Using Markets To Make Decisions

- There is abundant evidence that markets can predict events accurately, at least if the number of participants is large.

- How should we diagnose, and treat, the cases where markets do not work so well?
  - What do we mean by markets not working well.
Using Markets To Make Decisions

- Potential sources of market failure:
  - No information to aggregate
  - Information available, but biased processing
    - “Yes”-man effect in organizations
    - Institutional biases
    - Attitudes toward risk
Using Markets To Make Decisions

- One solution is to “bootstrap” the market, possibly using information from ancillary sources about the “true” distribution of types and their risk preferences (Chen, Fine & Huberman, 2001).

- Such bootstrapping allows one to consider the influence of a particular set of agents.
Using Markets To Make Decisions

- Bootstrapping -- at least non-parametric bootstrapping -- assumes that behavioral responses remain fixed.
- To go beyond this requires greater analytical structure.
Information Aggregation in Markets

- Characterizing behavior in asset markets: applications of some finance models.
  - Suppose that there are \( n \) risky assets
  - \( N_I \) “informed” traders; \( N_U \) “uninformed traders
  - Define: \( p_{it} \) = beginning period price of asset \( I \) and let \( \pi_{it} \) denote end-of-period price.
Notes on Information Aggregation in Experimental Asset Markets

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Abstract

Over the last decade considerable attention has been given to the possibilities of predicting political and other events by observing the outcome of appropriately designed experimental markets. In the Iowa Presidential Stock Market (cf. Forsythe et al. [1992]) the outcome of the U.S. presidential election was predicted successfully through observations of a stock market, where individual experimental subjects traded shares in each of the presidential candidates. Since then similar experimental markets have been used to predict different electoral outcomes in the U.S. as well as a number of other countries (Berg, Nelson, Rietz [2001]). A remarkable feature of these experimental asset markets is that they often yield forecasts more reliable than conventional techniques, including opinion polls, expert opinions and crude statistical analysis.

A number of reasons have been advanced why opinion polls and expert opinions may yield unreliable, even biased, predictions of electoral outcomes. Among these are judgement bias, inappropriate sampling of the population, and the problem of low voter turnout (cf. Forsythe et al. [1994] for a discussion of these and other possibilities). However, very little formal analysis has been devoted to the reasons why markets - appropriately designed - may yield reliable predictions.

In an experimental asset market each trader is subjected to a variety of information regarding the outcome of the event being predicted. In the case of an electoral outcome opinion polls are arguably the most prominent
information source, as is the general news obtained via the media. Not surprisingly, then, most traders in an experimental asset market base their investment decisions on these sources, while a number of traders presumably ignore such information and behaves rather erratically. Many of these information sources are private in nature (e.g. lunch chats) while others are largely publicly available (e.g. opinion polls). Thus, at any given time a distribution of knowledge exists in the markets among the different traders. When trades take place on the basis of this asymmetric information the resulting equilibrium price will to some extent reflect this distribution of information. The crucial question is to what extent the price will in some sense aggregate information possessed by the collection of agents. If markets are “efficient” in the sense of completely aggregating private information - in which case each individual trader acts as though he knows the entire collection of private information signals - markets will be accurate in predicting (electoral) outcomes. If no aggregation of information takes place markets presumably are not very efficient in transmitting information among the participating traders. Hence, it is crucial to empirically determine the amount of information aggregation taking place in experimental asset markets.

In the present study we consider the different ways the market price and quantity of assets transacted may reflect the different information sets possessed by individual traders in an experimental asset market. Borrowing freely from the modern theory of asymmetric information in asset markets (cf. Grossman and Stiglitz, [1980]) we consider empirical procedures of discriminating between various equilibrium concepts. This discrimination is achieved by considering the joint distribution of prices and quantities transacted. It is demonstrated that various informational assumptions (and resulting equilibrium concepts) impose different restrictions on the price-quantity distribution. These restrictions may, in turn, be used to empirically differentiate between the empirical validity of various informational assumptions.

