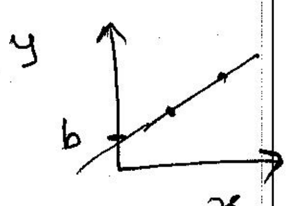
Systems

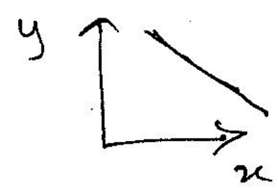
Static | Dynamic (time plays a role)

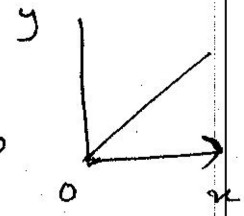
Dynamic: Discrete | Continuous.

Chap 1 modeling using Data (statistics)

3 models
- linear
- Exponential.
- Power.







$$\boxed{y = mx + b}$$

$b = 0$

m = slope

independent variable x y dependent variable

x_1 x_2 x_3 y

$y = f(x)$

$y = f(x_1, x_2, x_3)$

x_1 x_2 x_3 | y_1 y_2 ...

Given 2 points find the equation of the line.

$(x, y) = (2, 4) = (x_1, y_1)$

$(4, 8) = (x_2, y_2)$

$y_1 = mx_1 + b$

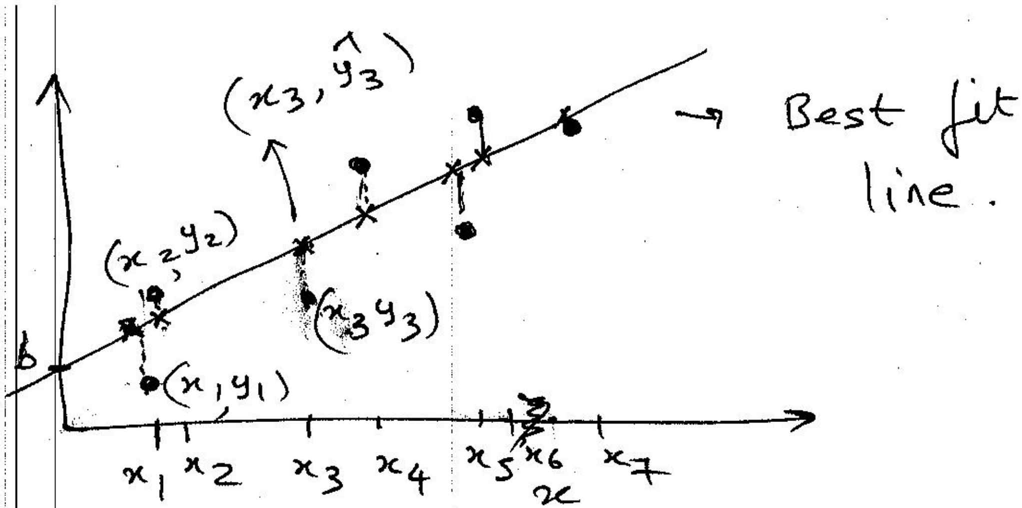
$y_2 = mx_2 + b$

$4 = 2m + b$

$8 = 4m + b$

$$\boxed{\begin{matrix} m = 2 \\ b = 0 \end{matrix}}$$

$$\boxed{y = 2x + 0}$$



hat → $\hat{y} = mx + b$

(Estimated y)

y − True y

$\hat{y}$ − Estimated y

$$J = \boxed{\text{Minimize} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2}$$

~~Total~~ sum of squared error

$$\frac{\partial J}{\partial m} = 0 \quad \rightarrow \quad m \sum x_i^2 + b \sum x_i = \sum x_i y_i$$

$$\frac{\partial J}{\partial b} = 0 \quad \rightarrow \quad m \sum x_i + nb = \sum y_i$$

n = # of data points.

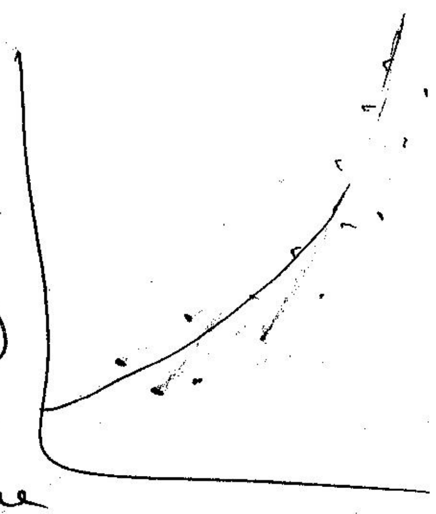
x	y	x^2	xy		$\hat{y}$	$y-\hat{y}$	$(y-\hat{y})^2$	$(y-\bar{y})$	$(y-\bar{y})^2$
				Fill the line $\hat{y} = mx+b$ $\boxed{\bar{y} = \frac{\sum y}{n}}$					
Σx	Σy	Σx^2	Σxy				Σ		Σ

