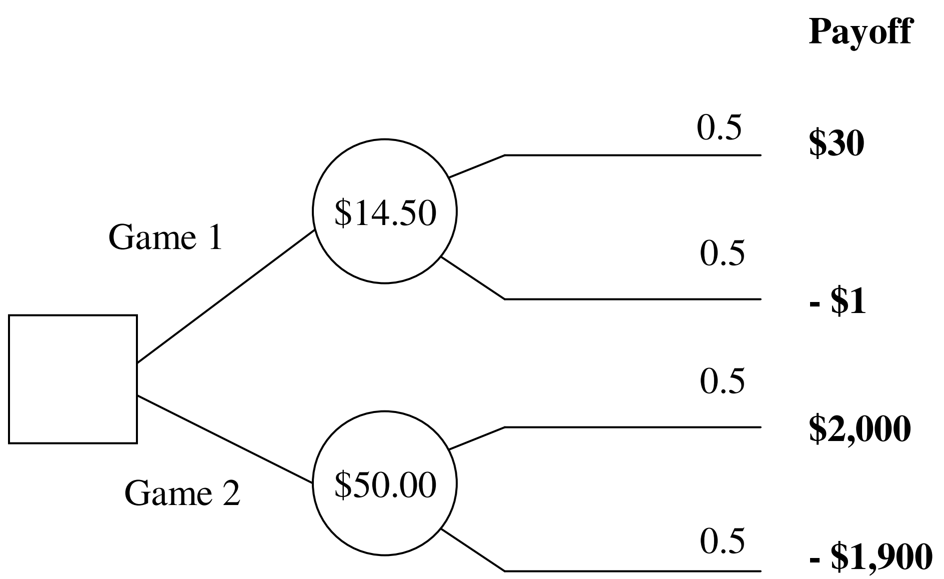
Risk and Utility

Risk - Introduction



EMV
30*0.5+(-1)*0.5= 14.5

EMV
2000*.5+(-1900)*0.5=
50.0

Which game will you play?
Which game is risky?

Going by expected monetary value (EMV) or the additive value function Game 2 has Higher EMV but also higher risk

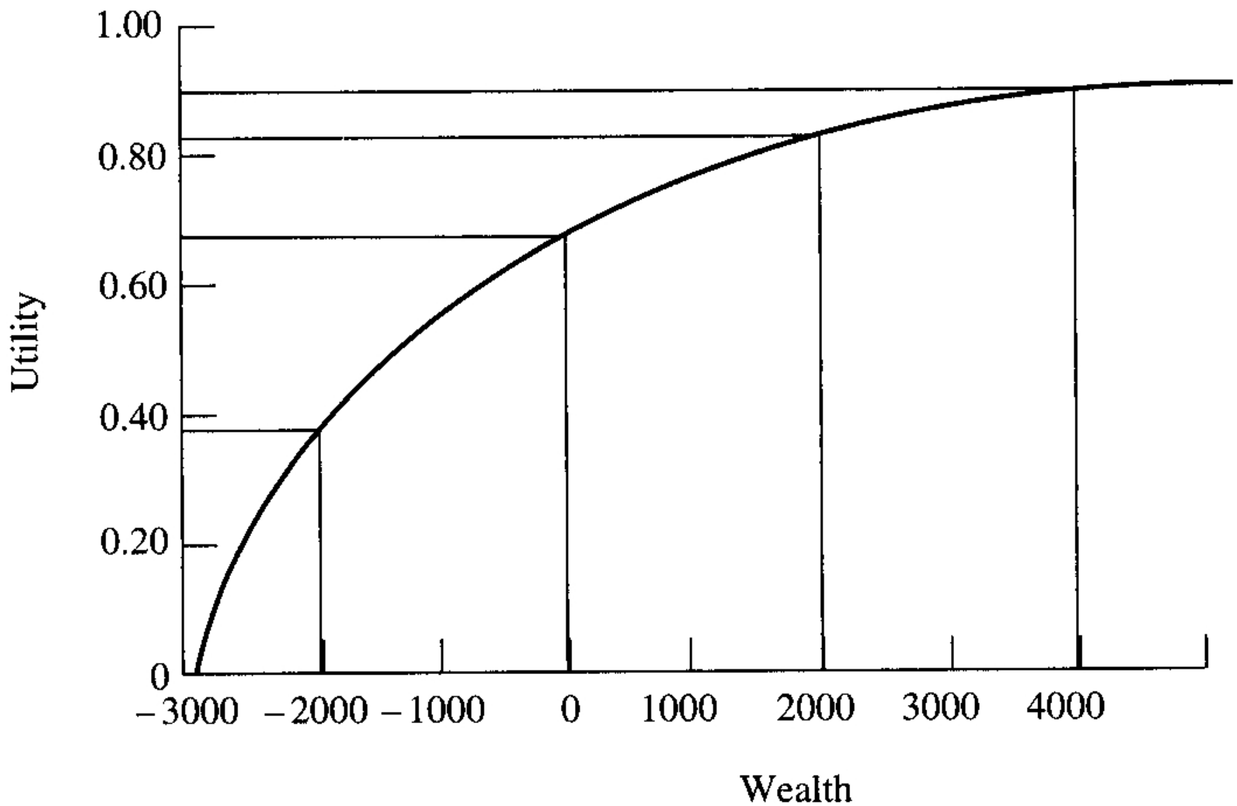
CONCLUSION: EMV alone is not enough for decision making. Risk is very important too

