Combinatoric Markets:
Application to policy analysis markets

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Markets and Decisions
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Outline

• Information markets do work. It helps if
  – securities are independent (Arrow-Debreu)
  – and price discovery is efficient.

• A long lived, useful policy tool needs
  – many securities (bets) if want fine grained detail
    • Lose liquidity and price discovery => less info.
  – few securities if interested in price discovery
    • Lose fine details => less info.

• Standard parallel markets can’t handle this well.
• Combinatoric Markets can help solve dilemma.
What are we trading?
What information do we want?

• Some examples of interesting data series
  – Countries - 12 or so
  – 5-10 series/ country
    • Economic health
    • Corruption levels
    • Military preparedness
    • U.S. forces deployed for the country
    • U.S. forces deployed against the country
What are we trading?
What information do we want?

• A security will be a bet on whether a data series attains a value in one of N ranges over a period of time.
  – Up, down, neutral (N=3)
  – During a specific quarter, e.g. 04Q2
An example: predictive advice on 3 series

**U3.4**

- **up**
- **neutral**
- **down**

03Q4 US troop deployments to area AB

6/5/02 (DARPA)

**A4.1**

- **up**
- **neutral**
- **down**

04Q1 A-land military deployments vs. B-land

**B4.1**

- **up**
- **neutral**
- **down**

04Q1 A-land GDP
So far so good

• For 10 data series on 12 countries we need $120 \times 3 = 360$ securities.
• But that isn’t enough.
• The market would like to elicit special information
• I would like to bid only on things I feel comfortable with.
Why isn’t 360 enough?
Interdependence

• Suppose
  – I believe that Pr\{X = A4.1- | Y = U3.4+\} = a
  – I don’t have a clue about Y.

• How can I bet on Pr(X|Y) and not on Pr(Y)?

• Will the market extract my information?

• Without additional securities I can’t and the market won’t.
Interdependence continued

- Since $\Pr\{X|Y\} = \Pr\{X\&Y\}/\Pr\{Y\}$, I believe that $\Pr(X) \geq \Pr(X\&Y) = a*\Pr(Y)$.
- So the bet: $(1X, -aY)$ has a positive expected value if $\text{price}(X) < a*\text{price}(Y)$.
- But I can’t avoid betting on Y and that bet doesn’t extract all my information.
  - Market can’t infer a from the prices of X and Y.
  - Market can only infer a bound on a.
The belief $\Pr(X|Y)=a$ implies only that $\text{price}(X) \geq a \times \text{price}(Y)$.
Solving the problem: Betting that $Pr(A-|U+)=a$

- To get the full information, we need to “issue” the security $[U+&A-]$

- I buy 1 of this for $p$ and get an expected value of $a*P(U+)-p$. But this is not enough to avoid $U+$.

- I also buy $x$ units of $[U=]$ and $x$ of $[U-]$ for $q1+q2=q$ and get an expected value of $a*P(U+)+x(1-P(U+))-(p+q)$

- If $1 \geq a \geq x \geq (p+q) \geq 0$, I am ahead for any $P(U+)$. 
So we create intersections

• Why create intersections?
  – More independence
  – Finer information
  – Traders more willing to bet, protected from states they don’t know about

• Why not create intersections?
  – $N^K (=27 \text{ vs. } 9) = 3^{120}$ (cognitive problem)
  – Thinness from fear of incomplete fulfillment
The new situation:

a partial view

(1 of these for each 3 of B4.1)

<table>
<thead>
<tr>
<th></th>
<th>A4.1+</th>
<th>A4.1=</th>
<th>A4.1- = X</th>
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</thead>
<tbody>
<tr>
<td>U3.4+ = Y</td>
<td>U+&amp;A+</td>
<td>U+&amp;A=</td>
<td>U+&amp;A-</td>
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<tr>
<td>U3.4=</td>
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<tr>
<td>U3.4-</td>
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Cross-hatched = pieces of bet on [A-|U+]
Thinness: from too many intersections

• If I want to bet on $X = A4Q1-$, I will have to buy 1 unit of each intersection with this security which is a total of $N^{(K-1)}$ securities.
  – In our example, I would need to buy $3^2 = 9$ pieces
  – This leads to a significant risk of losses from incomplete trades (partial fulfillment)
Partial Fulfillment implies less liquidity

• I want to bet on my belief $\Pr(A-)$. 
• I will buy 1 of $s1=[A-\&U+]$, 1 of $s2=[A-\&U=]$ and 1 of $s3=[A-\&U-]$ if $p1+p2+p3 < \Pr(A-)$. 
• Suppose I buy 1 of $s1$ and 1 of $s2$ even though $p1+p2 > \Pr(s1) + \Pr(s2)$, obviously hoping $p3 < \Pr(A-)-p1-p2$. 
• But then I find out that $p3 > \Pr(A-)-p1-p2$. 
• I am worse off than if I did not start. So I don’t
Specific Situation

