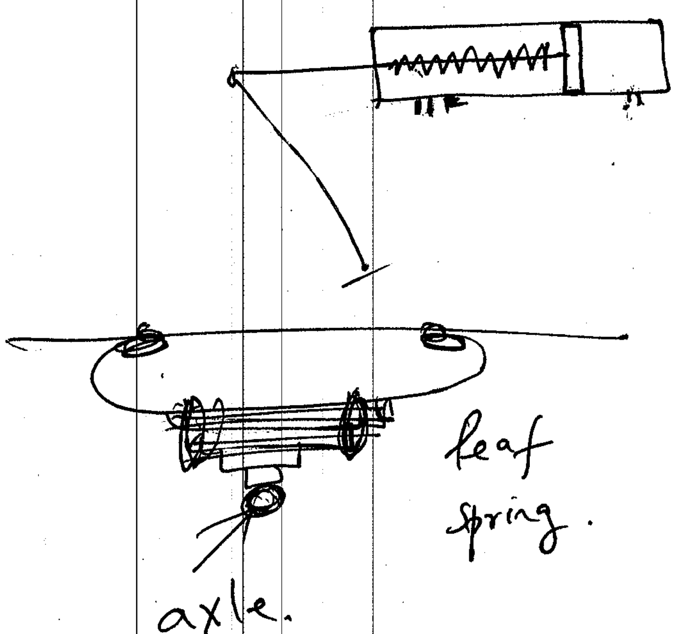
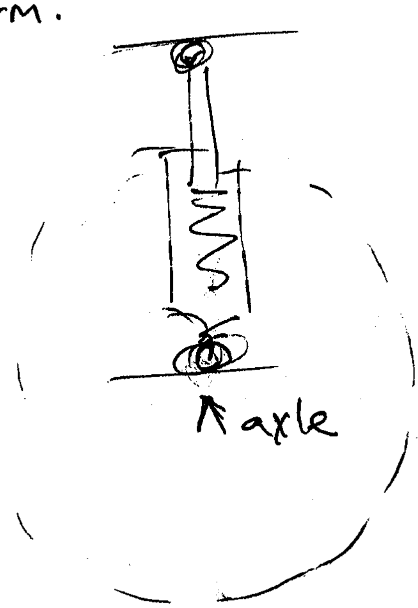
Chapter - 4 - springs & Dampers.

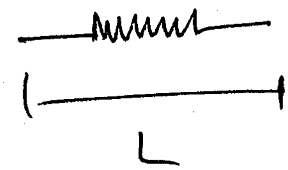
- Bodies that deform.





Compression Springs
Extension springs.

Free length of a spring. L

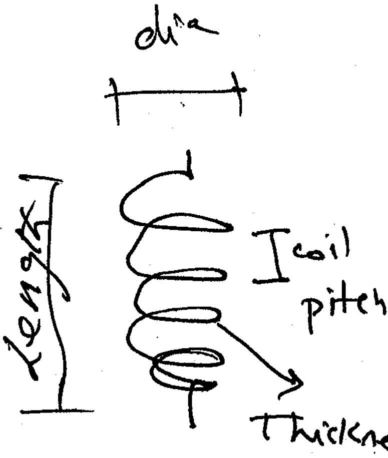


No load on the spring.

Length when manufactured.

Spring Constant k (N/m)

For coil springs | k depends on. Material
diameter of coil.
Thickness of spring.
Length, coil pitch



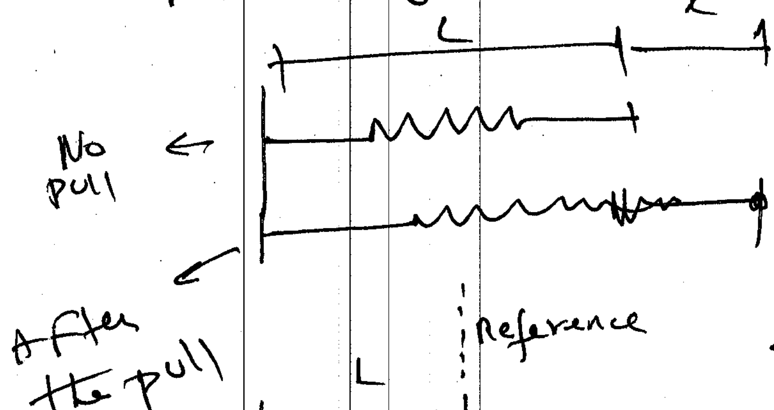
(2)

k is fixed for a spring as soon as it is manufactured.

