Nuclear Power

Basic Science

Chapter 3

3.1 INTRODUCTION

Some books on renewable energy do not go into the subject of nuclear energy, but it is important to include it based on the need to compare renewable energy technologies with the other available energy sources including nuclear. Moreover, a case can be made that nuclear is in fact a form of renewable energy. In this first of two chapters on nuclear energy we consider the basic science, an understanding of which is essential to the technological issues considered in the following chapter. The chapter begins with an historical overview, and then proceeds with the development of the basic science needed to understand nuclear energy; it also delves into a consideration of nuclear radiation, including its effects on humans.

3.2 EARLY YEARS

As with any new science, the early years of nuclear science were a period of confusion and accidental discovery. Although there were important contributions by many pioneers, we here highlight those by three individuals: Henri Becquerel, Marie Curie, and especially Ernest Rutherford. Antoine Henri Becquerel (who along with Marie and Pierre Curie was awarded the Nobel Prize in Physics in 1903) is generally acknowledged to be the discoverer of radioactivity. Becquerel's discovery was entirely accidental and occurred one day in 1896, while investigating phosphorescence in uranium salts (Becquerel, 1896). He happened to have placed some uranium salt above some photographic plates that were wrapped in very thick black paper to prevent light exposure. Becquerel found that the plates became fogged nevertheless. He also noted that

> If one places between the phosphorescent substance and the paper a piece of money or a metal screen pierced with a cut-out design, one sees the image of these objects appear on the negative. … One must conclude from these experiments that the phosphorescent substance in question emits rays which pass through the opaque paper and reduces silver salts (Becquerel, 1896).

At the time of Becquerel's discovery the nature of these "radioactive" emissions was completely unknown, as was their connection to the nucleus of the atom, whose existence would not be discovered for another decade. Becquerel's name is now attached to one important SI unit used in nuclear science, the Becquerel (Bq), which is defined as one nuclear disintegration or one decay per second.

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Marie Curie's important contributions to early nuclear science were her creation of a theory of the nature of radioactivity (a term she coined) and her realization that the phenomenon was due to the presence of several elements that were hitherto unknown. The first of these she named polonium, in honor of her native Poland, and the second she called radium. Marie Curie collaborated with her husband Pierre with whom she shared the Nobel Prize in Physics awarded in 1903. Remarkably, her daughter Irène Joliot-Curie later also shared a Nobel Prize with her husband Frédéric Joliot-Curie. Marie Curie (born Maria Skłodowska), who also won a Nobel Prize in Chemistry in 1911, was a truly remarkable woman and the first scientist ever to be awarded two Nobel Prizes. Marie Curie made her discoveries at the University of Paris, where she was the first female professor. Marie and Pierre's work to separate the element radium from the raw pitchblende that contained it involved physically difficult work conducted under unbelievably primitive conditions in a windowless unheated leaky shed (Figure 3.1).

BOX 3.1 MARIE CURIE'S OTHER LEGACY?

In most nations, the representation of females in physics is among the lowest in the sciences. For example, in the United States, around 18% of PhDs in physics were granted to women in 2007, according to data compiled by the American Institute of Physics (AIP, 2010). One 2005 AIP report looked at comparable statistics in 19 nations (AIP, 2001). Interestingly, the two nations where Marie Curie was born and did her great work (Poland and France) topped the list at numbers 2 and 1, with 23% and 27% women physics PhDs. Is this merely a coincidence?

One indication of the difficulty of the work is the fact that a ton of raw pitchblende was needed to extract a mere one-tenth of a gram of radium chloride. Of course, in those early years the dangers of radioactivity were not realized, a fact that later probably cost Marie Curie her life to what was likely cancer. In Curie's honor we have the radioactivity unit the Curie (Ci), which is 37 billion nuclear decays per second or Becquerels, the number corresponding roughly to the activity of 1 g of pure radium.

BOX 3.2 RADIATION-INDUCED CANCER?

Cancers caused by radioactivity are no different than those caused spontaneously, so an unambiguous claim that she died by a radiation-induced cancer cannot be made. However, it is also true that her work undoubtedly exposed her to very high levels of radiation, which would certainly increase the probability of cancer significantly. Moreover, throughout her adult life she was in a constant state of ill health (Coppes-Zatinga, 1998). It is ironic that while ionizing radiation can cause cancer, it is also used in its treatment—a field that Marie Curie pioneered.

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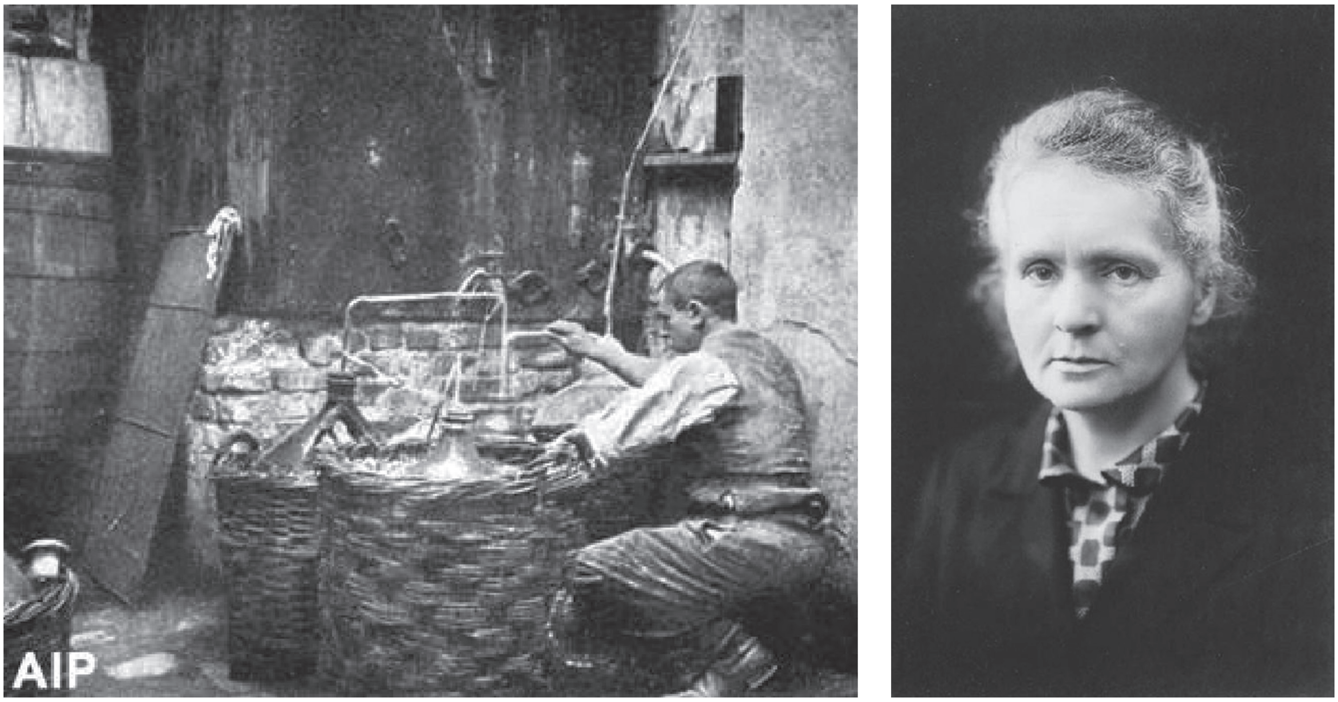

(a) (b)

Figure 3.1 (a) Extraction of radium in the old shed where Marie and Pierre Curie first obtained the element. Photo is taken from Marie Curie's autobiographical notes. (From Curie, M. and Curie, P., *Autobiographical Notes*, the Macmillan Company, New York, 1923; Image courtesy of AIP Emilio Segrè Visual Archives.) (b) Marie Curie (born Maria Salomea Skłodowska), Nobel Prize awardee in Chemistry and Physics. (Public domain image.)

3.3 DISCOVERY OF THE ATOMIC NUCLEUS

In the early years of the twentieth century, the concept of matter consisting of atoms corresponding to the various elements was reasonably well established based on arguments from chemistry, even though some scientists doubted the actual physical existence of atoms, and none understood their structure. Nevertheless, some physicists including J.J. Thomson did postulate models for the atom, most notably his so-called plum or raisin pudding model (Thompson, 1904). Thomson had previously discovered the negatively charged electron in 1897 (Thomson, 1897). Knowing that normally atoms were electrically neutral, he surmised that the electrons could be thought of as raisins in a static mass (the "pudding") of an equal amount of continuously distributed positive charge. This model had a number of attractive features, but only an experimental test could reveal whether it had any basis in reality. This task fell to the physicist Ernest Rutherford who led an experiment that is the prototype of much experimental work conducted today to reveal the properties of the fundamental particles of nature.