Goodness of fit Test

$$R^2 = 1 - \left[\frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2}\right]$$

$\bar{y} \rightarrow$ average or mean of y

coefficient of Determination



$$y_i - \hat{y}_i = (y_i - \bar{y}) - (\hat{y}_i - \bar{y})$$

$$\underbrace{(y_i - \bar{y})}_{\text{Total error}} = \underbrace{(y_i - \hat{y}_i)}_{\text{estimated error}} + \underbrace{(\hat{y}_i - \bar{y})}_{\text{Error due to regressors } (x)}$$

$\boxed{R^2 \text{ high means good fit}}$ $\quad 0 \le R^2 \le 1$

$\Sigma x = 2.96$

$\Sigma y = 35.4$

$\Sigma xy = 34.57$

$\Sigma x^2 = 4.041$

$4.041m + 2.96b = 34.57$

$2.96m + 4b = 35.4$

$$\underbrace{\begin{bmatrix} 4.041 & 2.96 \\ 2.96 & 4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} m \\ b \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 34.57 \\ 35.4 \end{bmatrix}}_{B}$$

$AX = B$

$X = A^{-1}B$

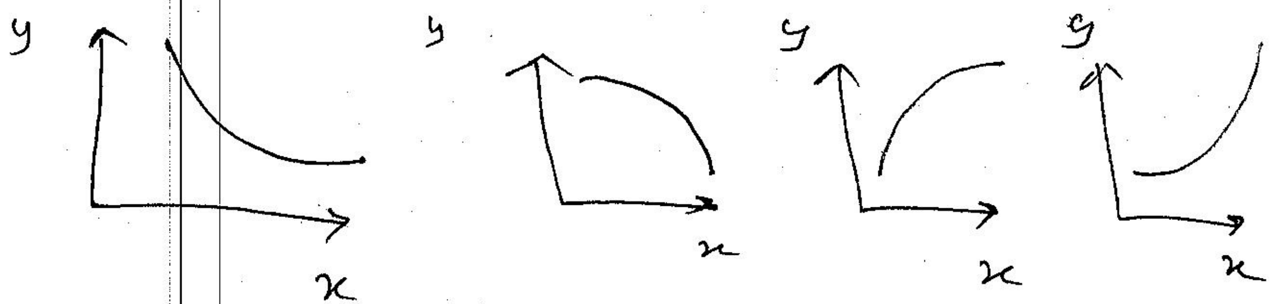
$$\boxed{\hat{y} = 4.52x + 5.50}$$

$$R^2 = 1 - \left[\frac{1.5}{39.4}\right]$$

$$\boxed{R^2 = 0.96}$$

Linear | $\hat{y} = mx + b$

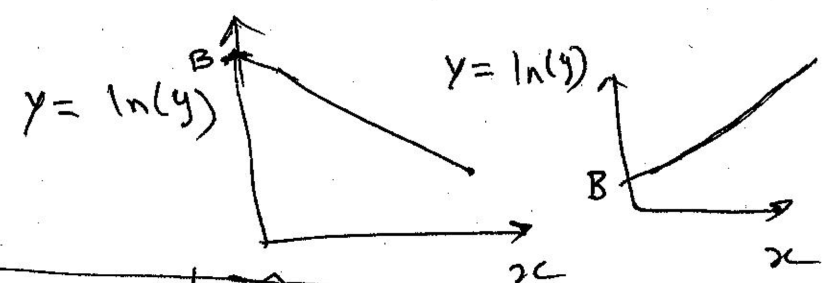
Exponential $\hat{y} = be^{mx}$ $b \neq 0$



Exponential Models <u>do not</u> pass through the origin.

$$\ln(y) = \ln(b) + mx$$

$$Y = B + mx \rightarrow \text{straight line.}$$



<u>Expo-Table</u>

x	y	Y: $\ln(y)$	x^2	xY		$\hat{y}$	$(y-\hat{y})^2$	$(y-\bar{y})^2$
					Find m, B. Also find $b = e^B$			
Σx	Σy	ΣY	Σx^2	ΣxY			Σ	Σ