Spring reaction force $= kx$

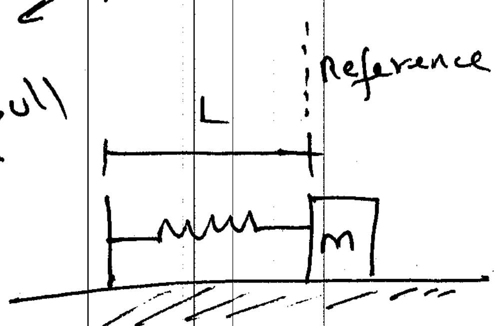
$x \rightarrow$ displacement of the spring.

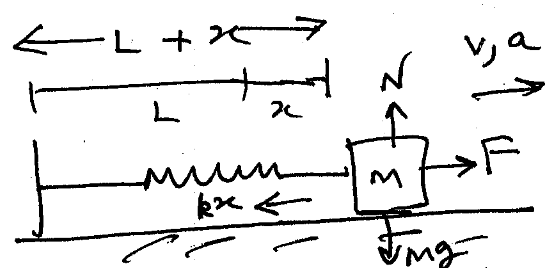
This force will oppose any applied force.



$kx = 0 \rightarrow$ Free length

$kx \neq 0$





$x = 0$

~~$F > kx$~~

No friction.

$F > kx$

$F - kx = ma$

$N = mg$

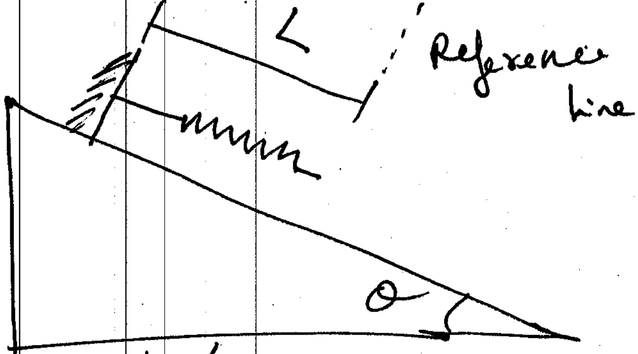
$F_f = \mu N$

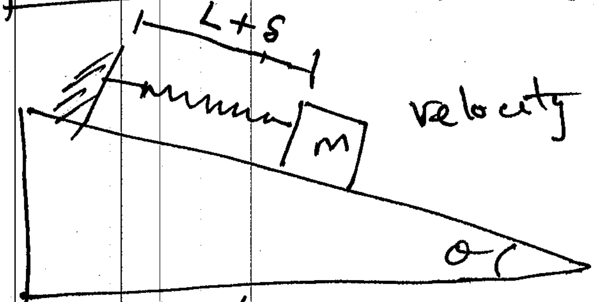
$= \mu mg$

with friction F_f

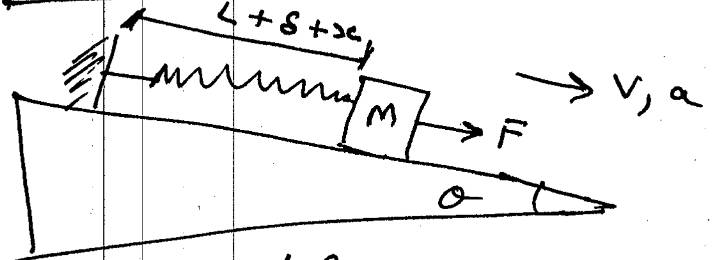
$F > kx + F_f$

$F - kx - F_f = ma$



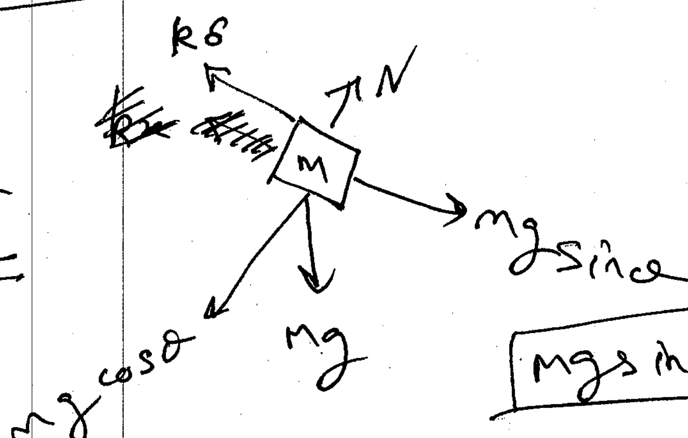


velocity after sliding δ is 0



[F causes x displacement]

No F



[mg sinθ causes δ displacement]