Having received the Nobel Prize in Chemistry in 1908, Rutherford conducted even more groundbreaking work the following year. Together with graduate students Hans Geiger (of later Geiger counter fame) and Ernest Marsden, Rutherford carried out the famous experiment that demonstrated the nuclear nature of atoms. The basic idea of the experiment is

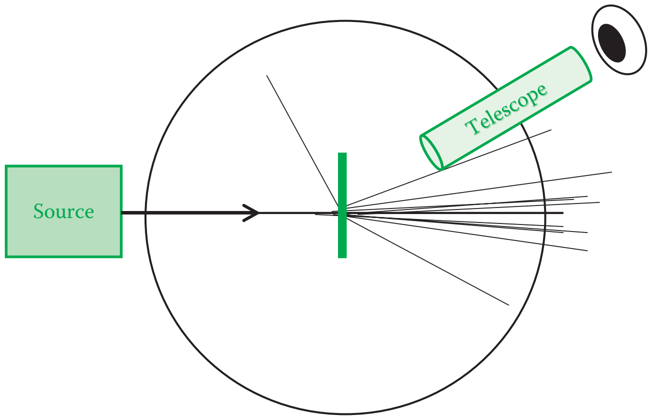

Figure 3.2 Drawing of the apparatus used in the Rutherford experiment. The telescope now making an angle of about 30° with the incident beam of alpha particles from a radioactive source is rotated about a vertical axis to count how many alphas are scattered through different angles after the beam strikes the thin gold foil target at the center of the apparatus.

quite simple. Rutherford sought to probe the structure of the atom using a collimated (directed) beam of particles fired at a thin sheet of material. Arranging to have a collimated beam was easy—by simply having a small hole in a thick lead container containing some radioactive radium. The so-called alpha particles that the radium emitted would then be reasonably well collimated, since only those alphas able to pass out of the narrow hole would escape the container. Rutherford chose gold as the atom to probe simply because a piece of gold foil could easily be made very thin (only a few atoms thick), which was essential so that the beam of alpha particles usually would encounter only one gold atom in close proximity in passing through the sheet (Figure 3.2).

Rutherford had earlier established that the electrical charge of the alpha particles was +2e (i.e., twice the charge of the electron in magnitude and opposite in sign), and its mass was roughly 4000 times greater. Alpha particles are now known to be the nuclei of helium atoms, which of course could not be known to Rutherford before he discovered the nucleus!

Rutherford wanted to observe how often a beam of alpha particles would be scattered through different angles when encountering gold atoms, and he planned to do this simply by counting the numbers of alphas deflected through different angles. In an age when no modern radiation detectors existed, measuring the angles along which deflected alpha particles traveled was challenging—certainly to the eyesight of his students Geiger and Marsden! Rutherford had earlier developed zinc sulfide scintillation screens, and he used them to detect the deflection angle when an alpha struck the screen placed at a given point and caused a brief flash of light there. What did Rutherford expect to find? Given the large mass of the alpha particles, and their high speed, he expected that the vast majority of alphas would be deflected through very small angles by the electrical

Table 3.1 Data Recorded by Geiger and Marsden for Alpha Particle Scattering Off a Gold Foil Showing the Number of Counts Recorded in 1° Intervals at Various Angles from 15° to 150°

Angle	150	135	120	105	75	60	45	37.5	30	22.5	15
Counts	33.1	43	51.9	69.5	211	477	1,435	3,300	7,800	27,300	132,000

Source: Geiger, H. and Marsden, E., The laws of deflexion (sic) of α particles through large angles, 25, 610, 1913, http://www.chemteam.info/Chem-History/GeigerMarsden-1913/GeigerMarsden-1913.html.

(Coulomb) force between an alpha and the nearest atom it encounters. In fact, to a first approximation on the basis of the Thomson raisin pudding model the deflection force would be almost zero, since the atom as a whole is electrically neutral, and its positive charge is diffuse.

Day after day Geiger and Marsden counted the numbers of flashes they saw at various angles of deflection, and their observations confirmed Rutherford's expectation that the vast majority would be at very small angles. However, there was one strange anomaly in the data. Some alphas (albeit only one in 8000) were found to be deflected by very large angles (over 90°). In fact, a very tiny percentage of alphas were almost deflected through 180°, i.e., directly backward. Table 3.1 shows the number of counts found at various angles.

The fractional numbers for the numbers of counts for some angles appears in the original paper. This seeming impossibility reflects the fact that for angles greater than 90°, longer periods of time had to be observed in order to obtain statistically significant numbers, and fractional numbers of counts result when adjusting for different counting periods.

Rutherford upon learning of Geiger and Marsden's observations that some counts were found at very large angles has been quoted as saying

> It was the most incredible event that ever happened to me in my life. It was almost as if you fired a 15-inch [cannon] shell at a piece of tissue paper and it came back at you" (Cassidy et al., 2002).

Rutherford realized that the explanation of this strange anomaly in the data was the existence in the atom of a small nucleus, which contained most of its mass. In that case the tiny massive nucleus would be capable of occasionally deflecting alpha particles backward in the event they were heading directly toward it. The rarity of these backward or near backward deflections implied that the nucleus had an extremely small size compared to the atom itself. The usual description of Rutherford's discovery of the atomic nucleus ends here, but it does not do justice to Rutherford's magnificent achievement. Any scientist if he or she is lucky can observe an anomaly in the data and formulate a new revolutionary theory based on it, but only a great scientist will take the next step and rule out alternative theories by showing that the data fully support the new theory in all their quantitative detail.

3.4 MATHEMATICAL DETAILS OF THE RUTHERFORD SCATTERING EXPERIMENT

Rutherford sought to explain the exact angular distribution of the alpha particles that his students Geiger and Marsden had recorded on the assumption that there existed a tiny massive nucleus at the center of the atom. The mathematics of this section is somewhat more challenging than most sections in this book, and some readers may wish to skip it on first reading focusing only on the result of Rutherford's derivation for the number of particles scattered through different angles (Figure 3.3).

BOX 3.3 CONCEPT OF THE SOLID ANGLE

The analog of an angle in three dimensions is known as a solid angle, and it is measured in steradians, rather than radians. Recall that the basic definition of an angle in radians is the length of an arc along a unit circle surrounding a point. The corresponding definition of a solid angle, universally represented by the symbol Ω, is the amount of area on a unit sphere surrounding a point. Obviously, the largest possible solid angle would be $\Omega = 4\pi$.

Rutherford assumed that individual alpha particles were deflected through different angles strictly based on their "impact parameter," b, defined as the perpendicular distance from the x-axis to the incident alpha particle velocity vector when the particle is very far from the target. Thus, alphas that headed directly toward a nucleus (having $b = 0$) would be deflected through 180°, while those that had a large

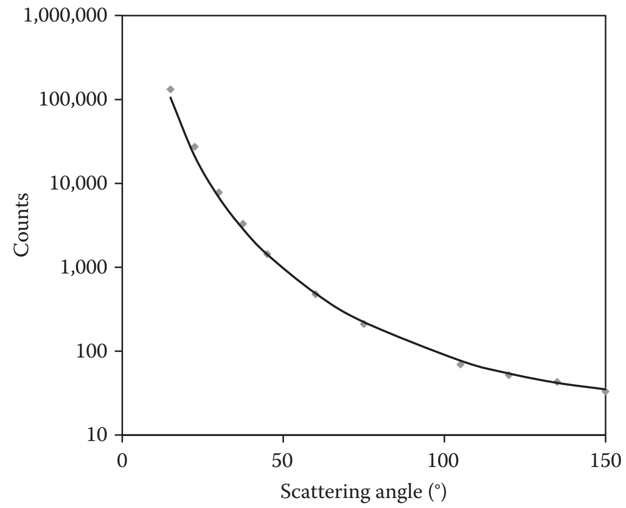

Figure 3.3 Rutherford experiment data and theory. The points are the number of counts observed in the experiment versus the scattering angle, and the smooth curve is a plot of the choice of $C \sin^{-4}(\theta/2)$, where C is a constant. The good agreement with the data for the appropriate choice of C is quite evident.

impact parameter would be deflected through a very small angle. Using classical mechanics and an inverse square Coulomb force, he was able to easily deduce the relationship between impact parameter, b, and scattering angle, θ, as (Goldstein et al., 2000)

$$b = \frac{kq_1q_2}{2E\tan(\theta/2)} \tag{3.1}$$

where
E is the kinetic energy of the alpha particle
$k = 9 \times 10^9$ N m^2/C^2 is the Coulomb force constant

the respective charges of the alpha particle and nucleus are $q_1 = +2e$ and $q_2 = +79e$.

