Rational Bar Bets

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The Basic Idea

Various "no trade" theorems suggest that rational agents who start from a common prior should not make speculative bets with each other. To support speculation, market microstructure models typically invoke agents with dynamic hedging demands. Thus a "bar bet", a simple bet on a risk-irrelevant topic negotiated in an informal but non-private social context, seems irrational.

We might, however, rationalize such bets in two different ways. First, we might assume they have the same purpose as a lot of similar behavior in similar social contexts: impressing an audience. By offering to bet on a topic, you might credibly signal to an audience containing potential associates (employers, lovers, friends, etc.) that you are informed about that topic. And these potential associates may prefer to associate with informed people.

Alternatively, we might assume that the purpose of such bets is to persuade. The audience may contain decision-makers who will make a decision in your favor only if they believe that some probability is above a certain threshold. In this case you might offer to bet in order to persuade decision-makers that you really have seen a signal favorable to your case.

Either way, we might explain "bar bets" as costly signals in a signaling game.

A Model of Impressing Associates

The following model describes how a bar bet might impress potential associates.

Let there be three players, one potential employer and two bettors: a challenger and an responder. The challenger and responder are "informed" with (independent) probability $\rho, \hat{\rho} \in (0, 1)$ respectively.

The challenger seeks employment, and the employer is looking to hire someone. The responder is already happily employed. If the employer does not hire anyone, she gets utility $\tilde{u}(0) = 0$. Otherwise she gets $\tilde{u}(v - w)$ where w is the wage paid and v is a value obtained. If the challenger is informed, then $v = \bar{v} > 0$, otherwise $v = \underline{v} < 0$, where $0 > \rho\bar{v} + (1 - \rho)\underline{v}$.

There are two states ω of the world, T and F, and two possible signals s, namely $\bar{s}$ and $\underline{s}$. Nature chooses the state ω according to probabilities $P(T) = P(F) = 1/2$, and then chooses independent signals for each bettor, with $P(\bar{s}|T) = P(\underline{s}|F) = q > 1/2$. Each bettor sees his signal only if he is informed, and does not see the other bettor's signal, nor whether that person is informed.

If b is the amount won or lost in a bet, then the utility of the responder is $\hat{u}(b)$. The utility of the challenger is $u(w-b)$, where w is the wage if employed. All of $u, \hat{u}, \tilde{u}$ are continuous strictly concave and increasing functions satisfying $u(0)=0$.

The game goes as follows. Upon learning if he is informed, and if informed of his signal, $\bar{s}$ and $\underline{s}$, the challenger publicly predicts the true state, T or F, and publicly offers a bet $\bar{b}, \underline{b}$. The challenger offers to pays $\underline{b} \geq 0$ if he is wrong, in exchange for $\bar{b} \geq 0$ if he is right. The responder, having learned if he is informed, and if informed of his signal, chooses whether or not to accept this bet.

The challenger then chooses a wage w which he offers to work for. The employer, having observed the challenger's offer to bet, decides whether or not to accept this wage offer. Finally, the state T or F is revealed and the bet is settled.

Theorem 1 *There is an intuitive equilibrium of this game where challenger offers a non-zero bet only when he is informed, and is hired if he offers.*

Proof: The intuitive equilibrium is as follows. The challenger only offers to bet when he is informed, and predicts T if his signal is $\bar{s}$, and F if his signal is $\underline{s}$. The responder only accepts if he is informed and his signal is contrary to the challenger's prediction. The challenger offers a wage $w=\bar{v}$, which the employer accepts. The employer believes any offer this large or larger could only come from an informed challenger, while any smaller offer could have come from either type.

Assume, without loss of generality, that the challenger sees signal $\bar{s}$ and predicts state T. Let $\hat{U}_1, \hat{U}_0$ be the expected utility of the responder if he accepts, given a contrary signal or a confirming signal respectively, and assuming the challenger is informed. We have

$$\hat{U}_1 = P(T|\bar{s}\underline{s})\hat{u}(-\bar{b}) + (1-P(T|\bar{s}\underline{s}))\hat{u}(\underline{b}),$$

$$\hat{U}_0 = P(T|\bar{s}\bar{s})\hat{u}(-\bar{b}) + (1-P(T|\bar{s}\bar{s})\hat{u}(\underline{b}).$$

where $P(T|\bar{s}\underline{s})=1/2$ and $P(T|\bar{s}\bar{s}) = 1/(1+((1-q)/q)^2) > 1/2$. Note that $\hat{U}_1 \geq \hat{U}_0$, and equal only when $\bar{b}=\underline{b}=0$; the responder does better with a more favorable signal.