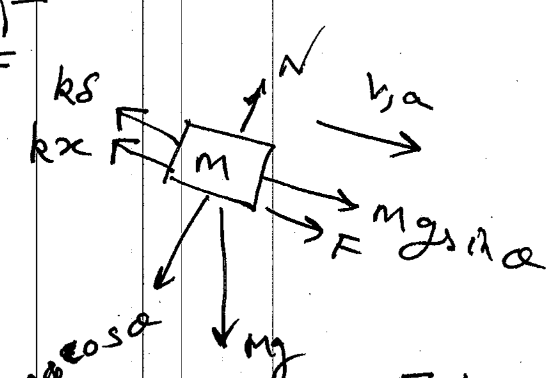
$$\boxed{mg\sin\theta = k\delta}$$

$$mg\cos\theta = N$$

<u>No friction</u>

Velocity = 0

With F



<u>No friction</u>

$$F + mg\sin\theta > k\delta + kx$$

$$F + \cancel{mg\sin\theta} - \cancel{k\delta} - kx = ma.$$

$$F - kx = ma$$

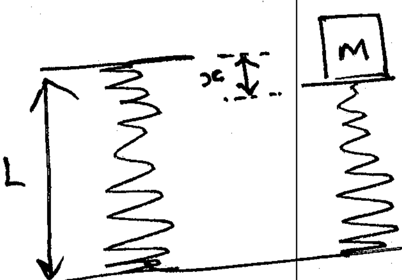
$$N = mg\cos\theta$$

with Friction

$$F - kx - F_f = ma$$

$$F_f = \mu N = \mu mg\cos\theta$$

Compression Springs



→ Compressed system comes to rest after compression by x

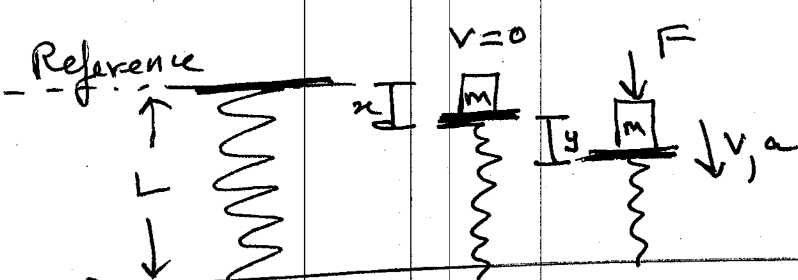
$L \rightarrow$ Free length

$mg \rightarrow$ causes the compressed.

$mg = kx$ when the system comes to rest

Load mg = reaction force by the spring.

This is a static system. $\underset{\downarrow}{mg}$ $\underset{\uparrow}{kx}$



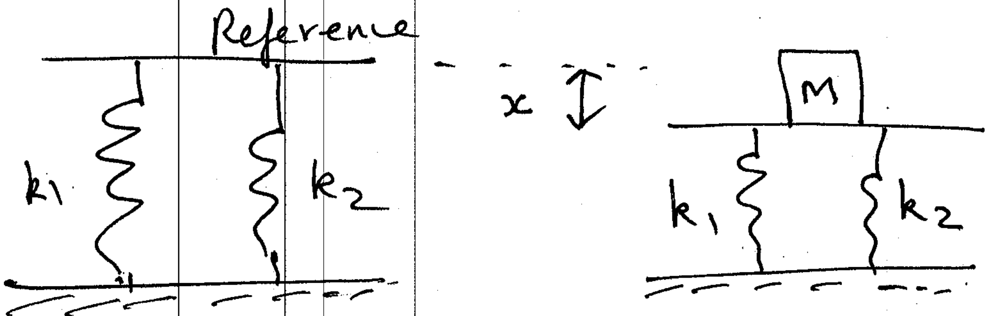
$mg = kx \rightarrow$ static

$F + \cancel{mg} > \cancel{kx} + ky$

$F - ky = ma$

y is due to F

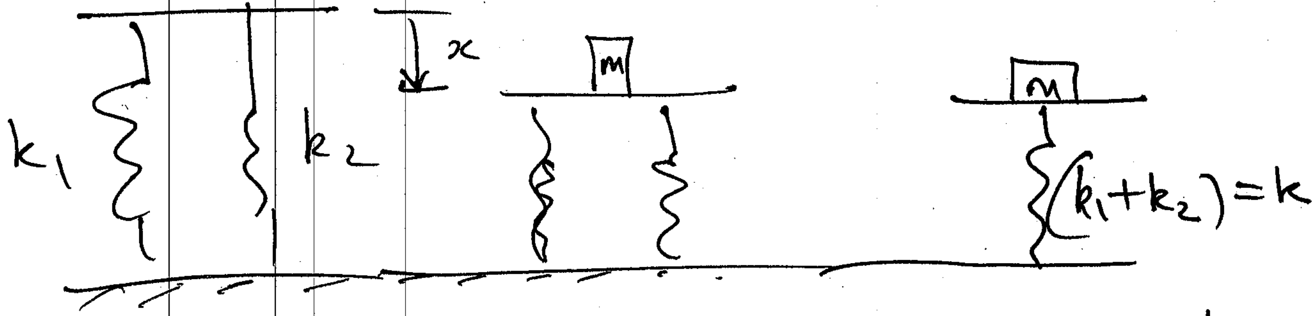
x is due to mg



Static (platform will come to rest after x displacement).