1 The model

Consider an asset market with \( n \) risky assets \((i = 1, \ldots, n)\) and one risk-free asset. In the market are \( N_I \) informed traders \((j = 1, \ldots, N_I)\), who base their investment decisions on private and public information, as well as \( N_U \) uninformed (noise) traders, whose investment decisions are completely random.
1.1 Informed traders

Let $p_{it}$ be the beginning of period price of risky asset $i$ ($i = 1, \ldots, n$), and let $\pi_{it}$ denote the end-of-period price of risky asset $i$ ($i = 1, \ldots, n$). Letting $w_{jt}$ denote the beginning-of-period wealth of trader $j$ in period $t$, and letting $r_t$ denote the rate of return of the risk free asset in period $t$, end-of-period wealth of the $j^{th}$ trader $w_{jt}$ is given by

$$
w_{jt} = \left( w_{jt} - \sum_{i=1}^{n} p_{it} x_{ijt} \right) (1 + r_t) + \sum_{i=1}^{n} \pi_{it} x_{it} \]

$$

(1)

where $x_{ijt}$ is the quantity of the $i^{th}$ asset purchased by agent $j$ in period $t$, $p_t = [ p_{1t} \ldots p_{nt} ]'$, $\pi_t = [ \pi_{1t} \ldots \pi_{nt} ]'$ and $x_{jt} = [ x_{1jt} \ldots x_{njt} ]'$. It is assumed throughout that the random end-of-period price $\pi_t$ is normally distributed with mean $\pi_t$ and positive definite covariance matrix $\Omega_\pi$; i.e.,

$$
\pi_t \sim N ( \pi_t, \Omega_\pi )
$$

(2)

It then follows that

$$
w_{jt} \sim N \left( \left( w_{jt} - x_{jtp_t} \right) (1 + r_t) + x_{jtp_t} \pi_t, \Omega_z x_{jtp_t} \right) \]

(3)

We furthermore assume that

$$
\pi_t = \pi + \nu_t
$$

(4)

where $\pi$ is the constant (fundamental) value vector, while $\{\nu_t\}$ is a Gaussian white noise process such that $\nu_t \sim N (0, \Omega_\nu)$ where $\Omega_\nu$ is a positive definite covariance matrix.

The $j^{th}$ informed trader is assumed to have a constant absolute risk aversion utility function of the form

$$
u_{j} (w_{jt}) = -e^{-\alpha_j w_{jt}}
$$

(5)

where $\alpha_j$ is the $j^{th}$ trader’s coefficient of absolute risk aversion (i.e., $\alpha_j = -\frac{w''(w_{jt})}{w'(w_{jt})}$). Given the $j^{th}$ trader’s information set at the beginning of period $t$, $I_{jt}$, the $j^{th}$ informed agent determines his portfolio by maximizing the
expected utility, conditional on the information set \( \mathcal{I}_{jt} \); i.e., the \( j^{th} \) trader solves the maximization problem

\[
\max_{x_{jt}} E [u_j (w_{jt}) | \mathcal{I}_{jt}]
\]  

(6)

The information structure of the model is specified as follows. At the beginning of each period the \( j^{th} \) informed agent receives a (private) signal \( q_{jt} \) given by

\[
q_{jt} = \pi_t + \varepsilon_{jt}
\]  

(7)

where \( \varepsilon_{jt} \) is a random error vector assumed to be independent of \( \varepsilon_{j't'} \) for \( j' \neq j \) and \( t' \neq t \) and such that

\[
\varepsilon_{jt} \sim N (0, \Omega_\varepsilon)
\]  

(8)

where \( \Omega_\varepsilon \) is a positive definite matrix.

Using standard properties of the normal distribution it follows immediately that the optimal portfolio of the \( j^{th} \) informed trader is given by

\[
x_{jt}(p_t) = \frac{1}{\alpha_j} \text{Var} \left[ \pi_t \mid \mathcal{I}_{jt} \right]^{-1} \left\{ E \left[ \pi_t \mid \mathcal{I}_{jt} \right] - (1 + r_t)p_t \right\}
\]  

(9)

Thus, the optimal portfolio depends on the market price \( p_t \), and we shall stress this relation by writing \( x_{jt} = x_{jt}(p_t) \). In addition, the optimal portfolio depends (though this is ignored in the notation) on the information set \( \mathcal{I}_{jt} \) available to him at the beginning of the period. Moreover, due to the normality assumptions and the assumption of constant absolute risk aversion, the optimal portfolio is linear in prices and does not depend on the level of initial wealth \( w_{jt} \).