- Three markets (short sales allowed in the one risk free asset)
- Three equally likely states with payouts

<table>
<thead>
<tr>
<th>Security</th>
<th>State X</th>
<th>State Y</th>
<th>State Z</th>
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<tbody>
<tr>
<td>A</td>
<td>170</td>
<td>370</td>
<td>150</td>
</tr>
<tr>
<td>B</td>
<td>160</td>
<td>190</td>
<td>250</td>
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<tr>
<td>NOTES</td>
<td>100</td>
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</tbody>
</table>
Specific Situation

• Endowment of risky assets and cash refreshed each period
  – E.g., 5 of A, 4 of B, and 400 cash
  – May vary across subject
  – Loan repayment of, say, 1900 at end of each period - (leverage!)

• Let them trade, then draw state, then pay $, then restart

• Subjects did not know market portfolio. So can’t use CAPM to predict prices.
Bossaerts/Kleinman/Plott

• Computer-based trade through a widely tested Multiple Unit Double Auction system (MUDA)

• Thin markets of 8-15 subjects
Converging slowly when thin
Bossaerts/Plott

- Trading through a web-based open book system, *Marketscape*, developed at Caltech (Plott)

- Large scale - up to 66 subjects
  - Cost = about $3000/run
A little faster when much thicker

Distance From CAPM Equilibrium: Thick-Market Yale Experiment

(N=40)
Conjecture: (Bossaerts/Plott)

• Why do thick markets fully equilibrate and why do thin markets not fully equilibrate?

• Because traders are interested in portfolios

• But they are forced to trade individual securities.
Proposed Solution

• Design a portfolio trading mechanism
  – Allow contingent bids
Simple Combined Value Market: how does bidding work?

• Allow CV bids (e.g. swaps)
  – I will pay (up to) $50 iff I buy 50 A and sell 40 B.
  – Could also allow other contingencies-(not here)

• A bid is \((q,x) \in R^k \times R^k\) and \(F \in [0,1]\)
  – Where \(x < 0\) means sell and \(x > 0\) means buy
  – and \(q\) is the per-unit “price” for \(x\)
  – and \(F\) is the minimum % acceptable fill.
    (This makes it computationally hard)
Simple Combined Value Market: how does matching work?

• Select a feasible collection of winning bids that maximizes the total gains from trade

• Max $\sum b \cdot f$ (revealed surplus)
  – Where $b = qx$
  – Subject to $\sum x \cdot f \leq 0$ (demand $\leq$ supply)
  – $\delta F \leq f \leq \delta$
  – $\delta \in \{0, 1\}$
Simple Combined Value Market: how does pricing work?

Pay your bid doesn’t seem to work very well (fitting in multi-lateral negotiations)
Use economics: double-auction, modified uniform price
Simple Combined Value Market: how does pricing work?

• Prices $p$ are chosen to satisfy
  – $q_x - p_x \geq 0$ for all accepted orders ($\delta > 0$)
  – $q_x - p_x \leq 0$ for all rejected orders ($\delta = 0$)

• Properties
  – Price improvement
  – Prices determined by marginal orders only

• Two issues: no prices, multiple-prices
What a CVM can do to a thin market!

(N=12)
Thin markets imply hard (state) price discovery

Backward
Should be red>bl>gr

6/5/02 (DARPA) Bossaerts-Plott
CVM: This is what should happen!
How CV works seamlessly

Where we place our bets

A- | U+

U=&A=

U-

Where market matching and pricing occurs

U+  U+  U+  U=  U=  U=  U-  U-  U-
A+  A=  A-  A+  A=  A-  A+  A=  A-
Possible solutions?

• All possible subsets could be provided
  – For our example we get $2^{((N-1)^K)}=2^{27}$ different securities
  – For the 10x12x3 example we get $2^{2^{120}}$
  – Not a solution
    • The cognitive and liquidity problems just got worse.
    • Coordinating prices across all sets is very difficult (arbitrage).
Possible solutions?

- A broker holds inventory and offers sets at a premium over market prices.
  - The price wedge still prevents desirable trades.
- Options are created
  - 1 if A- and U+, 0 if ~A- and U+, money back otherwise
  - Requires same intersections as underlying pieces.
Other solutions for “too many”

- Transmute from market to individual scoring rules (Hansen)

- Limited primary issuance of “intersections” - at least need contra-side interest (Net Exchange)