$$mg = k_1 x + k_2 x$$

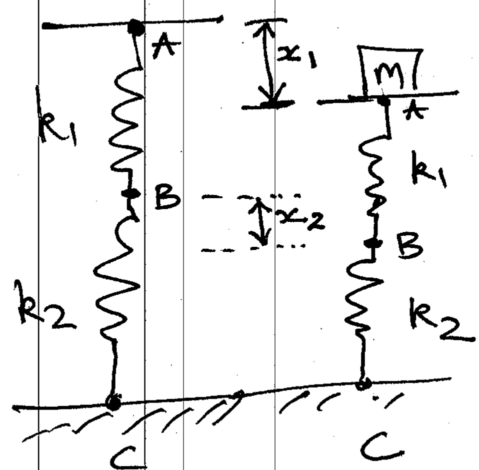
$$mg = \underbrace{(k_1 + k_2)}_{\text{spring constants get added}} x$$



$$mg = k_1 x + k_2 x \qquad mg = kx$$

$$mg = \underbrace{(k_1+k_2)}_{\text{Sp constants add up}}\, x$$

BC → free length of k_2
AB → free length of k_1



Spring k_2 was displaced (compressed) by x_2

Spring k_1 was displaced by $(x_1 - x_2)$

static

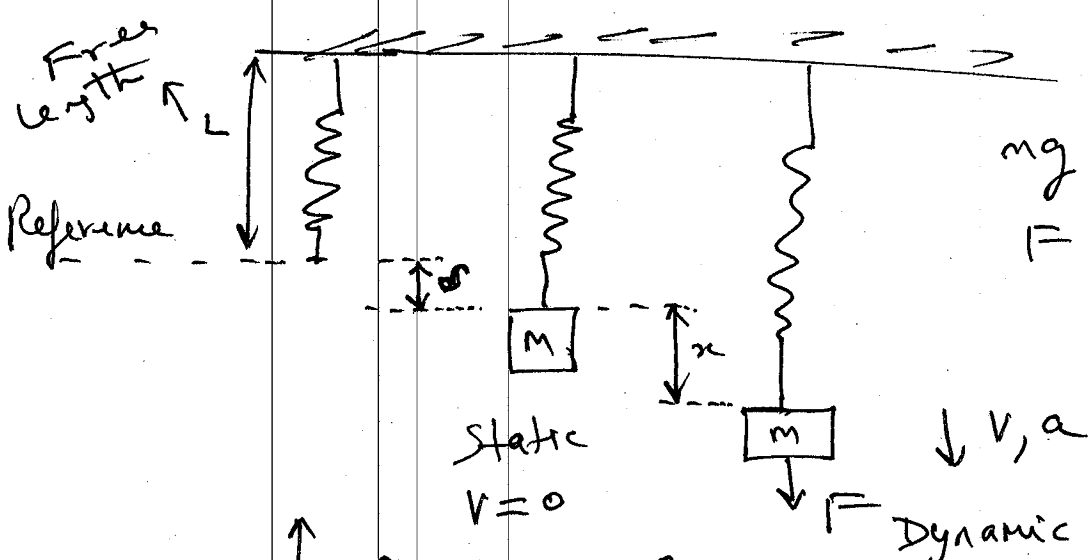
$$mg = k_1(x_1 - x_2) + k_2 x_2$$

$$mg = k_1 x_1 - k_1 x_2 + k_2 x_2$$

$$mg = k_1 x_1 + x_2 (k_2 - k_1)$$

$$\frac{mg}{k_1} = x_1 + x_2 \underbrace{\boxed{\frac{(k_2 - k_1)}{k_1}}}_{ke}$$

Extension springs



mg causes δ

F causes x

$\uparrow k\delta$

$\downarrow mg$

$Mg = k\delta$

$F + \cancel{mg} > \cancel{k\delta} + kx$

$F > kx$

$F - kx = ma$

If Force is released then the mass m will oscillate.

What is the frequency of oscillation?

