-750,000	500,000	14	219,000,000	1340	506,000,000	1940	37,500,000,000
-10,000	4,630,000	200	232,000,000	1400	562,000,000	1950	51,000,000,000
-5000	6,380,000	350	224,000,000	1500	733,000,000	1955	67,900,000,000
-4000	9,630,000	400	231,000,000	1600	963,000,000	1960	85,700,000,000
-3000	19,900,000	500	249,000,000	1650	1,022,000,000	1965	114,000,000,000
-2000	37,800,000	600	261,000,000	1700	1,248,000,000	1970	151,700,000,000
-1600	54,500,000	700	293,000,000	1750	1,606,000,000	1975	189,400,000,000
-1000	79,400,000	800	319,000,000	1800	2,190,000,000	1980	235,000,000,000
-800	121,500,000	900	396,000,000	1850	4,500,000,000	1985	281,000,000,000
-500	171,500,000	1000	441,000,000	1875	7,100,000,000	1990	344,000,000,000
-400	200,000,000	1100	495,000,000	1900	13,790,000,000	1995	421,000,000,000
-200	213,000,000	1200	468,000,000	1920	21,700,000,000	2000	513,000,000,000
		1250	445,000,000	1925	26,300,000,000		

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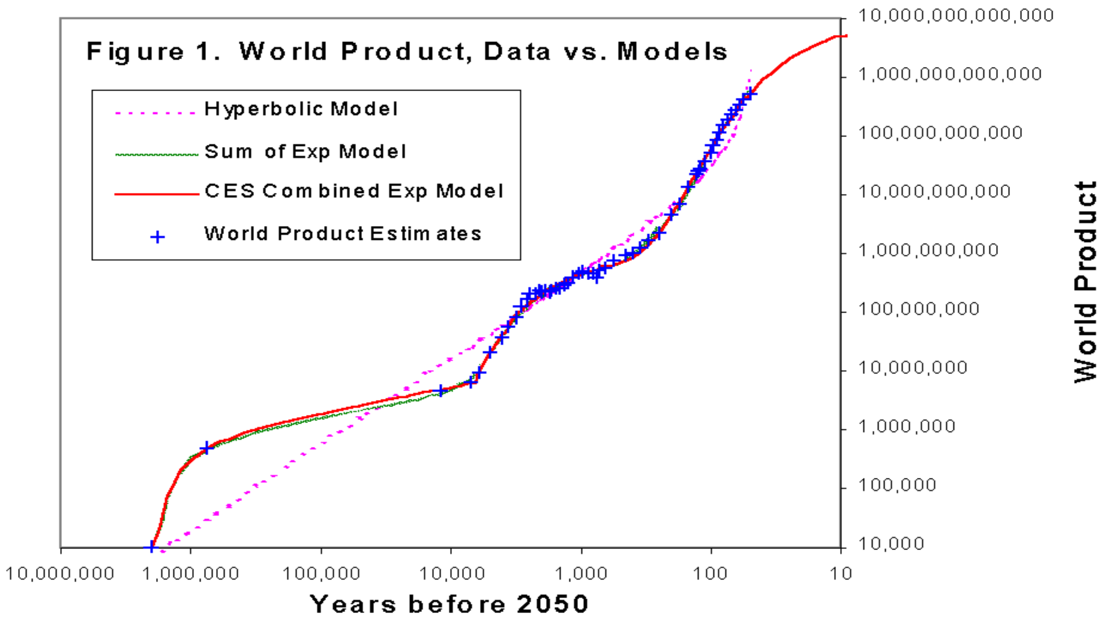
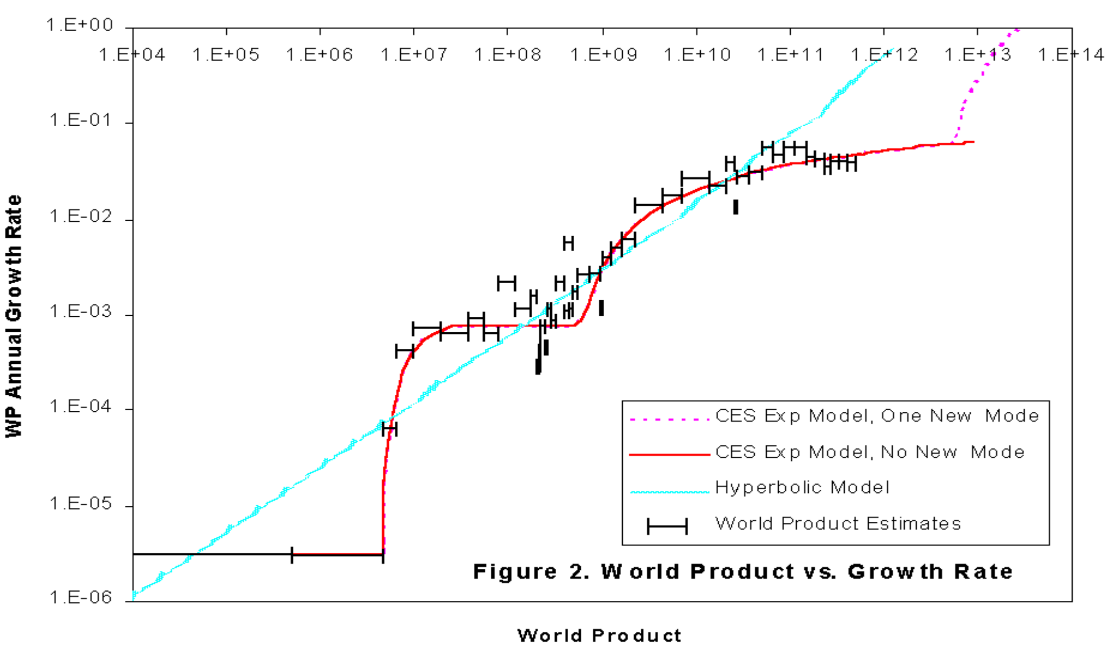


