Computational Challenges of Dynamic Channel Assignment for Military MANET

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***Abstract*—The electromagnetic spectrum available for military use is increasingly crowded and scarce. The military must efficiently allocate spectrum by reusing channels to the maximum extent possible, while also limiting the number of channel changes over time for systems that require manual channel configuration. The classic channel assignment problem is often used to provide exact allocation solutions, but solving this combinatorial optimization problem over multiple time steps while considering cumulative co-channel interference constraints is computationally challenging. Using realistic data sets from large U.S. Marine Corps combat scenarios, we illustrate the importance and difficulties of solving this problem. We examine current solution methods and describe their inadequacies when solving larger problem instances. We also provide a list of ideas for future research on how to address this important and timely challenge.**

I. Introduction

The United States military fields many different types of radios and other wireless systems that require vast swathes of electromagnetic (EM) spectrum, including wideband mobile ad-hoc network (MANET) radios, radars, jammers, satellite communications radios, and data links for unmanned aerial vehicles. These wireless systems offer tremendous capabilities, including high data transmission rates (in the case of communications devices) and high-fidelity portrayals of the operating environment (in the case of radars and other sensors). However, in general, the larger the amount of transmitted information, the more EM spectrum (i.e., *bandwidth*) is required.

Meanwhile, the U.S. military will continue to operate in environments with increasing restrictions on spectrum use, both in the U.S. and abroad. Wireless communications traffic from civilian, joint, and coalition networks will increasingly clutter the EM spectrum, and the Federal Communications Commission (FCC) is reassigning the military to new bands to share spectrum with the private sector [1], [2]. Efficient allocation of available *channels* (i.e., contiguous portions of EM spectrum) is required to ensure military forces are able to fully utilize new wideband transmission devices [3], yet current methods of allocation are woefully inadequate. Indeed, in a major study the U.S. Marine Corps found that with current allocation methods, Marine task forces will not have enough spectrum available to support the use of wideband MANET radios in major combat operations [4].

We consider the problem of a U.S. Marine Corps (USMC) *spectrum manager* who must determine an efficient spectrum allocation scheme to support multiple, mobile, independent MANETs operating on rough terrain over several days or weeks. Many Marine Corps EM systems, including single-channel radios, radars, jammers, and the independent MANETs we consider, do not automatically coordinate channel assignments because of security concerns, additional complexity, and communications overhead (i.e., bandwidth and processing required for coordination). They receive centralized channel assignments from a spectrum manager, and then a human operator manually configures the radios, thus creating a time lag. The spectrum manager knows the capabilities of each radio and their starting locations, and has a rough understanding of their future locations. Using this information and terrain elevation data, and being mindful of *co-channel interference* (unintentional electromagnetic transmissions between two or more radios assigned the same channel), the spectrum manager may choose to minimize the number of channels required to support communications, or minimize total interference given a fixed number of channels. We assume the spectrum manager has a few hours, and possibly as much as several days, to determine the best allocation using local computing resources.

Currently, spectrum managers use several software tools to inform spectrum allocation decisions, including the Systems Planning, Engineering, and Evaluation Device (SPEED) [5] and Spectrum XXI [6]. These tools provide radio coverage analysis reports, and the latter tool provides a database to deconflict assignments across a given operating area. Neither consider the interference among a large number of mobile transmitters over multiple time periods, nor do they provide a method for minimizing the number of required channels.

In a landmark paper, Hale [7] differentiates the *frequency assignment problem* (*FAP*) (where assigned frequencies may be non-contiguous) from the *channel assignment problem* (*CAP*) (where assigned frequencies are in a contiguous block) that we consider. Murphey et al. [8] observe that though there is extensive research into the CAP, it remains a notoriously difficult problem to solve. Metzger [9] first observes the possibility of using optimization techniques for solving CAPs. He relates the problem to the *graph-coloring problem*, which restricts any two adjacent nodes (i.e., radios) from being assigned the same color (i.e., channel). These pairwise constraints are used in the vast majority of research on the *interference-aware* CAP [10]–[13]. This basic form of the CAP was shown to be *NP-complete* [14], yet the more realistic interference constraints that must be considered for military MANETs are far more computationally challenging. In our application, there are many radios operating within a close distance of each other, so we must consider the cumulative effects of multiple

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sources of co-channel interference, rather than just interference between pairs of radios. In this way, our constraints can be represented using a *hypergraph*, where a *hyperedge* connects two or more nodes, and our optimization problem is thus a form of *hypergraph coloring* [15], [16].

In this paper, we build on previous research [4], [17] to examine in detail the computational difficulties of solving the cumulative co-channel interference-aware CAP for military MANETs. In the next section, we provide two simplified CAP formulations, and then describe our realistic datasets generated from USMC combat scenarios. In Section III we describe current solution methods and their shortfalls when faced with the computational challenges of our realistic datasets and constraints. In Section IV, we offer ideas for future research, and in Section V provide our conclusions.