$F = 0$ (it is released)

$-kx = ma$

Divide by m

$ma + kx = 0$

$a + \frac{k}{m}x = 0$

$\ddot{x} + \frac{k}{m}x = 0$

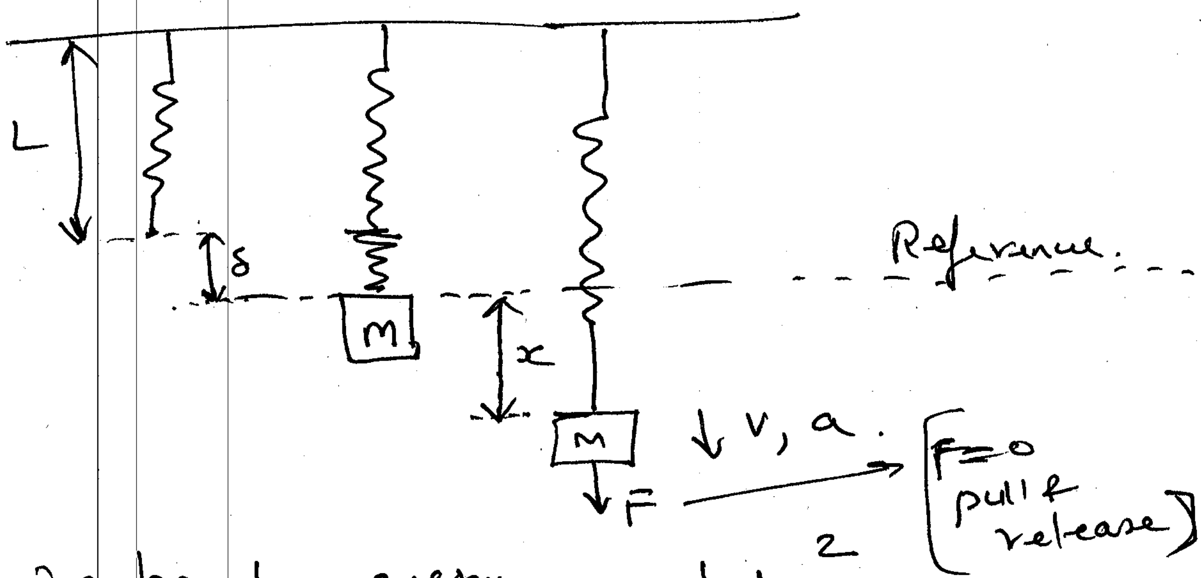
$\ddot{x} + \omega^2 x = 0$

ω = Frequency of oscillation

$\omega = \sqrt{\frac{k}{m}}$

$\boxed{\omega \text{ is independent of } x}$

Energy method



measured after x displacement

Spring kinetic energy $= \frac{1}{2} k x^2$

mass kinetic energy $= \frac{1}{2} m v^2$

Potential energy $= mg \times 0$ because $x = 0$ at the Reference line

Laws of conservation of energy.

$$mg \times 0 = \frac{1}{2} k x^2 + \frac{1}{2} m v^2$$

$\boxed{\text{At reference line } KE = 0}$

Energy At Reference = Energy at x while in motion

(PE + KE) at reference = (PE + KE) at x

$$0 + 0 = 0 + \frac{1}{2} k x^2 + \frac{1}{2} m v^2$$

$\frac{1}{2} kx^2 + \frac{1}{2} mv^2 = 0$

$\frac{1}{2} kx^2 + \frac{1}{2} m(\dot{x})^2 = 0$

Take derrivative w.r.t t

$\frac{1}{\cancel{2}} k \cancel{2} x \cancel{\dot{x}} + \frac{1}{\cancel{2}} m \cancel{2} \cancel{\dot{x}} \ddot{x} = 0$