Figure 13.1

What is an Utility function?

- A way to translate dollars into “utility units”
- It should help choose between alternatives by maximizing the expected utility
- Typical shapes of utility function include log, and exponential

Risk-Averse Utility Function

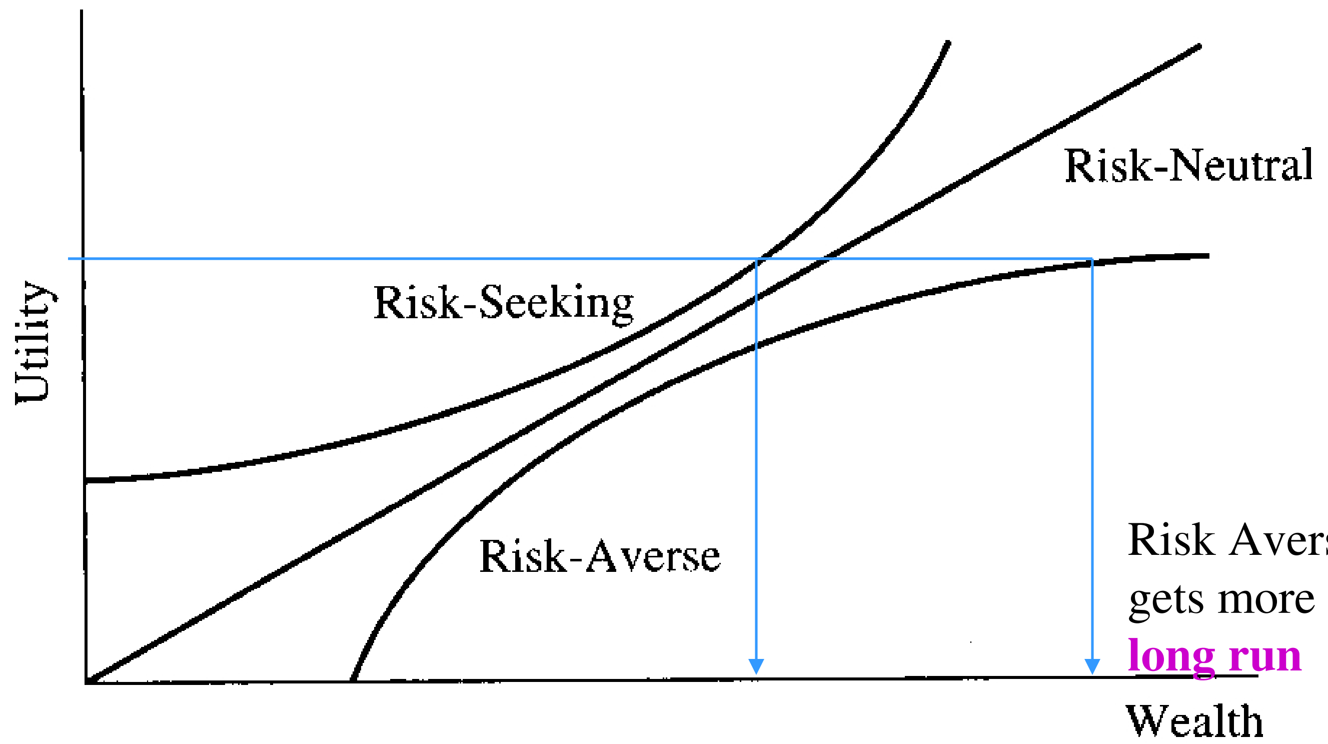


Note the Concave curve - this denotes Risk Averse - typical for most people

Risk averse person

- Imagine that you are gambling and you hit this situation
- Win $500 with prob 0.5 or lose $500 with prob 0.5
- A risk-seeking person will play the game but a risk averse person will try to trade in the gamble (try to leave the game) for a small penalty (example: pay $100 and quit).
- The EMV of the game is $0 and a risk averse person will trade in the gamble for an amount that is always less than the EMV value. In this case -$100 <$0