In deriving Equation 3.1, Rutherford assumed that the alpha particles in encountering an atom experienced a force almost exclusively due to the positively charged nucleus that was of sufficiently small size, so that when the alphas were inside a spherical cloud of many electrons they would exert no force on the alphas. Rutherford, by further assuming a random distribution of impact parameters, was able to deduce the fraction of particles scattered through each angle. In modern parlance, this is written in terms of the differential cross section

$$dA = \sigma(\theta) = 2\pi b db \tag{3.2}$$

which represents the area (of a ring) surrounding a target nucleus (the scattering center) that an incoming projectile need pass through to be deflected (scattered) by an angle θ, or more exactly into the interval from θ to $\theta + d\theta$.

Figure 3.4 illustrates the concept of differential cross section for particles scattered into a small angular range $d\theta$ or in three dimensions into a solid angle range $d\Omega = (2\pi \sin\theta)d\theta$. The number of particles, scattered into a small interval of solid angle is given by

$$dN = N_0\sigma(\theta)d\Omega = N_0\sigma(\theta)(2\pi\sin\theta)d\theta \tag{3.3}$$

where N_0 is the incident intensity (number of particles per unit area per unit time).

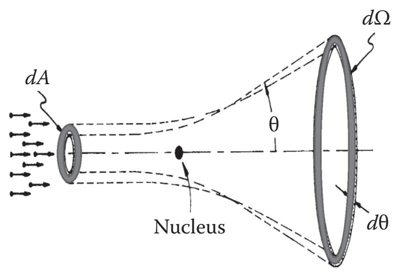

Figure 3.4 Collimated beam of particles incident on the ring area dA is scattered into a differential solid angle $d\Omega$ defined by this diagram.

Even when it is inappropriate to imagine particles traveling along trajectories (as in quantum mechanics), one can still define a measured cross section for scattering from Equation 3.3 using

$$\sigma(\theta) = \frac{1}{N_0}\frac{dN}{d\Omega} \tag{3.4}$$

However, recall that so far we have assumed a single target nucleus. In general, for a foil having N_t nuclei within the area of the beam, S, we have for the actual number of particles, dN scattered into a given solid angle range:

$$dN = N_t N_0 \sigma(\theta) d\Omega \tag{3.5}$$

The only unfinished business is finding the number of target nuclei N_t in terms of known quantities. It can easily be shown using dimensional analysis that the number of target nuclei in the foil lying within the area of the beam can be expressed in terms of the density of the foil ρ, its thickness d, Avogadro's number N_A, the area S, and the atomic weight of the material, A, i.e.,

$$N_t = \rho d N_A \frac{S}{A} \tag{3.6}$$

Note that Equations 3.2 through 3.6 apply to any force, but Equation 3.1 applies only to the inverse square force. Using Equation 3.1, and the conservation laws of classical mechanics, it can be shown that (Goldstein et al., 2000)

$$\sigma(\theta) = \frac{dN}{d\Omega} = \left(\frac{kq_1q_2}{4E}\right)^2 \frac{1}{\sin^4(\theta/2)} \tag{3.7}$$

The essential point of Equation 3.7 is that the number of alphas scattered per unit solid angle is proportional to the negative fourth power of half the scattering angle. Thus, for example, the number of particles through an angle of 60° should be 16 times greater than the number scattered through 180°. Thus, Rutherford had perfectly explained the alpha scattering data, which makes his claim of a nucleus to the atom, not just an explanation that qualitatively fits an anomaly in the data (presence of some alphas deflected through large angles) but a detailed quantitative description.

3.4.1 Example 1: Setting an Upper Limit to the Nuclear Size

Using the data recorded by Geiger and Marsden and the dominant energy of alpha particles emitted by radium (4.75 MeV), determine the experimental upper limit that Rutherford was able to set for the radius of the gold nucleus. Compare this value with the actual radius of (a) the gold nucleus and (b) the gold atom. Note that $1\,\text{eV} = 1.6 \times 10^{-19}$ J.

Solution

In order for the Rutherford scattering formula to fit the data even for scattering angles approaching 180°, the size of the nucleus would need to be smaller than the distance of closest approach, r. At its closest distance (for a 180° scattering), the alpha particle initial kinetic energy E would be entirely converted to electrostatic potential energy (since it is momentarily brought to rest), so that

$$E = \frac{kq_1q_2}{r} \quad (3.8)$$

Using $E = 4.75$ MeV, $q_1 = 2e$, and $q_2 = 79e$, we find $r = 4.79 \times 10^{-14}\ m = 47.9$ fm. According to data on the web, the true nuclear radius for gold is now known to be 7.3 fm, while that of a gold atom is 0.144 nm, making Rutherford's upper limit to the nuclear radius about 1/3000 the size of the gold atom—a truly tiny object.

In addition to the differential cross section that we have considered at length, one can also define the total cross section for any process by merely integrating overall angles:

$$\sigma_{TOT} = \int_0^{4\pi} \frac{d\sigma}{d\Omega} d\Omega \quad (3.9)$$

Note that the total cross section may be thought of as the effective size (cross-sectional area) of the target nucleus for any impact parameter. However, the total cross section, in general, can be different for different incident particles, and the meaning of the total cross section is not limited to the problem of elastic scattering, but it can be applied to any nuclear process induced by some projectile. The cross section is a measure of the probability that incident particles will cause that process to occur.

3.5 COMPOSITION AND STRUCTURE OF THE ATOM AND ITS NUCLEUS

Having established the existence of a tiny massive nucleus to the atom, Rutherford went on in 1911 to postulate his planetary model of the atom, whereby electrons orbited the nucleus, much like a miniature solar system (Rutherford, 1911). Two years later, Niels Bohr introduced his own model incorporating some new radical elements into the planetary model. These radical elements included quantum jumps between so-called stationary states (Bohr, 1913). Bohr's model, still taught in most introductory physics courses, was an important bridge on the road to a full understanding of atomic structure, based on quantum mechanics.

In the meantime, Rutherford and others continued their work on the structure and composition of the atomic nucleus. In 1919, Rutherford discovered that he could change ("transmute") one element into another by bombarding it with alpha particles. In subsequent experiments, Rutherford and others found that often during these nuclear transmutations hydrogen nuclei were emitted. Clearly, the hydrogen nucleus (now known as the proton) played a fundamental role in nuclear structure.

By comparing nuclear masses to their charges, physicists realized that the nuclear positive charge could be accounted for by an integer number of these protons. Ernest Rutherford in 1920 then postulated that there were neutral particles in the nucleus of atoms (now known as neutrons), which he thought of as electrons bound to protons (Rutherford, 1921). The need for these neutral particles having about the same mass as protons was the observed disparity between the atomic number (the charge Z) and the atomic mass A, which was often twice the former for many elements. It was not until 1932 that James Chadwick was actually able to detect Rutherford's neutron and confirm its existence (Chadwick, 1932).

With the experimental discovery of the neutron, the constituents of the atom were now apparently complete: Z electrons outside a nucleus containing most of the atom's mass, with the nucleus consisting of Z protons plus $N = A - Z$ neutrons.* A given element is characterized by the atomic number Z, which determines its chemical properties, and various isotopes of that element have different numbers of neutrons, or different A-values. Neutrons and protons shared many characteristics, including having the same mass (to within 0.1%) and the same spin, and hence they are collectively known as "nucleons."

3.6 NUCLEAR RADII

Recall that Rutherford was able to set an upper limit to the radius of the gold nucleus, based on his scattering experiment. By using alpha particles of somewhat higher energies, which could approach the nucleus even closer, it is possible to actually measure the size of the nucleus, and not merely set an upper limit. However, in practice, one usually uses electrons, rather than alpha particles in these experiments, since electrons have a strictly electromagnetic nuclear interaction, which is very well understood and they do not feel the strong force. Such experiments have been conducted for many different target nuclei, and they show a striking regularity. For all the nuclei whose radii have been measured, the following simple dependence on mass number A:

$$r = r_0 A^{1/3} \tag{3.10}$$

where $r_0 = 1.25$ fm, although admittedly it is a bit of a simplification to regard the nucleus as having a sharp well-defined surface. To understand the significance of this basic formula, simply calculate the volume of a spherical nucleus from its radius (Equation 3.10), and you will see that the volume is proportional to A. What does this fact imply?