$kx + m\ddot{x} = 0$

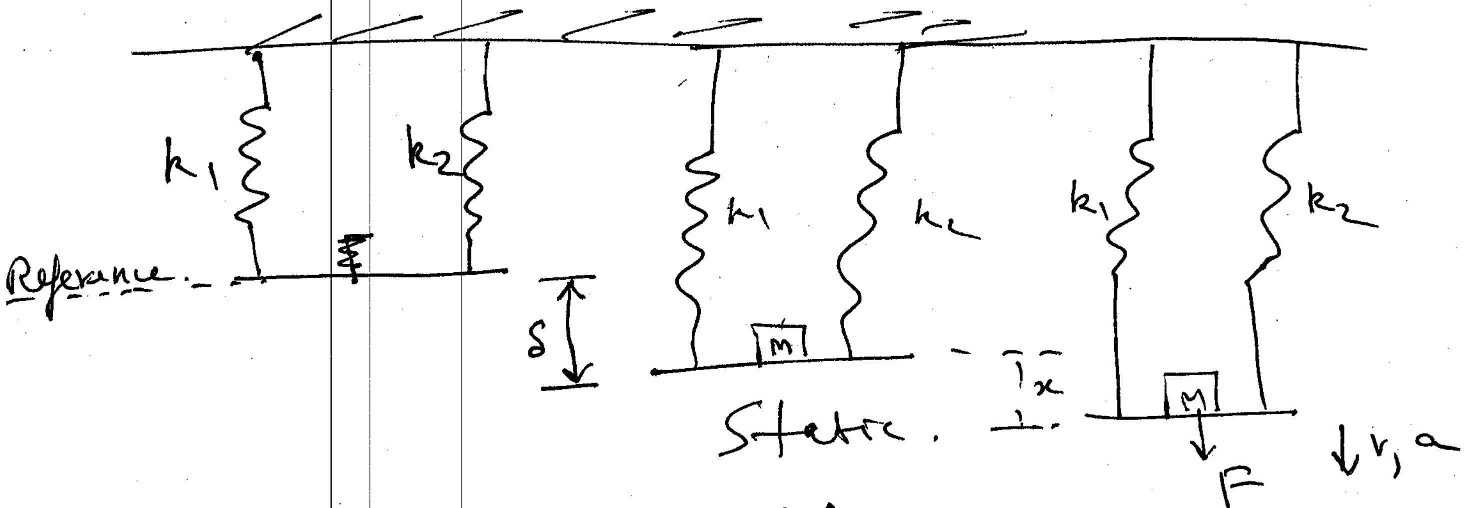
Divide by m

$\ddot{x} + \frac{kx}{m} = 0$

$\ddot{x} + \omega^2 x = 0$

$\omega = \sqrt{\frac{k}{m}}$

Extension springs.



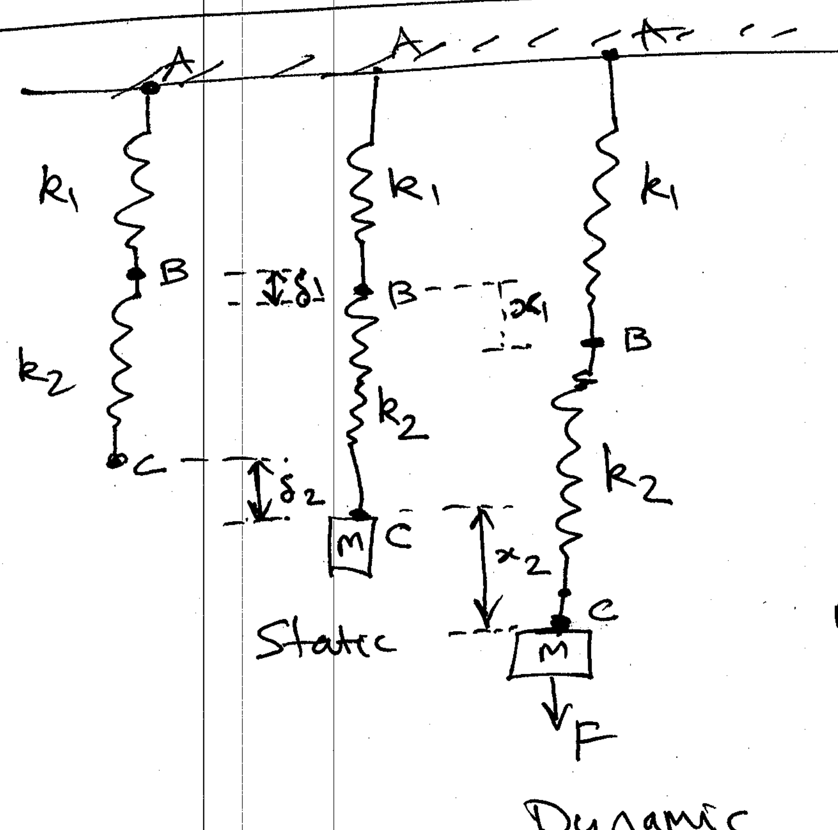
$mg \downarrow \quad k_1\delta \uparrow \quad k_2\delta \uparrow$

$$mg = k_1\delta + k_2\delta = (k_1+k_2)\delta$$

$$F + \cancel{mg} > \cancel{k_1\delta + k_2\delta} + k_1x + k_2x$$

$$F > (k_1+k_2)x$$

$$F - (k_1+k_2)x = ma$$



Static

Displacement for Spring $k_1 = \delta_1$

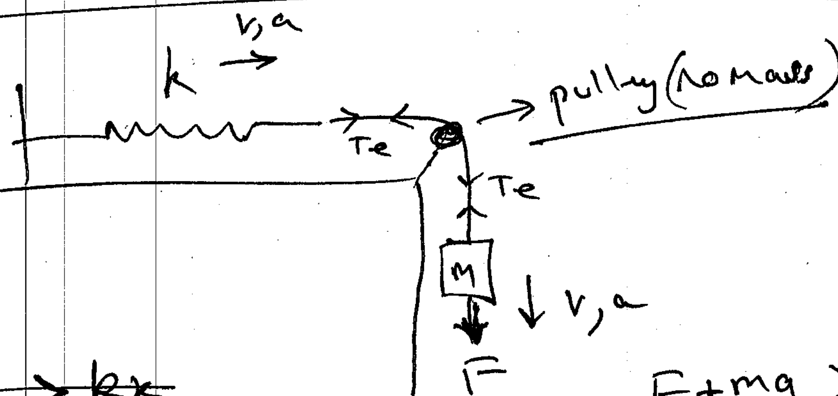
Displacement for spring $k_2 = \delta_2 - \delta_1$