### 1.2 Uninformed traders

Uninformed (noise) traders have a random inelastic demand for the risky assets \( x_{Ut} = [ x_{U1t} \ldots x_{Unt} ]' \). Thus, uninformed traders do not base their trading decisions on any formal private or public information, and serves as a source of ‘noise’ in the market.
1.3 Equilibrium

Equilibrium is obtained by equating demand \( \sum_{j=1}^{N_I} x_j(p_t) + x_{Ut} \) and supply \( s_t \), the latter assumed to be exogenous; i.e.,

\[
\sum_{j=1}^{N_I} x_j(p_t) + x_{Ut} = s_t
\]

(10)

where \( s_t = [s_{1t} \ldots s_{nt}]' \) and \( s_{it} \) is the supply of the \( i^{th} \) asset. This equilibrium condition is conveniently written as

\[
\sum_{j=1}^{N_I} x_j(p_t) = z_t
\]

(11)

where \( z_{it} = s_{it} - x_{Ut} \) is the net supply of the \( i^{th} \) asset and \( z_t = [z_{1t} \ldots z_{nt}]' \). Throughout the following it is assumed that \( z_t \) is normally distributed with mean \( z_t \) and positive definite covariance matrix \( \Omega_z \); i.e.,

\[
z_t \sim N(z_t, \Omega_z)
\]

(12)

Thus, the net supply (effect of noise traders) provide a source of uncertainty in the asset market.

2 The consequences of various equilibrium concepts

Depending on the specification of the informed agent’s information sets various equilibrium concepts result within the simple framework considered in the previous section. This is clear from expressions (9) and (11), from which it follows that the equilibrium portfolio and price depends crucially on the nature of the information sets considered. In the present section we consider the implications of various equilibrium concepts by considering the restrictions these concepts place on the joint distribution of prices and quantities transacted. The equilibrium concepts considered are

- Walrasian equilibrium: in this case each informed agent trades on the basis solely of his private information, ignoring the information contained in the equilibrium asset prices. In this case prices do not provide an information aggregation device.
• **Fully revealing rational expectations equilibrium**: this concept corresponds to the loose notion of an 'efficient market'. In this case each informed trader acts on the basis of his private information, as well as the information contained in equilibrium prices. In addition, it is assumed that the agent knows the net supply (i.e., the trading volume of the noise traders). In the case of a fully rational expectations equilibrium the equilibrium price provides a perfect information aggregation device, as the price is a 'sufficient statistic' for all information in the market. Thus, the traders act as though the know the entire collection of private signals in the market.

• **Noisy rational expectations equilibrium**: in this intermediate case informed agents base investment decisions on their private information as well as the market equilibrium price. However, net supply is not observed by each informed agent. Thus, a residual uncertainty is left for the informed agents to disentangle. In this case prices serve as information aggregators, though the aggregation is not perfect.

By focusing on the joint distribution of prices and quantities traded it is shown that the various informational assumptions imply different restrictions on the implied stochastic processes describing prices and quantities traded. These implications can, in turn, be used to empirically discriminate between the various equilibrium concepts when observing the outcomes of asset markets. Below, we discuss the results obtained for various equilibrium concepts, leaving the formal technical details to a number of technical appendices at the end of the paper.

Let

$$X_t = \frac{1}{2} \sum_{j=1}^{N_t} (\Delta x_{j,t} \ast \Delta x_{j,t})$$

(13)

be the $n \times 1$ vector of predictable components of aggregate trading volumes of informed traders, where $\ast$ denotes the Hadamard product.

For notational convenience, let

$$A_1 = (\Omega_\xi + \Omega_\pi)^{-1} \Omega_\pi \quad , \quad A_2 = \Omega_\pi (I - A_1)$$

$$A_3 = \sum_{j=1}^{N_t} \frac{1}{\alpha_j^2} \quad , \quad A_4 = \sum_{j=1}^{N_t} \frac{1}{\alpha_j}$$
Furthermore, let 
\[
\xi_{jt} = E[\pi_t \mid I_{jt}] \quad \text{and} \quad \bar{\xi}_t = \frac{1}{N_I} \sum_{j=1}^{N_I} \xi_{jt}
\]
denote the \( j^\text{th} \) trader’s posterior expectation of the unknown asset price (conditional on his information set \( I_{jt} \)) and the average expectation, respectively. In the following \( \Delta \) denotes the difference operator (e.g., \( \Delta p_t = p_t - p_{t-1} \)).

### 2.1 Walrasian (competitive) equilibrium

In the traditional Walrasian (competitive) equilibrium each informed trader bases his investment decision solely on his private information; i.e., \( I_{jt} = \{q_{jt}\} \), ignoring the additional information in the price vector \( p_t \).