Different Risk Attitudes

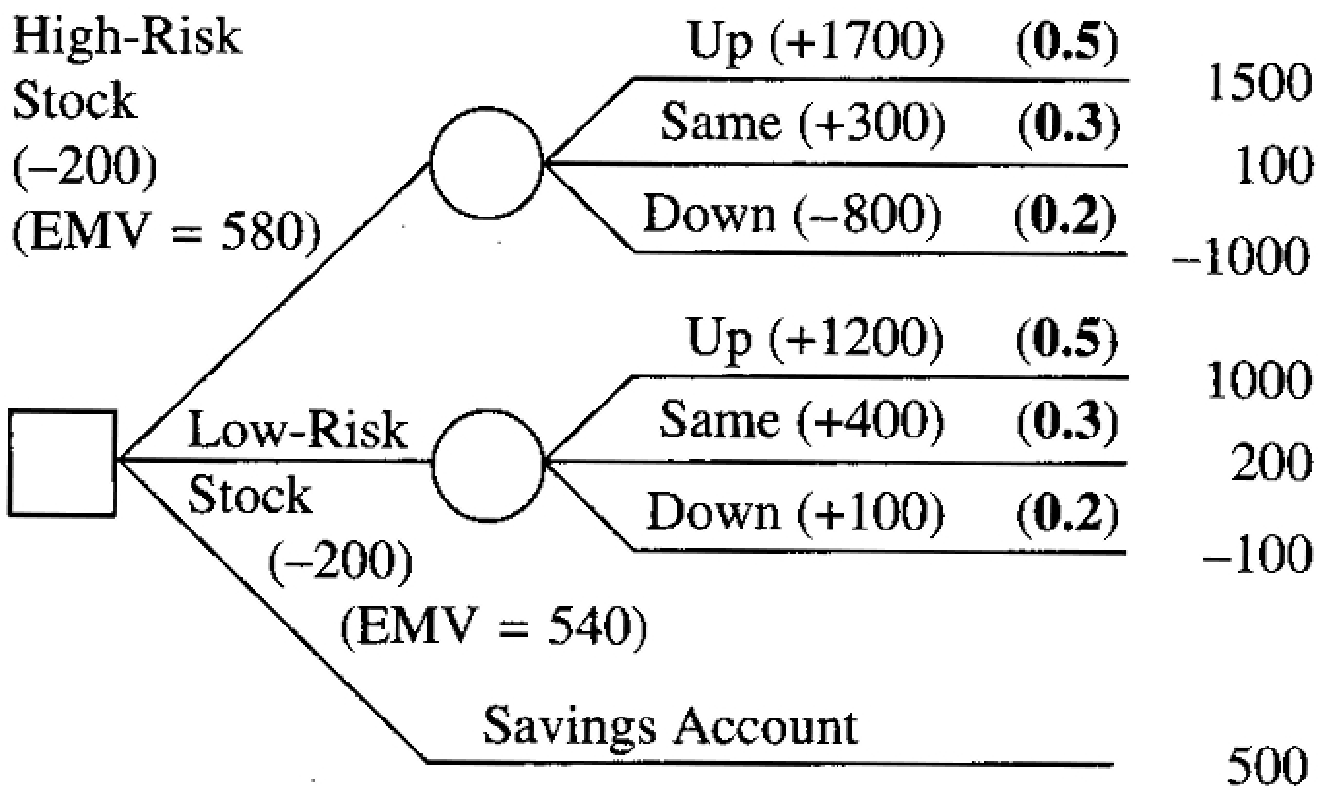


Ignores risk
Uses EMV

Risk Averse person gets more wealth **in the long run**

Different Risk Attitudes

Investing in the Stock Market



$200 brokerage fee

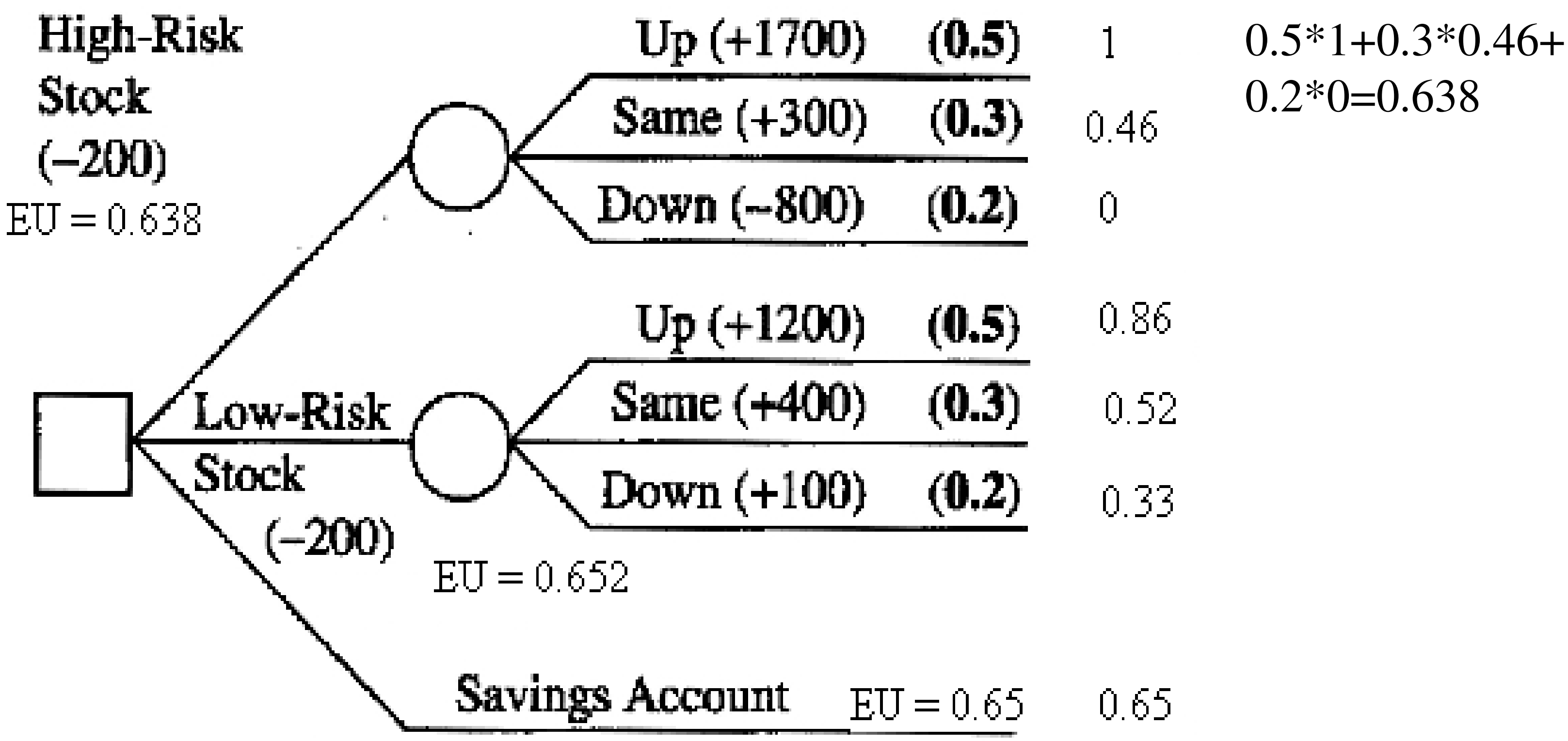
Solution to the decision tree is to invest in high-risk stock. Here risk is not incorporated

Utility function for investment

- Dollar value utility value

Dollar value	utility value
1500	1
1000	0.86
500	0.65
200	0.52
100	0.46
-100	0.33
-1000	0.00

Investing in the Stock Market



$200 brokerage fee

Solution to the decision tree is to invest in low-risk stock. Here risk is incorporated via utility function