* We here ignore the fact that neutrons and protons are now known not to be fundamental particles (like electrons), but are themselves made of quarks. There are many good popular-level books about quark theory, and the still more current "theory of everything" known as string theory.

Note that this behavior (volume proportional to the number of particles) is quite different from the atom outside the nucleus, since the volume of an atom is certainly not proportional to the number of electrons it contains. Thus, unlike electrons, nucleons seem to behave like incompressible objects that are packed together in close proximity. Another way to express the situation is to note that all nuclei have precisely the same density, which, using Equation 3.10, is found to be the astonishing value of 2×10^{17} kg/m^3, or 200 trillion times that of water. Does matter of such density exist anywhere in the universe, apart from the nucleus itself? The surprising answer is yes, inside of the strange astronomical objects known as neutron stars, which are the remnants of stars that have undergone supernova explosions toward the end of their lives.*

3.7 NUCLEAR FORCES

One issue that Rutherford and other nuclear scientists of his day wrestled with is the question of what holds the nucleus together? If only forces of an electromagnetic nature were present, clearly the positively charged protons would repel each other, so that no assemblage of protons could exist stably, even with the presence of neutrons that do not "feel" the electromagnetic force. Clearly, some attractive force must be present that is strong enough to overcome that Coulomb repulsion. This additional force has been given the unimaginative name of the "strong force." Table 3.2 summarizes the nature of the strong force in comparison with the more familiar Coulomb and gravitational forces, and the less familiar weak force.

The strength of 1 for the strong force is an arbitrary choice. Note the extreme weakness of the gravitational force compared to all the other three, which is why gravity is of no significance when considering nuclear reactions. There is one other fundamental force known as the "weak" force that does come into play inside the nucleus but, like the strong force, has no role outside it given its short range. In particle physics, all forces are assumed to be mediated by exchanged particles. Thus, as indicated in Table 3.2 the force of an electron on another electron is due to photons exchanged between them. The exchanged particles, however,

Table 3.2 Comparison between the Four Fundamental Forces

	Strong Force	Coulomb Force	Weak Force	Gravitational Force
Strength	1	1/137	10^{-13}	10^{-40}
Range	Around 1 fm	Infinite ($\sim 1/r^2$)	Around 0.01 fm	Infinite ($\sim 1/r^2$)
Sign	Always attractive	Attractive for opposite sign charges	Repulsive	Always attractive
Felt by	Nucleons	Any charged particle	Any particle	Any mass
Mediated by	Gluons	Photons	*W* and *Z* bosons	Gravitons

* A teaspoon full of nuclear matter would weigh 10 billion tons on Earth. Finding a material to make the spoon out of would be quite a challenge!

are not observed, and are referred to as "virtual" particles in contrast to "real" particles that are observed in detectors. In recent years, good arguments have been presented to show that at sufficiently high energies, all the four fundamental forces become unified.

BOX 3.4 BASIC FACTS ABOUT NUCLEAR ISOTOPES

1. All isotopes of an element have the same number of protons, Z, i.e., the same chemical identity. However, they have different numbers of neutrons and are distinguished by their atomic mass number, A, e.g., $^{235}_{92}U$ or $^{238}_{92}U$ two isotopes of uranium, the element with $Z = 92$, and $A = 235$ or 238.
2. Some isotopes are stable and some unstable (radioactive), but some "stable" isotopes just have half-lives too long to be observed—a half-life being the amount of time $\tau_{1/2}$ for half the original number of radioactive nuclei to decay.
3. Some elements have many stable isotopes, e.g., tin and xenon each have the most at 9, while other elements have none, e.g., uranium. As of 2010, the isotope with the longest half-life yet known is tellurium-128, with a half-life $\tau_{1/2} = 8 \times 10^{24}$ years; the one with the shortest half-life is beryllium-13, whose half-life $\tau_{1/2} = 2.7 \times 10^{-21}$ s.
4. Isotopes can be separated only by physical (not chemical) means that are sensitive to small nuclear mass differences.

3.8 IONIZING RADIATION AND NUCLEAR TRANSFORMATIONS

Radioactive nuclei by definition emit radiation when they decay. Often this radiation can penetrate matter and leave a trail of ionization, hence the term ionizing radiation, which is preferred to the looser term nuclear radiation. In fact, not all ionizing radiation, e.g., x-rays, emanates from the nucleus, and not all radiation that emanates from the nucleus, e.g., neutrinos, is ionizing. Although many forms of radiation can be harmful to biological organisms if the dose is high enough, ionizing radiation can be especially harmful. It is not simply its penetrating power (radio waves are also quite penetrating), but rather the cell damage associated with the trail of ions left in the wake of the radiation.

Three common types of ionizing radiation are known as alpha, beta, and gamma. As already noted, an alpha particle is a helium nucleus, which has $A = 4$ and $Z = 2$. Therefore, after a "parent" nucleus (A, Z) decays emitting an alpha particle, its "daughter" nucleus has atomic mass $A - 4$ and $Z - 2$, as illustrated in the case of the decay of the isotope $^{238}_{92}U$, which we would write as $^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He$. Beta rays are either electrons or their positively charged counterparts (known as positrons). During beta-plus nuclear emission, a proton transforms to a neutron and a positron and a third particle, the ghostly electron neutrino* according to the reaction: $p \rightarrow n + e^+ + \nu$, while in the beta-minus case it is a neutron that gets transformed into a proton, electron, and anti-neutrino, according to the reaction: $n \rightarrow p + e^- + \bar{\nu}$. Note that in both cases the identity of the nucleus containing the transformed n or p must change, since its atomic number changes by $\Delta Z = \pm 1$. Gamma rays also originate from nuclei when they undergo a change of energy level, but no change in their identity, i.e., their A or Z value.

You may be wondering under which conditions a proton is transformed into a neutron and when the reverse occurs in the cases of beta$^\pm$ decays. Nuclei spontaneously tend to transform themselves from less stable states to more stable states. In general, the most stable nuclei having a given mass number A tend to have a specific neutron–proton ratio, which is 50/50 ($N = Z$) for light nuclei, i.e., up to around $Z = 20$, and favoring neutrons ($N > Z$) to an increasing degree for heavier nuclei. These most stable nuclei lie along the "valley of stability"—the red region in Figure 3.5. Beta-plus decay (changing a proton into a neutron) occurs

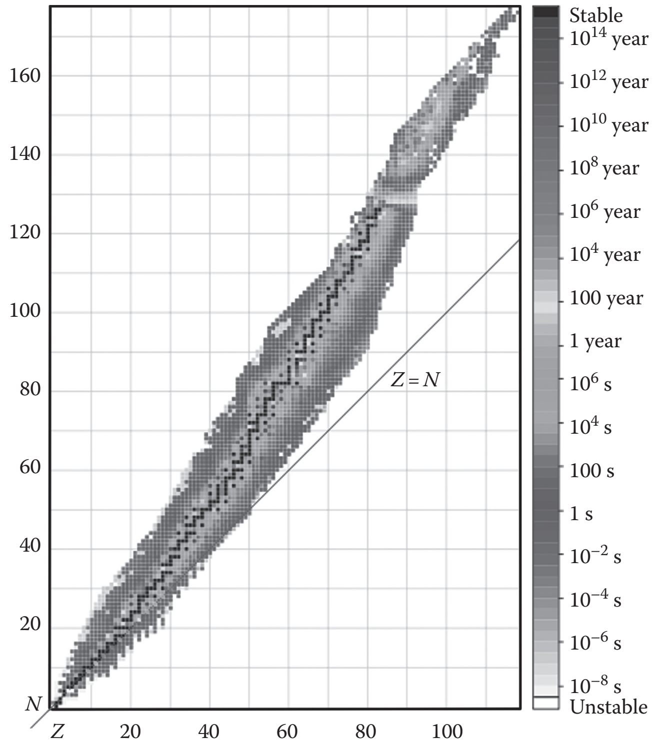

Figure 3.5 **(See color insert.)** Plot of number of neutrons N versus number of protons Z for all known nuclei, which have been coded by their lifetime. (Image created by BenRG and released into the public domain; http://commons.wikimedia.org/wiki/File:Isotopes_and_half-life.svg)

* At this very moment there are trillions of neutrinos passing through your body each second, but they cause no harm whatsoever, because neutrinos interact with matter so weakly. To detect neutrinos requires a truly massive detector, because of their weak interaction.

when a particular isotope is above and to the left of the "valley of stability," as they are too proton-rich, and conversely beta-minus decay (changing a neutron into a proton) occurs when a particular isotope is below or to the right of it.

