service and leaves the restaurant. It costs $c(s)$ dollars per period to serve customers at a rate s. Each customer spends R dollars, and the customer's food costs Hot Dog Queen $R - 1$ dollars to prepare.

c For each customer in line at the end of the minute, a cost of h dollars is assessed (because of customer inconvenience).

d The next minute begins.

Formulate a recursion that could be used to maximize expected revenues less costs (including customer inconvenience costs) incurred during the next T minutes. Assume that initially there are no customers present.

4 At the beginning of 2004, the United States has B barrels of oil. If x barrels of oil are consumed during a year, then consumers earn a benefit (measured in dollars) of $u(x)$. The United States may spend money on oil exploration. If d dollars are spent during a year on oil exploration, then there is a probability $p(d)$ that an oil field (containing 500,000 barrels of oil) will be found. Formulate a recursion that can be used to maximize the expected discounted benefits less exploration expenditures earned from the beginning of 2004 to the end of the year 2539.

5 I am a contestant on the popular TV show "Tired of Fortune." During the bonus round, I will be asked up to four questions. For each question that is correctly answered, I win a certain amount of money. One incorrect answer, however, means that I lose all the money I have previously won, and the game is over. If I elect to pass, or not answer a question, the game is over, but I may keep what I have already won. The amount of money I win for each correct question and the probability that I will answer each question correctly are shown in Table 13.

a My goal is to maximize the expected amount of money won. Use dynamic programming to accomplish this goal.

b Suppose that I am allowed to pass, or not answer a question, and still go on to the next question. Now determine how to maximize the amount of money won.

6 A machine in excellent condition earns \$100 profit per week, a machine in good condition earns \$70 per week, and a machine in bad condition earns \$20 per week. At the beginning of any week, a machine may be sent out for repairs at a cost of \$90. A machine that is sent out for repairs returns in excellent condition at the beginning of the next week. If a machine is not repaired, the condition of the machine evolves in accordance with the Markov chain shown in Table 14. The company wants to maximize its expected discounted profit over an infinite horizon ($\beta = .9$).

TABLE 13

Question	Probability of Correct Answer	Money Won
1	.6	\$10,000
2	.5	\$20,000
3	.4	\$30,000
4	.3	\$40,000

TABLE 14

	Next Week		
This Week	Excellent	Good	Bad
Excellent	.7	.2	.1
Good	0	.7	.3
Bad	0	.1	.9

a Use policy iteration to determine an optimal stationary policy.

b Use linear programming to detemine an optimal stationary policy.

c Perform two iterations of value iteration.

7 A country now has 10 units of capital. Each year, it may consume any amount of the available capital and invest the rest. Invested capital has a 50% chance of doubling and a 50% chance of losing half its value. For example, if the country invests 6 units of capital, there is a 50% chance that the 6 units will turn into 12 capital units and a 50% chance that the invested capital will turn into 3 units. What strategy should be used to maximize total expected consumption over a four-year period?

8 The Dallas Mavericks trail by two points and have the ball with 10 seconds remaining. They must decide whether to take a two- or a three-point shot. Assume that once the Mavericks take their shot, time expires. The probability that a two-point shot is successful is TWO, and the probability that a three-point shot is successful is THREE. If the game is tied, an overtime period will be played. Assume that there is a .5 chance the Mavericks will win in overtime. (*Note:* This problem is often used on Microsoft job interviews.)

a Give a rule based on the values of TWO and THREE that tells Dallas what to do.

b Typical values for an NBA team are TWO = .45 and THREE = .35. Based on this information, what strategy should most NBA teams follow?

9 At any time, the size of a tree is 0, 1, 2, or 3. We must decide when to harvest the tree. Each year, it costs \$1 to maintain the tree. It costs \$5 to harvest a tree. The sales price for a tree of each size is as follows:

Tree Size	Sales Price
0	\$20
1	\$30
2	\$45
3	\$49

The transition probability matrix for the size of the tree is as follows:

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} .8 & .2 & 0 & 0 \\ 0 & .9 & .1 & 0 \\ 0 & 0 & .7 & .3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

For example, 80% of all size 0 trees begin the next year as size 0 trees, and 20% of all size 0 trees begin the next year