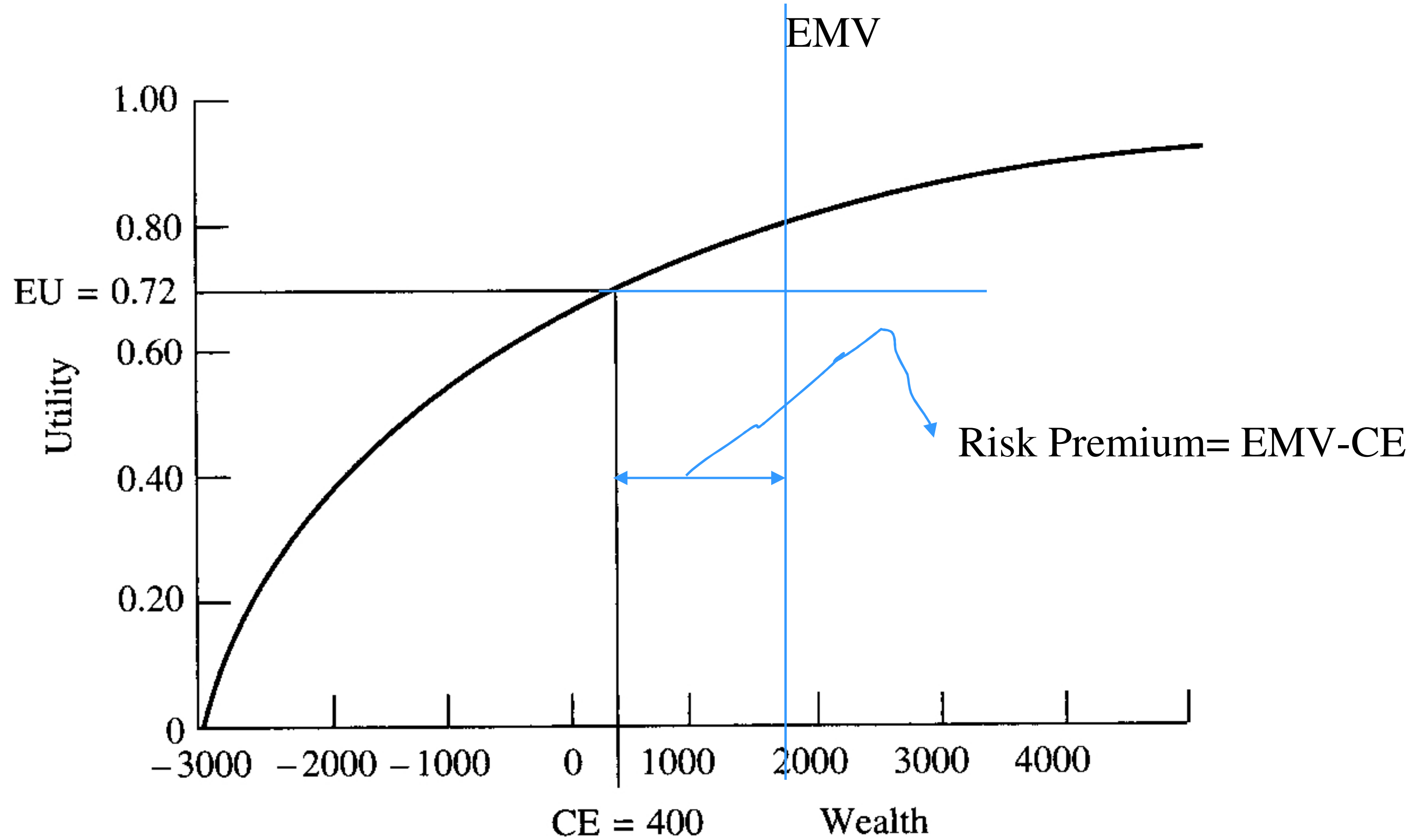
Definitions: Certainty Equivalents and Utility

- A *Certainty Equivalent* is the amount of money you think is equal to a situation that involves risk.
- The *Expected (Monetary) Value - EMV* - is the expected value (in dollars) of the risky proposition
- A Risk Premium is defined as:

  Risk Premium = EMV – Certainty Equivalent

- The *Expected Utility (EU)* of a risky proposition is equal to the expected value of the risks in terms of utilities, and *EU(Risk) = Utility(Certainty Equivalent)*