The nuclei that are stable are represented by the dark dots in Figure 3.5, and they lie along a curve known as the "valley of stability." As one moves away from the valley on either side, the nuclei get less and less stable (shorter half-lives). These gray regions are where the beta-plus and beta-minus emitters lie. Very massive nuclei at the upper end of the valley tend to be alpha emitters, as this process allows them to shed mass very efficiently. The reader may have wondered why it is that up to around $Z = 20$ nuclei tend to have equal numbers of neutrons and protons, i.e., $N = Z$, while for $Z > 20$, neutrons are increasingly favored over protons. The reason is that as with electrons in an atom, neutrons and protons in a nucleus fill a set of energy levels starting with some lowest level, but each of these particles fills its own set of levels (two to each level just like electrons). Thus, suppose there were 10 protons in a nucleus but only 6 neutrons, in this case it would be energetically favored for two of the protons to convert to neutrons and fill the lowest vacant neutron levels, giving rise to equal numbers of neutrons and protons. However, the situation changes above $Z = 20$, because the repulsion protons feel for one another becomes increasingly important as Z increases. The reason is because the number of proton–proton interactions varies as $Z(Z - 1)/2 \sim Z^2$ while the nearest-neighbor short-range strong attraction only varies as Z. The increasing importance of proton–proton repulsion over attraction as their number Z increases has the effect of raising the proton energy levels over those of neutrons.

3.9 NUCLEAR MASS AND ENERGY

In any of the three decay processes so far discussed, the amount of energy that is released when the parent nucleus decays is enormous—about a million times that of chemical processes on a per atom or per kg basis. Yet despite the enormous energy, Rutherford, the "father" of nuclear science is famously reputed to have said about his nuclear studies "…Anyone who expects a source of power from the transformation of these atoms is talking moonshine" (Hendee et al., 2002). Remarkably, his comment was made in 1933, on the verge of the discovery of nuclear fission. However, Rutherford's failure to appreciate the practical impact of his work is understandable, because the types of processes known to him would indeed not be appropriate for generating large amounts of power. In order to understand the origin of the energy released in nuclear reactions, we need to consider Einstein's 1905 special theory of relativity, and what is perhaps the most famous equation ever written: $E = mc^2$. One way to interpret this equation is to note that mass (m) is a form of energy (E), and the "conversion factor" between mass (in kg) and energy (in Joules) is the quantity c^2, with c being the speed of light, 3×10^8 m/s.* An equivalent way to understand $E = mc^2$ is to note that since mass is just a form of energy, in any reaction in which energy E is released, the net mass of the reactants must decrease by an amount: $m = E/c^2$. Given the enormous size of the quantity c^2, such mass changes will only be noticeable in cases where a truly prodigious amount of energy is released, e.g., in nuclear processes. In such processes the total number of nucleons always stays constant; however, despite this fact the mass of the system does change in accordance with $E = mc^2$ because of differences between the binding energies of the initial and final states.

3.10 NUCLEAR BINDING ENERGY

The strong force holds the nucleus together against the much weaker Coulomb repulsion (between protons), so it should not be surprising that it would require a massive amount of energy to disassemble a nucleus into its constituent nucleons. The energy of total disassembly represents the "binding energy" of the nucleus, which simply equals the difference in mass between all the constituents (Z protons and N neutrons) and the original nucleus times c^2, or as in the following equation:

$$E_B = (Zm_P + Nm_n - M)c^2 \qquad (3.11)$$

Consider a plot of E_B/A (binding energy per nucleon), shown in Figure 3.6.

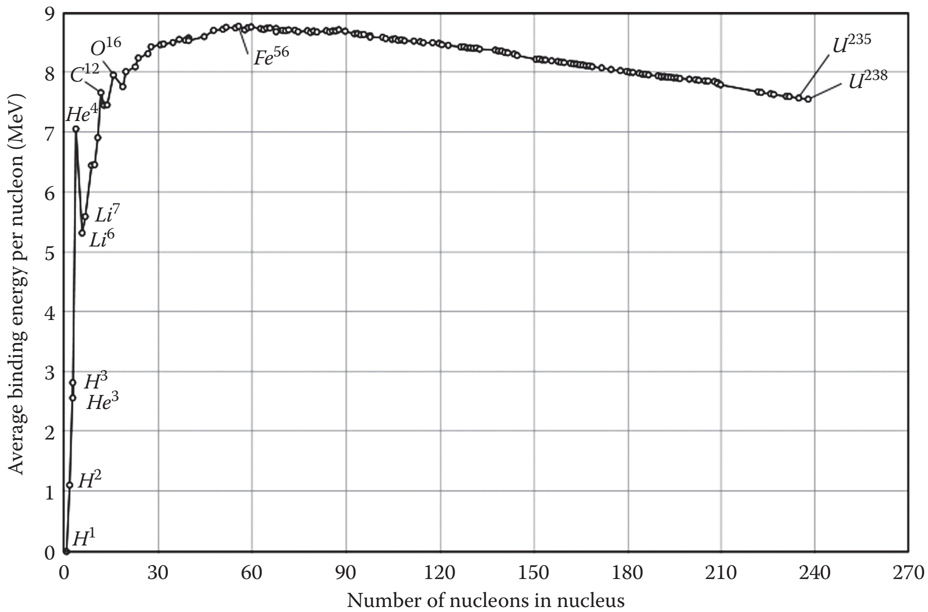

Figure 3.6 Curve of nuclear binding energy per nucleon, or E_B/A versus number of nucleons A. Image created by Mononomic and released to the public domain. (From http://en.wikipedia.org/wiki/File:Binding_energy_curve_-_common_isotopes_with_gridlines.svg)

* Were it possible to somehow convert 1 kg of "stuff" entirely into energy, the amount available would be 9×10^{16} J, or enough to supply all New York City's electricity for nearly 2 years.

Nuclei having the largest values of E_B/A tend to be the most stable, and hence they are the most tightly bound. These nuclei also have the smallest nuclear mass in relation to that of the constituents by virtue of Equation 3.11. According to Figure 3.6, the most stable nucleus is iron (Fe^{56}). Many students are confused by the sign of the binding energy. Obviously, it must be positive based on its definition as the work needed to disassemble a bound system (the nucleus). However, the potential energy responsible for binding the system together is negative earlier—see Figure 3.8.

3.11 ENERGY RELEASED IN NUCLEAR FUSION

The shape of the curve of binding energy suggests a way of extracting nuclear energy during two types of processes: fission and fusion. Very heavy nuclei have less binding energy per nucleon than those closer to iron, and therefore were a heavy nucleus such as uranium to split (fission) into two lighter ones, the combined mass of the two lighter ones would be less than the original parent nucleus, with the mass loss converted into the released energy. In a similar manner, if two light nuclei were to combine (fuse), energy would also be released by exactly the same argument. To illustrate, consider the "d–t" fusion reaction, where d and t stand for the hydrogen isotopes known as deuterium and tritium, respectively, which are also often written as H_2 and H_3. The d–t reaction can be written as $H_2 + H_3 \rightarrow He_4 + n_1$, where n_1 is a neutron. Given the known respective binding energies of the initial nuclei, i.e., 2.2 and 8.5 MeV, and the final nuclei, i.e., 28.3 and 0 MeV, we find that the reduction in binding energy is 17.6 MeV, so that mass lost in the reaction is 17.6 MeV/c^2 and hence the energy released is 17.6 MeV. Note that it is convenient here to consider the c^2 as simply being part of the units of mass, i.e., MeV/c^2.

3.11.1 Example 2: Estimating the Energy Released in Fusion

How does the energy released in this hydrogen fusion reaction compare with the ordinary burning of hydrogen?

Solution

If 1 kg of hydrogen is burned the energy released is 130 MJ, but the energy equivalent of the original 1 kg by $E = mc^2$ is $9 \times 10^{16} = 9 \times 10^{10}$ MJ. Thus, the fractional change is a mere 1.5×10^{-7}%. In contrast, consider the d-t fusion reaction where 17.6 MeV is liberated. The original two nuclei have a combined mass $A = 5$, whose energy equivalent is approximately 5 × 938 MeV = 4690 MeV, for a percentage change of 0.38%. A mass decrease of 0.38% may not sound like a lot, but it is roughly 2.5 million times more energy released (per reaction) than for ordinary hydrogen combustion!