$$mg = k_1\delta_1 + k_2(\delta_2 - \delta_1)$$

Static

$$F + \cancel{mg} > \cancel{k_1\delta_1 + k_2(\delta_2 - \delta_1)} + k_1 x_1 + k_2(x_2 - x_1)$$

$$F - k_1 x_1 - k_2(x_2 - x_1) = ma$$



$$\cancel{T_e > kx}$$

$$\boxed{T_e = kx + k\delta}$$

$$F + mg > T_e$$

$$F + mg - T_e = ma$$

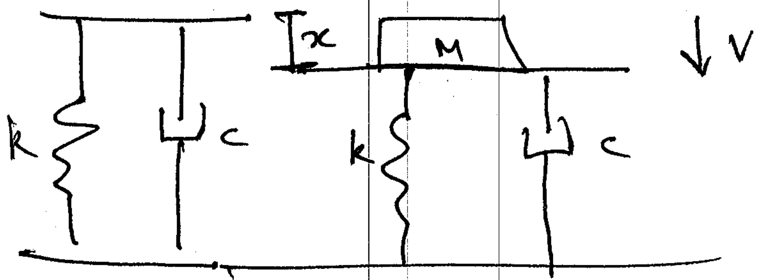
Suppose There is No Force $F = 0$

$$mg = k\delta$$

$$F + mg - (kx + k\delta) = ma$$

$$F + \cancel{mg} - kx - \cancel{k\delta} = ma$$

$$F - kx = ma$$

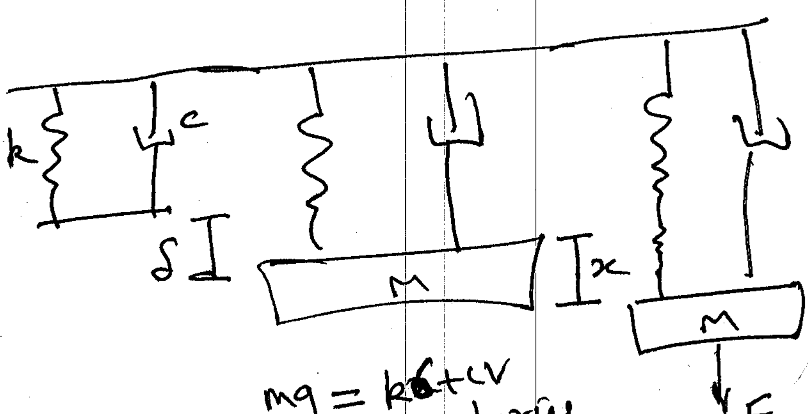


Compression

Static.

$mg = kx + \underbrace{cv}_{\text{during motion}}$

After stopping $mg = kx$. because $V = 0$



Extension.

v, a.

$mg = k\delta + cv$ during motion

$mg = k\delta$ after coming to a stop (static)

$V = 0$

with F it is now dynamic.

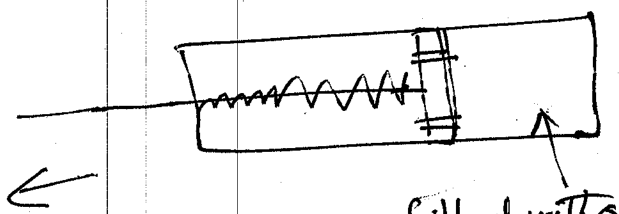
$\cancel{mg} + F > \cancel{k\delta} + kx + cv$

$F - kx - cv = ma.$

$F - kx - c\dot{x} = m\ddot{x}$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{F}{m}$$

Dampers.

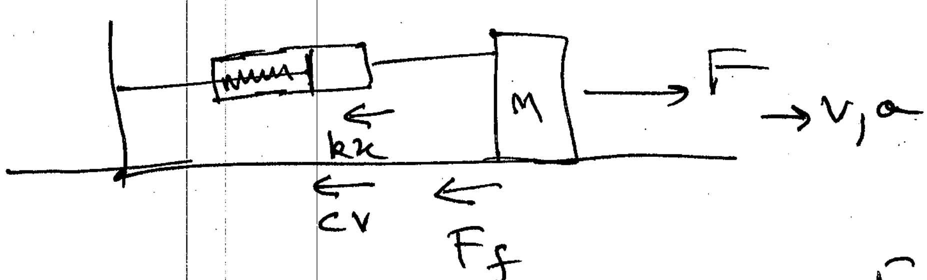


Reaction force of a Damper $= cv$

$c \rightarrow$ Damper coefficient

$v \rightarrow$ velocity

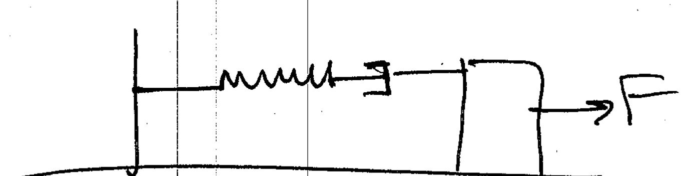
$c = \frac{Ns}{m} \rightarrow$ unit.

