store	Demand	Prob of demand
1	1	0.6
	2	0
	3	0.4
2	1	0.5
	2	0.1
	3	0.4
3	1	0.4
	2	0.3
	3	0.3

Figure 1: Demand for Milk

1 Probabilistic or Stochastic DP- Finite Horizon

There are two types of problems: 1) The next state is certain but the reward/cost obtained in the current state is stochastic, 2) both the next state and the reward/cost obtained in the current state are stochastic.

1.1 Example 1: Milk Distribution

This is a probabilistic resource allocation problem. The next state is certain but the reward/cost obtained in the current state is stochastic.

The purchase price of milk is \$1/gal. The selling price is \$2/gal. A supermarket chain has 3 stores. It buys 6 gal from a dairy and sells to its customers via the 3 store outlets. Any unsold milk must be bought back by the dairy from the supermarket chain at \$0.5/gal. See Figure 1 for demand and demand probability. Obtain an allocation policy to maximize revenue. [1]

1.2 Example 2: Stochastic Inventory Control

In this problem, both the next state and the reward/cost obtained in the current state are stochastic. In general

$$f_t(i) = \min_x [Expected\ Cost\ c(i,x) + \sum_j p(i,x,j) f_{t+1}(j)], \tag{1}$$

See hand out

1.3 Example 3: Gambling Game

A gambler has \$2. She has 4 chances to play. Her goal is to get \$6. If she bets b dollars she will increase (decrease) her capital position by b dollars if she wins (loses) (Prob of winning 0.4 and losing is 0.6). She cannot bet more that what she has on any game. She can bet \$0 too. Determine a betting strategy that will maximize the probability of reaching her goal.

References

[1] W. L. Winston. *Introduction to Mathematical Programming, Vol 1.* Thompson, 2003.