3.12 MECHANICS OF NUCLEAR FISSION

The fission of a heavy nucleus into two lighter ones can either take place spontaneously, as in the case of $^{238}_{92}U$, but with an extremely long half-life (4.5 billion years), or it can be induced, usually with the absorption of a neutron. Neutrons are especially effective at inducing a nucleus to fission, because unlike positively charged protons they can easily penetrate the nucleus unhindered by any Coulomb repulsion, and unlike electrons they feel the strong nuclear attractive force. An artist's conception of a neutron-induced fission of a heavy nucleus is illustrated in Figure 3.7. The oscillations of the dumbbell shape—with the long dimension first oriented horizontally (not shown) and then vertically are very much like the oscillations that can actually occur in a liquid drop in a weightless environment such as a space shuttle. One way to understand the source of the energy released is to consider that when the dumbbell is most elongated during its oscillation, the Coulomb repulsion between the two pieces drives them apart and wins out over the very short range strong force, causing a complete rupture, followed by an acceleration of the pieces due to the Coulomb repulsion between them.

The emission of two or three neutrons following a nuclear fission is required by the fact that heavier nuclei tend to have a greater percentage of neutrons than lighter ones. Thus, when they fission into two fragments, these nuclei will tend to be too "neutron-rich" and will in very short order emit neutrons to reach a more stable nucleus.

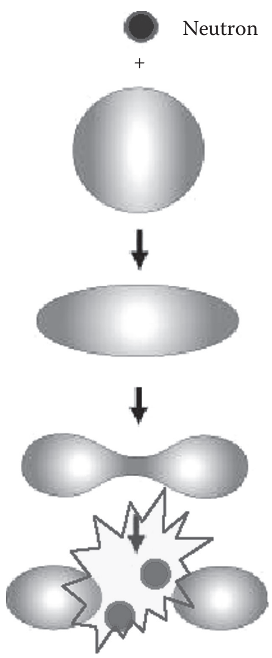

Figure 3.7 Time sequence of events leading to the fission of a large nucleus induced by the absorption of a neutron, and ending with the formation of two daughter nuclei, two neutrons, and the release of energy. This drawing hints at how during an intermediate stage of the process the parent nucleus undergoes oscillations forming a "dumbbell" shape prior to the actual fission. In practice, the two fission fragments (or the daughters) tend to be of unequal size, unlike the drawing.

3.12.1 Example 3: Estimating the Energy Released in Fission

Consider the spontaneous fission of a $^{238}_{92}U$ nucleus. Imagine that it simply fissions into two equal mass fission fragments. Use Figure 3.6 to estimate the energy released in such an event, and compare your estimate with the usually reported value. Hint: a straight line approximation to the curve in Figure 3.6 between $A = 90$ and $A = 240$ would approximately go from an E_B/A value of 8.7 to 7.6.

Solution

Using the values given in the hint we see that E_B/A changes by 1.1 MeV when A changes by 150 units. Given an assumed constant slope over this portion of the curve if the $^{238}_{92}U$ nucleus splits into equal size pieces having $A = 119$, the change from the original A is also 119, so that the E_B/A rise during fission would be $(1.1 \times 119)/150) = 0.87$ MeV per nucleon. Given a total of 238 nucleons in the original nucleus, we find an increase in binding energy of $0.87 \times 238 = 207$ MeV when the $^{238}_{92}U$ splits; 207 MeV is therefore also the estimated energy released, which is very close to the usual estimate during $^{238}_{92}U$ fission.

3.13 MECHANICS OF NUCLEAR FUSION

This emission of neutrons accompanying fission is extremely important, because it makes possible the concept of a chain reaction, and is the key to generating large amounts of nuclear energy. Claims of "cold fusion" aside, nuclear fusion unlike fission cannot be initiated without heating the atoms to be fused to extremely high temperature, comparable to that at the center of the sun.

BOX 3.5 COLD FUSION?

In 1989, electrochemists Martin Fleischmann and Stanley Pons announced to the world that they had a tabletop method of producing nuclear fusion at close to room temperature (Fleischmann and Pons, 1989). The experiment involved electrolysis of heavy water on a palladium electrode, and their claim was based on (a) the anomalous heat production ("excess heat") and (b) the observation of small amounts of nuclear by-products, including neutrons and tritium. There are many theoretical reasons for disbelieving this claim, and it later transpired that there was no convincing evidence for nuclear reaction by-products allegedly produced. Following a number of attempts by others to confirm these results—some positive, and some negative, the U.S. Department of Energy (DOE) convened a panel that same year to review the work (DOE, 1989). A majority of the panel found that the evidence for the discovery of a new nuclear process was not persuasive. Moreover, a second 2004 DOE review panel reached conclusions similar to the first (DOE, 2004). Cold fusion has now been renamed by the true believers, low-energy nuclear reactions (LENR) or condensed matter nuclear science, to escape the disrepute the field is held by most physicists.

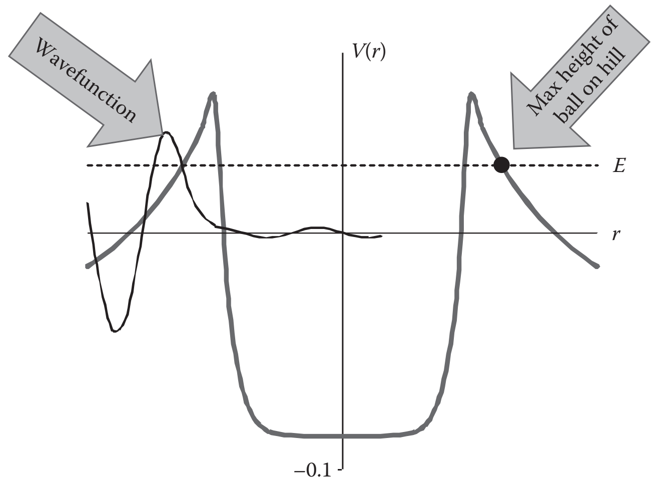

Figure 3.8 Potential energy *V*(*r*) as a function of r for one deuteron approaching another shown with a thick curve. At distances greater than a few fm (10^{-15} m), *V*(*r*) has the form of the Coulomb repulsive potential 1/*r*, but at shorter distances (inside the potential well), the potential is dominated by the strong attractive potential—represented by the steep walls. If the approaching deuterons are described by classical mechanics refer to right half of figure; if they obey quantum mechanics refer to the left half.

The need for very high temperatures to initiate fusion follows from the Coulomb repulsion between the positively charged nuclei that you seek to fuse. This repulsion can only be overcome if the nuclei collide with sufficiently high speed and energy. Let us consider the d–d fusion reaction $H_2 + H_2 \rightarrow He_4^*$, where He_4^* represents an extremely short-lived nucleus that decays virtually instantly into one of three pathways. When one deuterium nucleus approaches the other head-on from afar, it sees the potential energy function shown in Figure 3.8.

This potential energy graph (shown with a thick curve) is plotted symmetrically around $r = 0$ (the location of the target deuteron). For r greater than a few fm (at the cusps of the potential), $1/r$ Coulomb repulsion between the deuterons acts to repel them. The left and right halves of the Figure 3.8 contrast what would happen if classical or quantum mechanics described the interaction. For the classical case (right half of figure), the approaching deuteron, based on the value of its energy (height of the dotted line) stops at the point indicated by the small circle and then turns back—much like a ball rolling up a hill of this shape. Only if the energy were above the top of the hill would a classical deuteron come within the range of the strong (attractive) force, and fuse with the other one.

Now consider the correct quantum mechanical description suggested in the left half of Figure 3.8. Here a deuteron approaching a second one from the left is described by a wavefunction. The amplitude of the wavefunction exponentially decays as it tunnels through the forbidden region (where $V > E$), but its nonzero amplitude inside the well indicates that fusion is possible, even though classical mechanics would not allow it for this energy E.

3.13.1 Example 4: Find the Temperature Needed to Initiate d–d Fusion

Solution

At very high temperatures, as in the core of the sun, matter is in a state known as plasma with nuclei and electrons moving at random, somewhat like the molecules in a gas. In such a state, we may define the temperature using the relation from kinetic theory:

$$E = \frac{3}{2} k_B T \tag{3.12}$$

where

E is the average energy of the nuclei or electrons
T is the absolute temperature
k_B is the Boltzmann constant

The two deuterium nuclei have radii given by Equation 3.10. In the case of $A = 2$, the radius works out to be $r = 1.57$ fm. Thus, fusion is guaranteed if the two nuclei approach each other head-on with a center-to-center separation equal to 3.14 fm. Let us stick with the head-on collision case, which makes the calculation simplest. If the two nuclei have the same energy E when very far apart, and the kinetic energy is entirely converted into electrostatic potential energy when they just make contact. Thus, we have

$$2E = \frac{kq^2}{r} = 2 \times \frac{3}{2} k_B T \tag{3.13}$$

so that

$$T = \frac{2kq^2}{3k_B r} = \frac{2(9 \times 10^9)(1.6 \times 10^{-19})^2}{(3 \times 1.38 \times 10^{-23})(3.14 \times 10^{-15})} = 3.54 \times 10^9 \text{ K} \tag{3.14}$$

The calculated temperature 3.54 billion K for "ignition" to occur for the d–d reaction is much higher than the value reported in the literature, i.e., "only" 180 million K. As previously noted, the discrepancy arises because it is necessary to use quantum mechanics, not classical mechanics, here.