Finding a Certainty Equivalent given utility curve

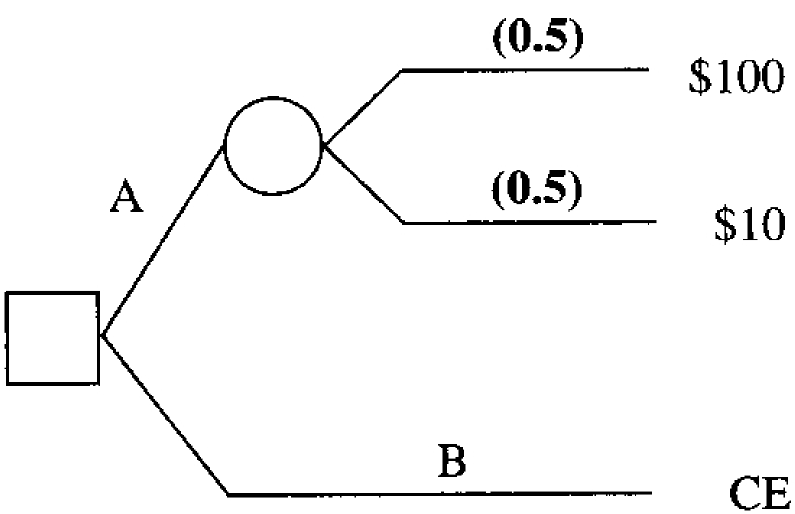


How to find the utility curve?

- Using Certainty Equivalent

Assessing Utility Using Certainty Equivalents

Lottery Example

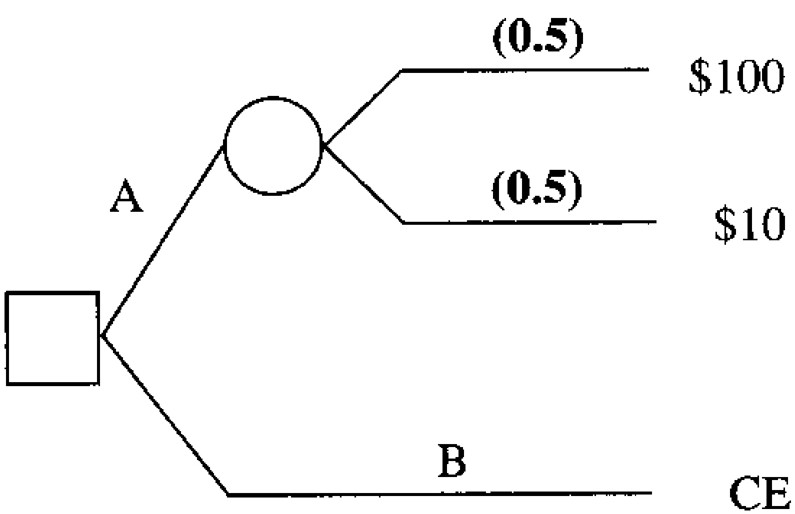


In a Reference Lottery, you can:

- Vary the probabilities
- Vary the payoffs associated with the risk
- Vary the Certainty Equivalent

In all cases, you must set all of the other values to find the one you want

Assessing Utility Using Certainty Equivalents

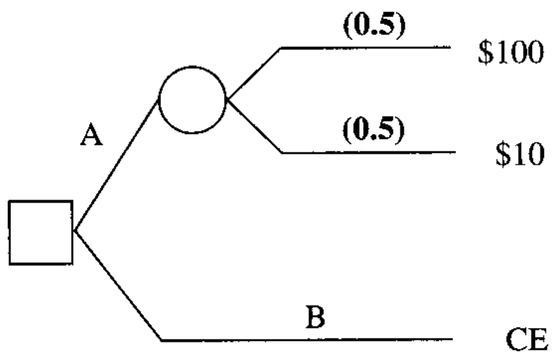


Let utility for $100 be 1 and for $10 be 0
The EMV is $55.