Figure 1. World Product, Data vs. Models
Hyperbolic Model
Sum of Exp Model
CES Combined Exp Model
World Product Estimates
10,000,000,000,000
1,000,000,000,000
100,000,000,000
10,000,000,000
1,000,000,000
100,000,000
10,000,000
1,000,000
100,000
10,000
World Product
10,000,000
1,000,000
100,000
10,000
1,000
100
10
Years before 2050


1.E+00
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
1.E-06
1.E+04
1.E+05
1.E+06
1.E+07
1.E+08
1.E+09
1.E+10
1.E+11
1.E+12
1.E+13
1.E+14
WP Annual Growth Rate
CES Exp Model, One New Mode
CES Exp Model, No New Mode
Hyperbolic Model
World Product Estimates
Figure 2. World Product vs. Growth Rate
World Product

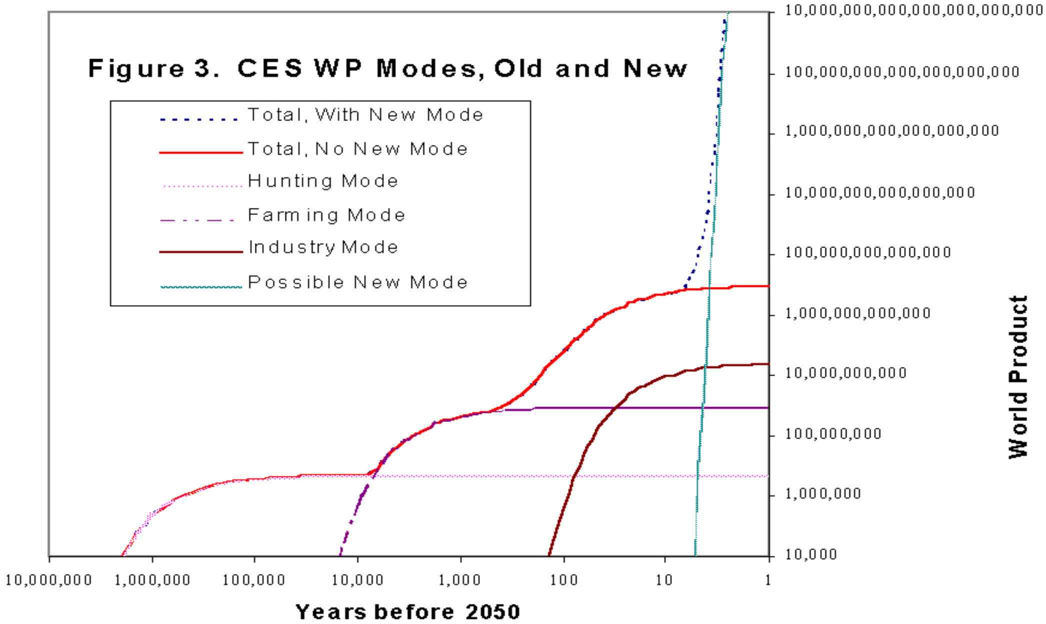