3.14 RADIOACTIVE DECAY LAW

Radioactive decay is a completely random process, implying that a radioactive nucleus has no "memory" of how long it has been waiting to decay. In fact, the number of decays per second for a given radioisotope depends only on the number of nuclei present at a given time. As a consequence, the number of nuclei N that survive to a time t can be expressed in terms of the initial number N_0 and either the decay constant λ or the so-called mean lifetime T, according to

$$N = N_0 e^{-\lambda t} = N_0 e^{-t/T} \tag{3.15}$$

Still another way to express this result is in terms of the number of half-lives $n = t/\tau_{1/2}$:

$$N = N_0 2^{-n} \tag{3.16}$$

Differentiation of Equation 3.15 shows that the activity at any given time also satisfies the exponential decay law:

$$\frac{dN}{dt} = -\lambda N = -\frac{0.693N}{\tau_{1/2}} \tag{3.17}$$

3.15 HEALTH PHYSICS

Health physics refers to the field of science concerned with radiation physics and radiation biology, with special emphasis on protection of personnel from the harmful effects of ionizing radiation. The field of health physics is complicated by its use of various units—both current SI units and an older set of units that still appear in many books and articles (Table 3.3).

BOX 3.6 BIOLOGICALLY EFFECTIVE DOSE

The biologically effective dose is the dose adjusted for the type of radiation. For example, the ionization trail left by some particles tends to be more localized, which is sometimes more harmful than if it is not. Thus, for an equivalent dose, neutrons are roughly 10 times as harmful as gamma rays. The dose rate can also be important when considering biological effects. Since cell repair can occur spontaneously at low dose rates, a given total dose is likely to be more harmful if received rapidly.

Table 3.3 Some More Important Units for Various Radiation Quantities

Name	SI Unit (Abbreviation)	Old Unit	Conversion
Activity (decays/s)	Becquerel (Bq) 1	Curie (Ci) 3.7×10^{10}	
Radiation dose (energy absorbed)	Gray (Gy) 1 J/kg	rad 100 erg/g	1 cGy = 1 rad
Biologically effective dose	Seivert (Sv)	rem	1 cSv = 1 rem
Dose rate	Gray/s	rad/s	

3.16 RADIATION DETECTORS

It has often been noted that we can neither see, feel, smell, or sense by any other means the presence of ionizing radiation, which may be one reason it is so greatly feared. It is therefore necessary to rely on various kinds of radiation detectors such as the well-known Geiger counter, which essentially counts the number of particles per second that pass through a detecting tube.

One can easily imagine Hans Geiger's motivation for inventing the Geiger counter: he and Marsden spent many days observing counts by looking at the scintillation screens in the experiment they performed under Rutherford's direction (Figure 3.9).

Nowadays, radiation detectors come in a variety of forms—some small enough to fit on your keychain. There is even a simple app to convert your smart phone into a radiation detector! Apparently, all you need to do is install the app and stick some opaque black tape such as electrician's tape over the camera lens. Since the sensors used in smart phone cameras do not just pick up visible light but also gamma and x-rays from radioactive sources, then covering the lens only allows those to make it to the sensor. The application then counts the number of impacts the sensor receives and translates it into a value in microsieverts per hour.

3.17 RADIATION SOURCES

Ionizing radiation is continually present in the natural environment. The three primary natural sources of radiation are (a) from space—in the form of cosmic rays (mostly shielded by atmosphere); (b) from the

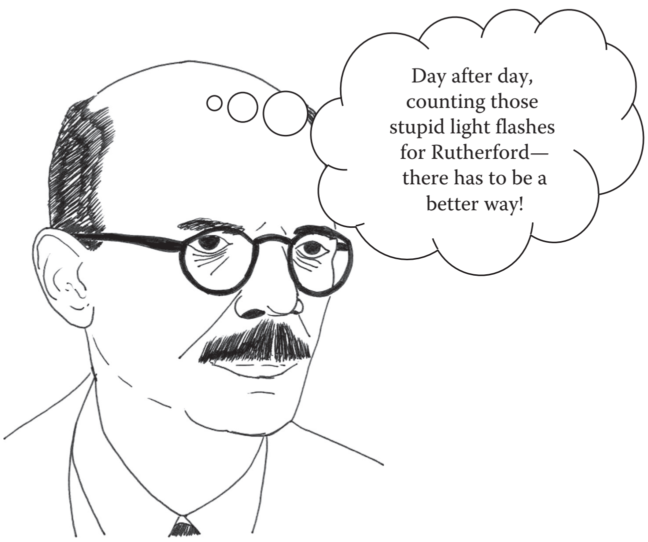

Figure 3.9 Drawing of Hans Geiger by Kevin Milani, included with his permission, and modified by adding the "thought balloon."

Earth—both food* and water, and building materials; and (c) from the atmosphere—mostly in the form of radon gas that is released from the Earth's crust. Both the level of radon and the amount of cosmic rays to which you are exposed can greatly depend on your circumstances; thus the former increases significantly with your altitude, and the latter depends on the local geology. Radon is usually the largest of the natural sources, amounting to around 200 mrem or 2000 μSv annually in a typical case, but with very large variations.

The most significant radiation you receive from man-made sources is received in medical tests or treatment. These medical exposures can vary enormously depending on their purpose, and responsible physicians will always weigh the diagnostic or treatment benefits against the risks of receiving the radiation exposure. For example, while a simple chest x-ray might expose you to the equivalent of 3 days normal background radiation, a CT scan of your abdomen might give you the equivalent of 4.5 years worth. These estimates assume that the technician administering the test adheres to accepted guidelines, and that the machine is not defective, which regrettably is not always the case.

3.17.1 Example 5: Comparison of Two Radioactive Sources

A 1 Curie source with a half-life of 1 week has the same number of radioactive nuclei at time *t* as a second source whose half-life is 2 weeks. What is the activity of the second source at this time? What will be the activities of the two sources after 4 weeks have elapsed? Which source is more dangerous to be around?

Solution

Since the two sources have the same number of radioactive nuclei, by Equation 3.17, the second source must have half the activity of the first or 0.5 Ci. The activity of both sources exponentially decays based on their respective half-lives. Therefore, at the end of the 4 week period, the activity of the first source has declined to 1/16 Ci and that of the second source has reached 1/8 Ci. Initially, and up to 2 weeks, the first source had the higher activity (was more dangerous), but after 2 weeks it was the second—see Figure 3.10. The total dose received integrated over time up to a period of many weeks would be the same for both sources if you were continually in their presence.

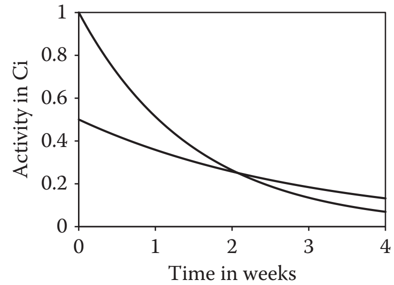

Figure 3.10 Activity in Curies versus time in weeks for the two sources discussed in Example 5.

* Sometimes food is exposed to extremely high levels of radiation that kill all the bacteria they contain, but this exposure does not make the food itself radioactive. In fact, irradiated meat can even be left outside a refrigerator, and stored on a shelf.

3.18 IMPACTS OF RADIATION ON HUMANS

In general, the danger of being in the presence of a radioactive source depends on many factors:

- The length of time you are in the presence of the source
- Whether the source is in a well-shielded container
- How far away you are from the source
- Type of radiation (alpha, beta, or gamma)
- Ventilation (in the event of a radioactive gas such as radon)
- Your own situation (whether you might be pregnant, for example)

It is also very important whether the radioactive source has become internalized to your body either by breathing a radioactive gas such as radon or eating food that was contaminated by radiation. Short-term, very large exposures can cause radiation sickness and death, and longer-term exposures can cause both genetic mutations and cancer. In the case of cancer it is important to note that the risk increases with increasing dose, and the occurrence of cancers do not appear for many years after radiation exposure.