**Theorem 1** Under the Walrasian equilibrium we have
\[
\Delta p_t = A_1 \frac{1}{1 + r_t} \Delta \pi_t - \frac{1}{A_4} A_2 \frac{1}{1 + r_t} \Delta z_t \tag{14}
\]
and
\[
X_t \ast X_t = \frac{A_3}{4N_I} \sum_{j=1}^{N_I} A_2^{-1} \Delta (\xi_{jt} - \bar{\xi}_t) \ast \Delta (\xi_{jt} - \bar{\xi}_t) A_2^{-1} \\
\quad + \frac{1}{4} \{ A_4 (\Delta p_t \ast \Delta p_t) - A_4 (\Delta \bar{\xi}_t \ast \Delta \bar{\xi}_t) \} + u_t \tag{15}
\]
as \( N_I \to \infty \), where \( u_t \) is an error term such that \( E[u_t] = 0 \).

In a competitive Walrasian equilibrium asset prices thus adjust to changes in average expected asset prices as well as to changes in the net supply of the risky assets. Quantities traded depends on two different sources: the first term in (15) shows that the more diverse price expectations among agents, the greater the quantity traded. The second term reflects the fact that changes in net supply impacts on the quantity traded.
2.2 Fully revealing rational expectations equilibrium

In the fully revealing rational expectations equilibrium the informed traders base their investment decision on his private information \( q_{jt} \), the information contained in the equilibrium price \( p_t \), as well as the net supply vector \( z_t \); i.e., \( I_{jt} = \{q_{jt}, p_t, z_t\} \). In the fully revealing rational expectations equilibrium we thus assume that the net supply is observable to all informed traders.

**Theorem 2** Under the fully revealing rational expectations equilibrium we have

\[
\Delta p_t = \Delta \pi_t
\]  
(16)

and

\[
X_t \ast X_t = \frac{1}{4} \Delta z_t \ast \Delta z_t
\]  
(17)

as \( N_I \to \infty \).

In a fully revealing rational expectations equilibrium prices constitute a 'sufficient statistic' for all private and public information. Hence, all informed trader’s expectations reduce to this common value \( \pi_t \), and prices must equal this value, as all informed traders are indifferent between selling and buying at this value. Hence, trading volume changes only to accommodate known changes in the net supply of assets and is uncorrelated with the changes in prices.

2.3 Noisy rational expectations equilibrium

Somewhat intermediary between the informational assumptions imposed above we consider a noisy rational expectations equilibrium, where the information based traders base their investment decisions on private information \( q_{jt} \) as well as the additional information contained in equilibrium prices \( p_t \). In this case, however, the net supply is not observed by the informed traders; i.e., \( I_{jt} = \{q_{jt}, p_t\} \).

**Theorem 3** Under the noisy rational expectations equilibrium there exists a positive definite matrix \( C_1 \) and a negative definite matrix \( C_2 \) such that

\[
\Delta p_t = C_1 \Delta \pi_t + C_2 \Delta z_t
\]  
(18)
and

\[ X_t * X_t = \frac{A_3}{4N_I} \sum_{j=1}^{N_I} A_2^{-1} \Delta (\xi_{jt} - \bar{\xi}_t) * \Delta (\xi_{jt} - \bar{\xi}_t) A_2^{-1} \]

\[ + \frac{1}{4} \{ A_5(\Delta p_t * \Delta p_t) + A_6(\Delta \xi_t * \Delta \bar{\xi}_t) \} + v_t \]  

(19)

as \( N_I \to \infty \), where

\[ \xi_{jt} = B_0 + B_1 q_{jt} + B_2 p_t \]  

(20)

and

\[ A_5 = C_1^{-1} \left\{ I_n + (B_1 + 2B_2 C_1)^{-1} C_1 B_2 B_2^t C_1' \right\} C_1'^{-1} \]

\[ A_6 = -C_2^{-1} (B_1 + 2B_2 C_1)^{-1} C_1 C_2'^{-1} \]

and \( v_t \) is an error term such that \( E[v_t] = 0 \).

It is demonstrated in the appendix that the matrix \( A_5 \) is positive definite, while the matrix \( A_6 \) is negative definite. Under noisy rational expectations informed traders posterior expectations \( \xi_{jt} \) are different even after prices are observed, since the net supply is not observed by informed traders. Hence, trading volume again depends on the dispersion of posterior expectations as well as on changes in net supply of the risky assets.