Also, let U_1, U_0 be the expected utilities of the challenger who offers to bet when he is and is not informed respectively, assuming the responder only accepts on a contrary signal. We have

$$U_1 = (1-\hat{\rho}P(\underline{s}|\bar{s}))u(\bar{v}) + \hat{\rho}P(\underline{s}|\bar{s})[P(T|\bar{s}\underline{s})u(\bar{v}+\bar{b}) + (1-P(T|\bar{s}\underline{s}))u(\bar{v}-\underline{b})],$$

$$U_0 = (1-\hat{\rho}P(\underline{s}))u(\bar{v}) + \hat{\rho}P(\underline{s})[P(T|\underline{s})u(\bar{v}+\bar{b}) + (1-P(T|\underline{s})u(\bar{v}-\underline{b})].$$

where $P(\underline{s}|\bar{s}) = 2q(1-q) < 1/2$, $P(T|\bar{s}\underline{s}) = 1/2$, $P(\underline{s}) = 1/2$ and $P(T|\underline{s}) = 1-q < 1/2$. Note that $U_1 \geq U_0$, and equal only when $\bar{b}=\underline{b}=0$; the challenger does better when he is informed.

If only the informed challenger offers to bet, then it is rational for that informed challenger to offer the maximal wage $w = \bar{v}$, and for the employer to accept. Given this and the above inequalities, all that remains to be shown is that there exists a betting offer $\bar{b}, \underline{b}$ such that $U_0 = \hat{U}_1 = 0$, so that the uninformed offers a zero bet, and responder is just willing to sometimes accept the offer. Since there are two variables and two distinct constraints here, there are no remaining degrees of freedom to consider.

Since $\hat{u}$ is strictly increasing and continuous, we can solve $\hat{U}_1 = 0$ for $\bar{b}(\underline{b}) = -\hat{u}^{-1}(-\hat{u}(\underline{b}))$. By the concavity of $\hat{u}$, this function $\bar{b}(\cdot)$ is also strictly increasing and concave, and satisfies $\bar{b}(\underline{b}) \leq \underline{b}$, equal only when $\underline{b} = 0$.

Substituting $\bar{b}(\underline{b})$, we get a strictly concave function $U_0(\underline{b})$ maximal at $U_0(0) = \bar{v} > 0$ on R^+. This implies the existence of a $\underline{b}^* > 0$ satisfying $U_0(\underline{b}^*) = 0$. Thus there exist $\bar{b}(\underline{b}^*), \underline{b}^*$ solving $U_0 = \hat{U}_1 = 0$, and so this equilibrium exists as claimed.

This equilibrium is intuitive because the employer believes that, out of equilibrium, only informed challengers would offer $\bar{b}, \underline{b}$ so that $U_0 < 0$. QED.

A Model of Persuasion

A model of persuasion can be constructed by small modifications of the above model of impressing associates.

We replace the employer with a decision-maker who must choose an action $a \in \{T, F\}$, with utility $\tilde{u}(a, \omega)$ given by $\tilde{u}(T,T) = 1, \tilde{u}(F,F) = \tilde{q}/(1-\tilde{q})$ and $\tilde{u}(T,F) = \tilde{u}(F,T) = 0$, where $\tilde{q} \in (1/2, q)$.

The challenger has an interest in this decision, getting a cash-equivalent of $\bar{v}$ if $a = T$, and 0 otherwise. The game is otherwise exactly the same.

Theorem 2 *There is an intuitive equilibrium of this game where challenger offers a non-zero bet only when he is informed with signal $\bar{s}$, and the decision-maker chooses action T only if the responder does not accept this offer.*

Proof: The intuitive equilibrium is as follows. The challenger only offers to bet when he is informed his signal is $\bar{s}$, in which case he predicts T. The responder only accepts if he is informed and his signal is contrary to the challenger's prediction. The decision-maker chooses $a = T$ only if a bet offer is made and not accepted, and believes any offer this large or larger could only come from an informed challenger, while any smaller offer could have come from either type.

The proof is almost the same as before. The decision-maker will choose $a = T$ if she believes $P(T) > \tilde{q}$, and will choose $a = F$ if she believes $P(T) < \tilde{q}$. Only the case where the challenger sees signal $\bar{s}$ is relevant now, however, as otherwise he simply cannot credibly signal to the decision-maker that $P(T) \geq \tilde{q}$.

Expected utilities $\hat{U}_0, \hat{U}_1$ have exactly the same form as before, and the only change in U_0, U_1 is the loss of $\bar{v}$ from the cases where the bet offer is accepted. For example, we now have

$$U_1 = (1 - \hat{\rho}P(\underline{s}|\bar{s}))u(\bar{v}) + \hat{\rho}P(\underline{s}|\bar{s})[P(T|\bar{s}\underline{s})u(\bar{b}) + (1 - P(T|\bar{s}\underline{s}))u(-\underline{b})]$$

But since the proof before didn't depend on this detail, the same proof structure applies, and so the claimed intuitive equilibrium exists. QED.

Discussion

This model can be straight-forwardly extended to cases where transaction cost of bet, or where only a chance that audience is seeking to associate.

The models here are admitedly simple - the point is just to explicitly demonstrate that there are more possible motivations for speculative trades than are commonly considered.

I don't want to argue strongly that trading in familiar financial markets are dominated by these types of motivations, though it may be possible to explicitly design new markets in which such motives are more important.