Find R^2

Power

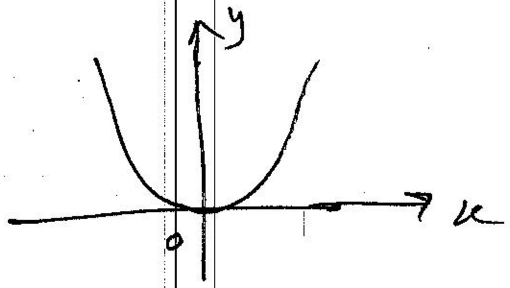
linear $\hat{y} = mx + b$

Expo $\hat{y} = be^{mx}$

$$\hat{y} = b x^{m} \quad \underline{\underline{b \neq 0}}$$

$$y = x^2 \quad \begin{array}{l} b = 1 \\ m = 2 \end{array}$$

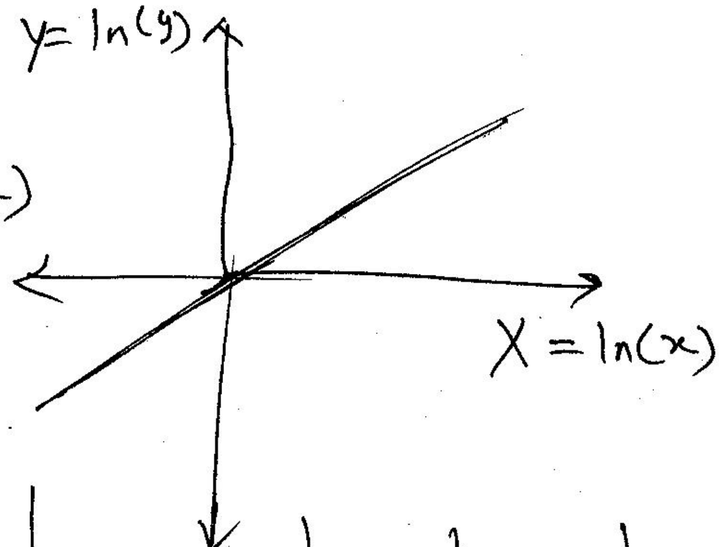
Power models go through the origin



$$\ln(y) = \ln(b) + m\ln(x)$$

$$Y = B + mX$$

$\underline{\underline{\text{St line}}}$



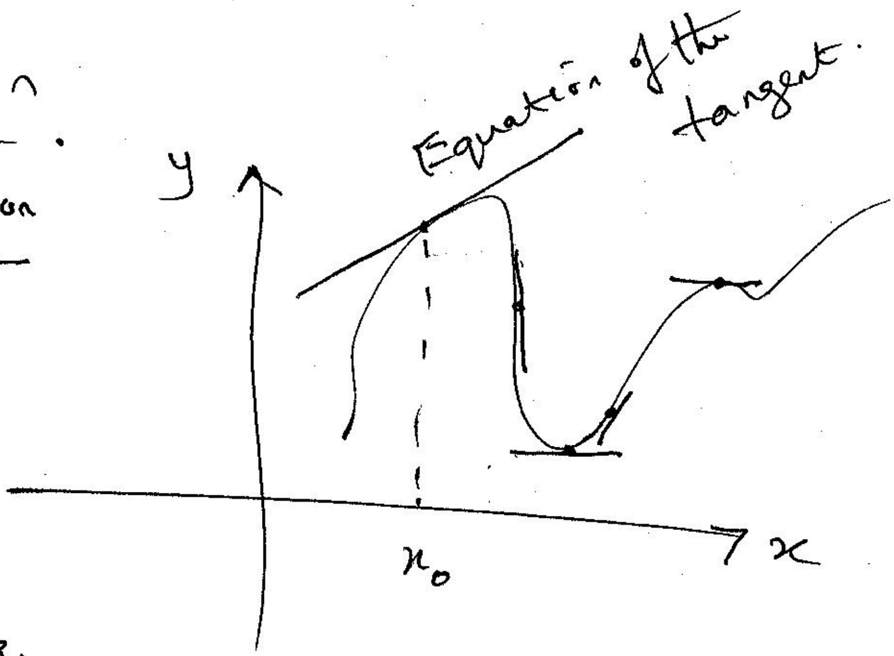
x	y	X $\ln(x)$	Y $\ln(y)$	XY	X^2		$\hat{y}$	$(y-\hat{y})^2$	$(y-\bar{y})^2$
						Solve for m, B Find $b = e^B$			
		ΣX	ΣY	ΣXY	ΣX^2			Σ	Σ

$$\bar{y} = \frac{\Sigma y_i}{n}$$

$n \rightarrow$ # of datapoints.

Linearization

Linear approximation



Taylor series.

$$y = f(x) = f(x_0) + f'(x_0)(x - x_0) + \dots$$

First derivative of y at x_0 gives the slope of the tangent at x_0

$y = f(x) = x^2 - 3x + 5$ Find the linear approx at $x_0 = 1$

$$\frac{dy}{dx} = f'(x) = 2x - 3$$

$$f'(x_0) = f'(1) = 2 - 3 = -1 \text{ (slope)}$$

$$f(x_0) = f(1) = 1 - 3 + 5 = 3$$

$$f(x) = 3 + (-1)(x - 1)$$

$$f(x) = 3 - x + 1 = 4 - x$$

Eqn of tanget at x_0