

year	cumulative demand	cost per plant
	# of plants	in \$ M
2011	1	5.4
2012	2	5.6
2013	4	5.8
2014	6	5.7
2015	7	5.5
2016	8	5.2

Figure 1: Capacity Expansion

1 Capacity expansion

This problem is a little different from the last week's power plant capacity generation problem which was solved as an inventory control problem.

The building of a nuclear power plant takes approx. 1 year. The fixed cost to build one or more plants in a year is \$ 1.5 M, and the demand and variable costs are given in Fig 1. In a year atmost 3 plants can be built. Obtain a plan for construction that minimizes the total cost.

2 Multiple resource allocation

2.1 Two Resources

If c_t and d_t are 2 types of available resources, $r_t(x_t)$ is the cost function at t , then

$$f_t(c_t, d_t) = \max_{x_t} [r_t(x_t) + f_{t+1}(c_t - h_t(x_t), d_t - g_t(x_t))], \quad (1)$$

where $h_t(x_t)$ and $g_t(x_t)$ are the quantity allocated from c_t and d_t respectively at t . Note: state space explosion!

2.1.1 Capacity Allocation:

ABC oil needs to build enough capacity to refine 5000 barrels of oil per day and 10,000 barrels of gasoline per day. They can build the capacity at 4 locations. The cost of building a refinery at location t that has the capacity of refining x barrels of oil and y barrels of gasoline is $c_t(x, y)$. Formulate a DP to find how much capacity should be located at each site. [1].

3 Objective captured as a state

This problem is a hybrid between the inventory control and investment problem. A farmer has \$5000 in cash and 1000 bushels of wheat. The cost of buying and selling 1 bushel of wheat in a month t is p_t . Develop a DP model to decide how many bushels to buy or sell for the next 6 months under the following constraints with the objective that the cash on hand is maximized at the end of month 6. [1]

1. In any month the money spent on buying cannot exceed the cash available at the beginning of the month
2. In any month you cannot sell more than what is available in the inventory at the beginning of a month
3. The holding capacity in the warehouse at the end of the month is at most 1000 bushels.

Hint: 2-D system state to track cash position and inventory. If you sell (buy) then the inventory decreases (increases) but cash position increases (decreases).

References

- [1] W. L. Winston. *Introduction to Mathematical Programming, Vol 1*. Thompson, 2003.