3.18.1 Safe Radiation Level and Cancer Risks

It is often stated that there is no known safe level of radiation, which can be translated as there being no known level of radiation below which it is known that no harm whatsoever results. The reason that it is so difficult to establish whether a threshold for harm exists is that the harmful effects of radiation are so small at very low levels. An observation would need to have extraordinary statistics to reveal anything meaningful. For example, consider the question of whether having a chest x-ray (dose about 2 mrem) increases your risk of dying from cancer? Even though it is not morally possible to do experiments to see how humans are affected by doses of any given magnitude, extensive data have been compiled on the survivors of the Hiroshima–Nagasaki bombings during World War II (Preston et al., 2007). Based on these data, a dose of 1 Sv or 100 rem would increase your chances of dying from cancer by about 50%. If the harm done is assumed to be linearly proportional to the dose received, your chances of dying from cancer would increase by 0.001%. Since roughly 25% of the population normally dies from cancer anyway, such an increase would be impossible to establish.

Even for much larger exposures than 2 mrem it is difficult to pin down whether a threshold for harm exists. For example, the largest source of natural radiation exposure is from radon gas seeping up from the ground. In fact, radon is second only to smoking as a cause of lung cancer. Radon levels vary considerably depending on the local geology. A study performed by physicist Bernard Cohen has reported that in U.S. counties having higher average radon levels the rate of lung cancer tends to be lower, which some have interpreted as being evidence for the controversial hypothesis of "radiation hormesis" (Cohen, 1997). This interpretation, however, can be challenged based on the possibility of confounding variables, which cannot be definitively ruled out in an epidemiological study of this type.

Hormesis, which is well established for agents or substances such as sunshine, iodine, and iron, is the notion that while very high levels are harmful to health, low doses are actually beneficial. Whether at very low levels the effects of radiation are in fact harmful, beneficial, or neutral, the risks of radiation need to be assessed in comparison to other risks. For example, taking a plane ride or moving to Colorado will increase your exposure to radiation very slightly, but few people would let this factor dictate their decision-making about these activities. In the former case, the risks of flying (or more importantly driving to the airport) likely far outweigh the extra risk of dying from cancer, and in the latter case, moving to Colorado, despite its slightly higher background radiation level, will probably improve your health in view of the climate, lack of smog, and healthy lifestyle of the populace.

BOX 3.7 THE "RADIATION PARADOX"

The highest level of natural background radiation recorded in the world is from areas around Ramsar, in Iran where levels can reach 200 times greater than the worldwide average level. Most of the radiation in the area is due to dissolved radium-226 in hot springs. This high level of radiation has not had any observed ill effects on the residents, who live healthier and longer lives than average. This strange fact and similar reports from other high natural radiation areas, as well as studies like Bernard Cohen's has been called the "radiation paradox." They may not definitively prove that radiation hormesis is correct, but they certainly call into question the validity of linear no-threshold hypothesis, which is currently the basis of all radiation regulations.

3.18.2 Relative Risk

The possibility of hormesis aside, it is useful to have some sense of the relative risk to one's life resulting from various radiation exposures as compared to other hazardous situations. The following table is highly instructive in this regard. It shows how much the average person's life tends to be shortened as a result of various causes. Concerning the last table entry it is assumed that the nuclear plant functions normally, and that you live your whole life adjacent to it (Table 3.4).

Table 3.4 Comparison of Shortened Life Expectancy for Various Hazards

Health Risk	Shortened Life Span
Smoking a pack of cigarettes/day	6 years
Being 15% overweight	2 years
Consuming alcohol	1 year
Being a farmer	1 year
All accidents	207 days
All natural hazards	7 days
Receiving 300 mrem per year	15 days
Living next to a nuclear plant	12 h

3.19 SUMMARY

This chapter reviewed the highlights of the science needed to understand how nuclear reactors operate. It began with a brief overview of early nuclear history, including Rutherford's discovery of the nucleus, and then described the properties of the atomic nucleus, including nuclear forces, and the role they play in nuclear transformations, such as alpha, beta, and gamma decay. It considered the relation between mass and energy, and how to estimate the energy released in the processes of fission and fusion, which is perhaps a million times greater than for chemical reactions. The chapter concluded with a discussion of the topic of health physics, namely, the biological effects of radiation.

PROBLEMS

1. Show by direct integration that the total cross section, according to the Rutherford formula, is infinite. Hint: Up to angles of say 15° the small angle approximation holds very well, which allows the integration of Equation 3.7 to be easily performed over that interval.
2. Suppose that in the Rutherford experiment he observed 1000 scattering events for a 1° interval centered on 30° in a given time interval, how many events would he have found for a 1° interval centered on 90° in the same time interval?
3. Find on the web the data contained in Geiger and Marsden's original paper for thin foils made of silver rather than gold and show they do not fit the Rutherford scattering formula.
4. What energy alpha particles would have to be used in Rutherford's experiment for them to come within range of the strong force (about 1 fm) for a $b = 0$ scattering from a gold nucleus? Hint: First find the approximate radii of an alpha particle and a gold nucleus, and remember that $b = 0$ implies a head-on collision.
5. (a) Prove that the number of target nuclei of a sheet of material of thickness d lying within an area S is given by $N_t = \rho d N_A S/A$ (Equation 3.6). (b) Using this equation show that the number of gold nuclei/m^2 in a thin sheet of thickness dx is $5.9dx \times 10^{28}\,\mathrm{m}^{-2}$.

6. A radioactive source consisting of a single radioisotope has an activity of 1000 Bq at a certain time and 900 Bq after 1 h. What is its half-life? What will be the activity after 10 h have elapsed?
7. Verify the 10^{-40} figure in Table 3.2 by considering the relative strengths of the gravitational and Coulomb forces between a pair of electrons any given distance apart.
8. The process of alpha emission from a nucleus has been explained on the basis of quantum tunneling. How might the explanation go? How did an alpha particle get to be in the nucleus? Why is alpha emission from nuclei observed, but proton emission is virtually never observed?
9. Another possibility instead of there being neutrons in the nucleus to account for the fact that for many nuclei $A = 2Z$ is to have electrons inside the nucleus instead. Why according to the uncertainty principle of quantum mechanics is this not credible?
10. Do the calculation mentioned at the end of Section 3.6 to find the density of nuclear matter.
11. Why do alphas emitted by a particular radioisotope have a fixed energy (line spectrum), but betas have a continuous spectrum? Hint: How many particles are present after the decay in each case?
12. When a uranium-238 nucleus decays via alpha emission, use the known masses of the parent and daughter nuclei to determine the amount of energy liberated. How much of this energy is given to the alpha and how much to the daughter nucleus?
13. What form of radiation is a nucleus having $Z = N = 50$ likely to emit?
14. Look up on the web how much electricity New York City uses annually and do the calculation in the footnote in Section 3.9.
15. Show that if the activity of a source is merely proportional to the number of radioactive nuclei present that the exponential decay law must follow.
16. Using the longest known half-life found to date (for beryllium-13), $\tau_{1/2} = 8 \times 10^{24}$ years, estimate what fraction of the original amount has decayed in a time equal to the age of the universe.
17. Very slow neutrons are especially effective in causing nuclei to fission. Can you think of a reason why the total cross section for nuclear absorption of neutrons might vary inversely with their velocity at low velocities? Hint: How does the likelihood of a neutron inducing a fission depend on the time it is close to the nucleus?
18. Verify the figure 0.00056% mentioned in Box 3.17 regarding the excess cancer deaths due to a chest x-ray.
19. Suggest an alternative plausible hypothesis to "hormesis" that might explain the results Bernard Cohen found in his radon study in Section 3.17. Hint: The leading cause of lung cancer is smoking.
20. Why would you not expect the data in the Rutherford Experiment to fit the formula for very small angle scattering—say 0.01°?
21. Consider two decays in sequence A → B → C, having two different half-lives. Derive an expression for the amount of nucleus B remaining after a time *t*, in terms of the initial number of A nuclei and the two half-lives.

22. Suppose you are inadvertently in the presence of a radioactive source whose half-life is 3 h for a 4 h interval. Calculate your total exposure over the 4 h if the initial dose rate was 5 rad/h.
23. Estimate the percentage increase in your long-term risk of cancer from a medical exposure equal to 3 months worth of the average background radiation—assuming the linear no-threshold hypothesis is true.
24. Based on Figure 3.6, estimate the relative amount of energy released per kg in fission and fusion. Note that the vertical axis is in units of BE/A, not BE.

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