Figure 3. CES WP Modes, Old and New
Total, With New Mode
Total, No New Mode
Hunting Mode
Farming Mode
Industry Mode
Possible New Mode
10,000,000,000,000,000,000,000
100,000,000,000,000,000,000
1,000,000,000,000,000,000
10,000,000,000,000,000
100,000,000,000,000
1,000,000,000,000
10,000,000,000
100,000,000
1,000,000
10,000
World Product
10,000,000
1,000,000
100,000
10,000
1,000
100
10
1
Years before 2050

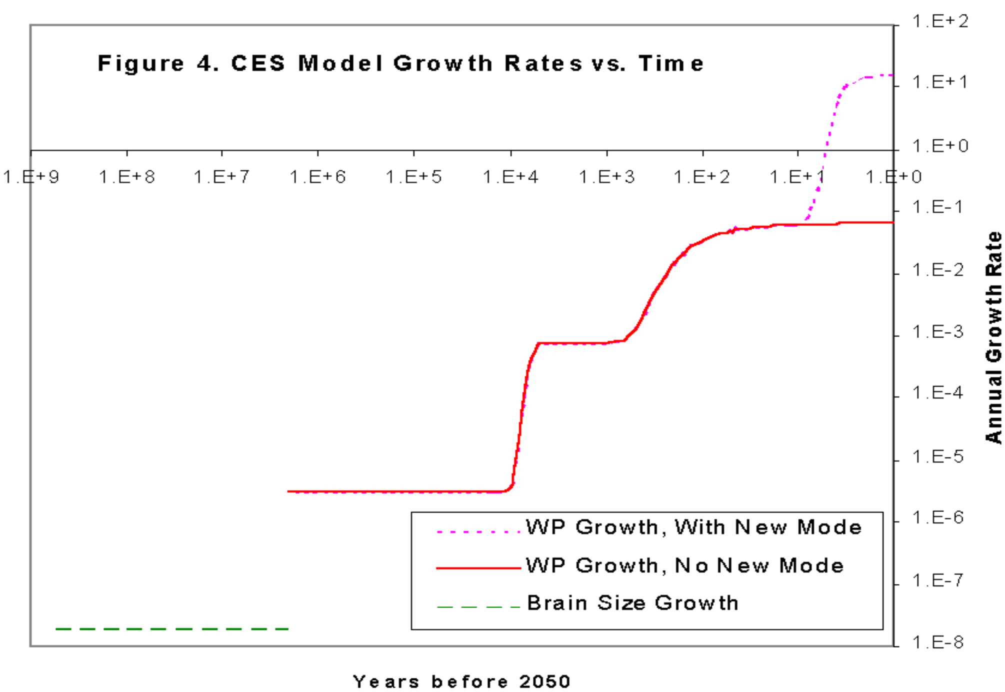


Figure 4. CES Model Growth Rates vs. Time
1.E+2
1.E+1
1.E+0
1.E-1
1.E-2
1.E-3
1.E-4
1.E-5
1.E-6
1.E-7
1.E-8
1.E+9
1.E+8
1.E+7
1.E+6
1.E+5
1.E+4
1.E+3
1.E+2
1.E+1
1.E+0
Annual Growth Rate
WP Growth, With New Mode
WP Growth, No New Mode
Brain Size Growth
Years before 2050