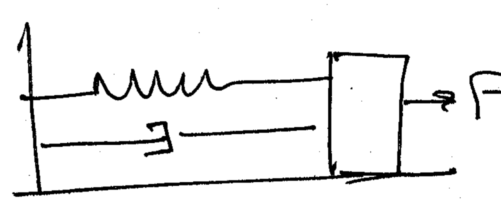


$F_f \rightarrow$ Frictional force.

$F > kx + cv + F_f$

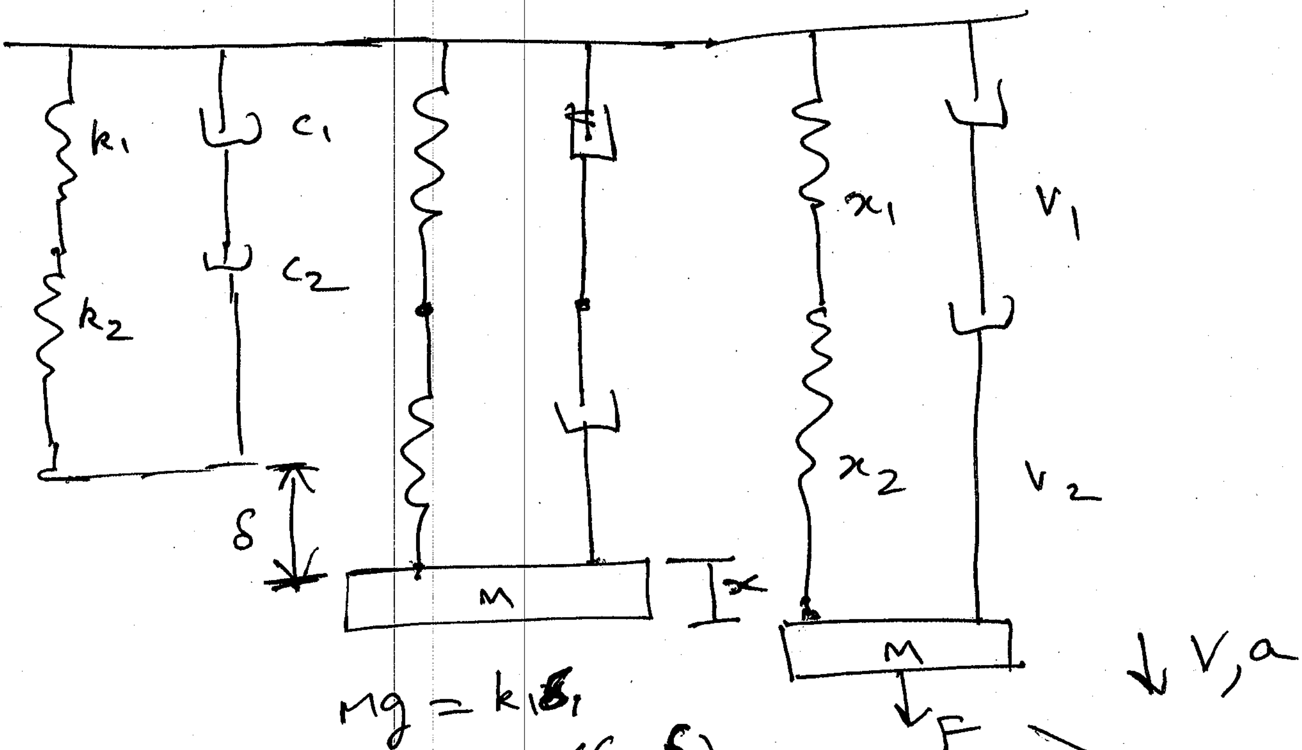
$F - kx - cv - F_f = ma$





$\rightarrow$ Damper

$\rightarrow$ spring.



$$Mg = k_1\delta_1 + k_2(\delta_2 - \delta_1)$$

True extension of spring k_2

Static (System comes to a stop)

$$F > k_1x_1 + k_2(x_2 - x_1) + c_1V_1 + c_2(V_2 - V_1)$$

True Displacement of k_2

True velocity of c_2

$$F - k_1x_1 - k_2(x_2 - x_1) - c_1V_1 - c_2(V_2 - V_1) = ma$$