As a risk averse person you will not play the game and accept a value less than $55. Let that be $30 (this is a subjective value and can differ from person to person)

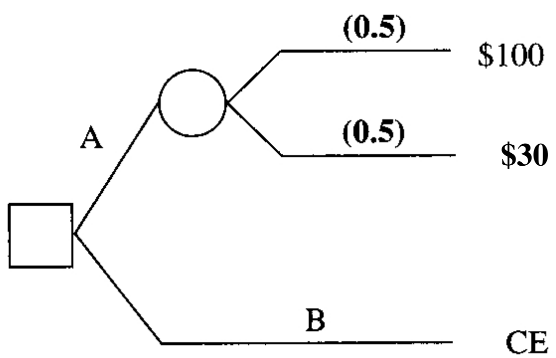
Make the
Expected Utility (EU) of option A = EU of option B
That is the options are indifferent in terms of EU

Eliciting a Utility Curve



CE = $30;
U($100) = 1 and U($10) = 0;
therefore,
U($30) =1*0.5+0*0.5= 0.5

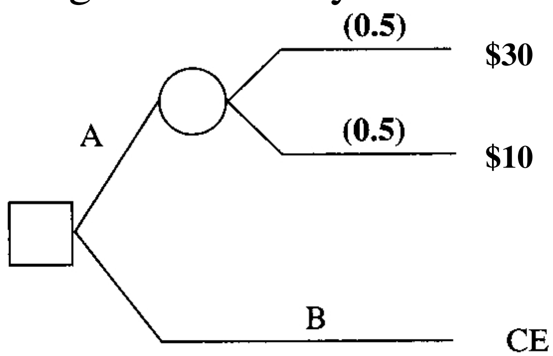
Replace dollar $10 with $30 and play a new gamble. Now the EMV is 100*.5+30*.5 =65
For what dollar value will you trade this gamble? Say $50 (again this is subjective)



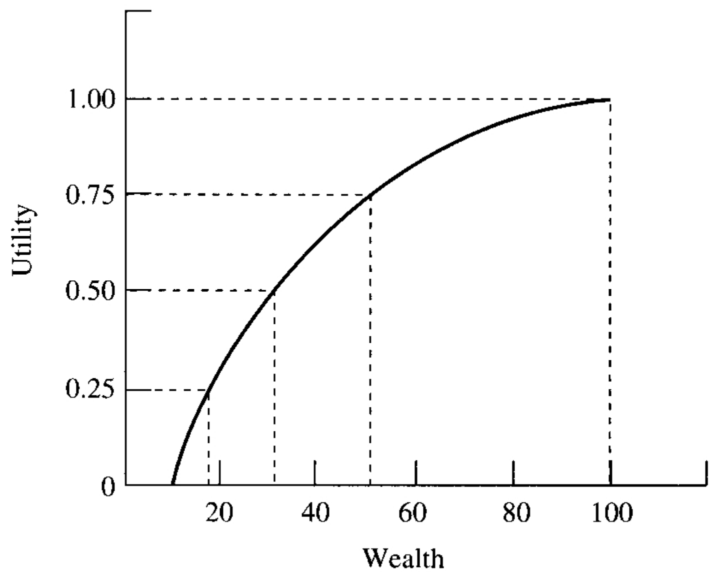
Make EU(A)=EU(B)
CE = $50;
U($100) = 1 and U($30) = 0.5;
therefore,
U($50) = 0.5(1) + 0.5(.5) = 0.75

Eliciting a Utility Curve (Cont.)

Repeat the process another time, say with \$10 and \$30. Find EMV. This is now \$20. Trade in the gamble for say \$18.



CE = \$18;
U(\$30) = 0.5 and U(\$10) = 0;
therefore,
U(\$18) = 0.5(.5) = 0.25



Plot Utility curve

Utility function matching the two assumed bounds (\$10 and \$100) and the three points that were elicited (the 25th, 50th, and 75th percentiles)

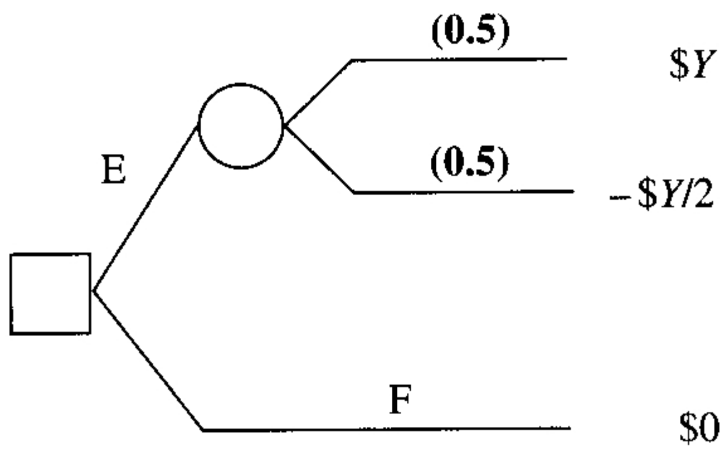
The Exponential Utility Function

- We assume the utility function can match an exponential curve

$$U(x) = 1 - e^{-x/R}$$

- *R* will affect the shape of the exponential curve, making it more or less concave ⇒ more or less risk averse, thus
- *R* is the *risk tolerance*
- There is an approximation that can be used to estimate the risk tolerance

