as size 1 trees. Assuming the discount factor for cash flows is .9 per year, determine an optimal harvesting strategy.

10 For $50, we can enter a raffle. We draw a certificate containing a number 100, 200, 300, . . . , 1,000. Each number is equally likely. At any time, we can redeem the highest-numbered certificate we have obtained so far for the face value of the certificate. We may enter the raffle as many times as we wish. Assuming no discounting, what strategy would maximize our expected profit? How does this model relate to the problem faced by an unemployed person who is searching for a job?

11 At the beginning of each year, an aircraft engine is in good, fair, or poor condition. It costs $500,000 to run a good engine for a year, $1 million to run a fair engine for a year, and $2 million to run a poor engine for a year. A fair engine can be overhauled for $2 million, and it immediately becomes a good engine. A poor engine can be replaced for $3 million, and it immediately becomes a good engine. The transition probability matrix for an engine is as follows:

	Good	Fair	Poor
Good	.7	.2	.1
Fair	0	.6	.4
Poor	0	0	1

The discount factor for costs is .9. What strategy minimizes expected discounted cost over an infinite horizon?

Group B

12 A syndicate of college students spends weekends gambling in Las Vegas. They begin week 1 with W dollars. At the beginning of each week, they may wager any amount of their money at the gambling tables. If they wager d dollars, then with probability p, their wealth increases by d dollars, and with probability $1 - p$, their wealth decreases by d dollars. Their goal is to maximize their expected wealth at the end of T weeks.

a Show that if $p \geq \frac{1}{2}$, the students should bet all their money.

b Show that if $p < \frac{1}{2}$, the students should bet no money. (*Hint:* Define $f_t(w)$ as the maximum expected wealth at the end of week T, given that wealth is w dollars at the beginning of week t; by working backward, find an expression for $f_t(w)$.)

Group C

13 You have invented a new product: the HAL DVD player. Each of 1,000 potential customers places a different value on this product. A consumer's valuation is equally likely to be any number between $0 and $1,000. It costs $100 to produce the HAL player. During a year in which we set a price p for the product, all customers valuing the product at $\$p$ or more will purchase the product. Each year, we set a price for the product. What pricing strategy will maximize our expected profit over three years? What commonly observed phenomenon does this problem illustrate?

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