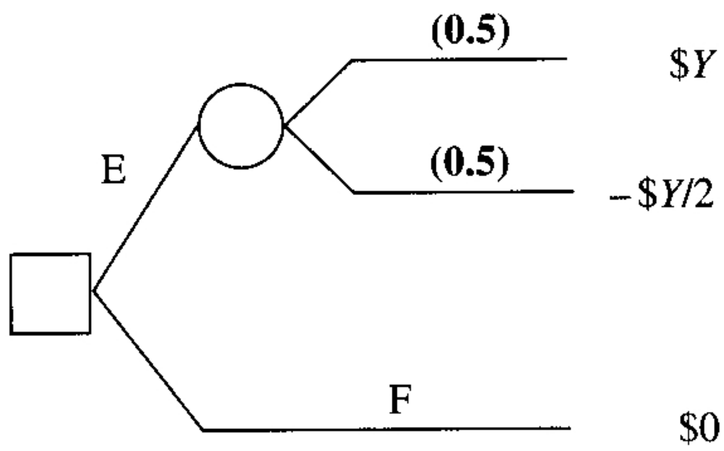
The Risk Assessment Lottery



$$U(x) = 1 - e^{-x/R}$$

- For what value of Y will you play the game? Or at what Y soes the game become risky?
- The highest value of Y at which you are willing to take the lottery (bet) [instead of remaining with $0 loss or gain] is approximately equal to *R*

The Risk Assessment Lottery



$$U(x) = 1 - e^{-x/R}$$

- Choose Y to be $900 (again subjective)
- The highest value of Y at which you are willing to take the lottery (bet) [instead of remaining with $0 loss or gain] is approximately equal to *R*.. *Hence R =Y = 900*

The Risk Assessment Lottery

- Choose Y to be $900 (again subjective)
- The highest value of Y at which you are willing to take the lottery (bet) [instead of remaining with $0 loss or gain] is approximately equal to *R.. Hence R =Y = 900*
- *Using the above data find EU and CE for this game*
  - Win $2000 with prob 0.4
  - Win $1000 with prob 0.4
  - Win $500 with prob 0.2

Plug in R=900 in $U(x) = 1 - e^{-x/R}$

Find U(x) for x = 2000, 1000, and 500

The Risk Assessment Lottery

- *Using the above data find EU and CE for this game*
  - Win $2000 with prob 0.4
  - Win $1000 with prob 0.4
  - Win $500 with prob 0.2

  Plug in R=900 in $U(x) = 1 - e^{-x/R}$

  Find U(x) for x = 2000, 1000, and 500

EU = 0.4 U(2000) + 0.4U(1000) + 0.2 U(500) = 0.7102

To find CE 0.7012 = 1- e $^{(-x/900)}$. $CE := -R \cdot (\ln(1 - .7102))$

Solve for x, which is $1114.71

Summary: This CE is calculated using exponential Utility Function

Alternate CE Calculations using Expected value and Variance of Payoffs.

From page 544 of Clemens, we have a lottery with payoffs $x := \begin{bmatrix} 2000 \\ 1000 \\ 500 \end{bmatrix}$

and probabilities $p := \begin{bmatrix} .4 \\ .4 \\ .2 \end{bmatrix}$. The Risk Tolerance R is assumed to be \$ 900.

Using either method, you must compute $\mu = \sum_i p_i \cdot x_i = 1300$ and

$Var := \sum_i p_i \cdot \left(x_i - \mu\right)^2$ to get $\sigma = 600$ So we have alternative calculations:

$$CE := 1300 - \frac{0.5 \cdot Var}{R} \quad \text{or} \quad CE = 1100$$

With calculators that have ln functions or Excel, I think that the more precise answer is about as easy to calculate.