II. Problem Formulation and Datasets

A. Channel Assignment Problem

In order to describe the computational challenges of cumulative co-channel interference, we build on [4], [11], [17] to present both *minimum-order* (*MO*) (i.e., minimizing the number of channels) and *minimum-interference* (*MI*) CAPs for groups of multiple, independent MANETs. CAPs with alternate objectives, such as maximizing service, will suffer the same computational challenges we describe.

Let $r \in R$ (alias s) represent each MANET *radio*. Each radio is permanently assigned to a MANET *unit* $u \in U$, indicated by the set of *logical arcs* $(r, u) \in L$. A unit may represent a tactical military organization, such as an infantry company or battalion headquarters. Let the set of *nodes* N (indexed by n) comprise both radios R and units U, i.e., $n \in N = R \cup U$.

Let a channel $c \in C$ be a contiguous range of EM frequencies, where C is the set of available orthogonal (i.e., non-interfering) channels. Each unit u and the radios assigned to it require a channel assignment. Let $X_n^c \in \{0, 1\}$ indicate whether node n (either a radio or a unit) is using channel c:

$$X_n^c = \begin{cases} 1, & \text{if node } n \text{ uses channel } c \\ 0, & \text{otherwise} \end{cases} \quad \forall n \in N, c \in C. \tag{1}$$

All radios in a unit use the same channel, so:

$$X_r^c = X_u^c \quad \forall c \in C, (r, u) \in L \tag{2}$$

and each unit u is assigned only one channel, so:

$$\sum_{c \in C} X_u^c = 1 \quad \forall u \in U. \tag{3}$$

Let $Y^c \in \{0, 1\}$ indicate whether channel c is being used:

$$Y^c = \begin{cases} 1, & \text{if channel } c \text{ is used} \\ 0, & \text{otherwise} \end{cases} \quad \forall c \in C \tag{4}$$

which is enforced via:

$$X_u^c \leq Y^c \quad \forall u \in U, c \in C. \tag{5}$$

Let $(r, s) \in W$ indicate the set of all *wireless arcs* between all radios $r, s \in R$. These arcs represent both intentional EM transmissions between radios assigned to the same unit, and

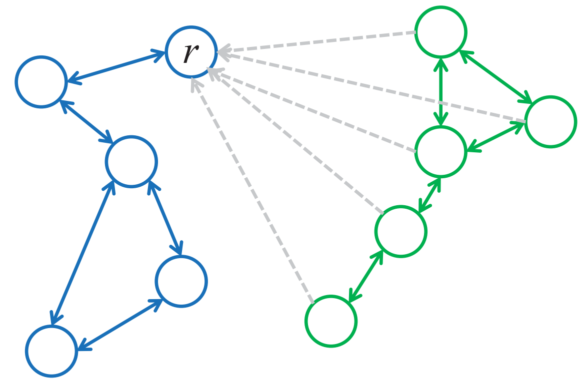

Fig. 1. Two MANETs supporting separate units (indicated in blue and green) but assigned to the same channel. Solid arrows indicate wireless arcs between radios in the same MANET; dashed arrows indicate co-channel interference.

unwanted interference from all other radios assigned to the same channel $c \in C$. These arcs exist in both directions, and each radio can receive transmissions from any other radio, so $|W| = |R|(|R| - 1)$. Each unit $u \in U$ forms a separate MANET among its assigned radios using the available wireless arcs $(r, s) \in W : (r, u) \in L, (s, u) \in L$. Fig. 1 shows two separate units (indicated in blue and green) and their assigned radios. The solid arrows indicate bidirectional wireless arcs $(r, s) \in W$ between radios assigned to the same unit. All radios are subject to co-channel interference from all other radios assigned to different units but operating on the same channel, indicated by dashed gray arrows directed to radio r (other lines withheld for clarity). In our scenarios, there are no connections between units; that is, disparate MANETs are not connected via a *backhaul network*.

We use a basic *signal-to-interference ratio* (*SIR*) model to calculate both co-channel interference and the strength of desired wireless transmissions between intra-unit radios. We consider only co-channel interference, as adjacent-channel and other harmonic interference are negligible due to orthogonal separation and white space between channels. We calculate the *received signal strength* (*RSS*) along all wireless arcs $(r, s) \in W$ in watts using the standard link budget formula [18]. While SIR is far from the only consideration in determining radio performance, it is often the limiting factor in determining the ability to reuse a channel [19], [20], especially in our scenarios where radio propagation is greatly affected by rough terrain and radio mobility [4], [21].

Common methods of calculating signal propagation include the Irregular Terrain Model (ITM) [22] and Hata-COST 231 [23]; we use Systems Toolkit (STK) [24] and the Terrain Integrated Rough Earth Model (TIREM) [25] to instantiate our scenarios and calculate total path loss.

For each radio, we pre-calculate the maximum allowable interference $\mathit{max\_interference}_s^c$ before the radio is disconnected from its MANET (see [4] for details). The magnitude of co-channel interference along all arcs $(r, s) \in W$ for each available channel $c \in C$ is indicated by $\mathit{interference}_{rs}^c$. Pairwise interference between radios r and s may be modeled as:

$$\mathit{interference}_{rs}^c X_r^c X_s^c \leq \mathit{max\_interference}_s^c \quad \forall (r, s) \in W, c \in C. \tag{6}$$

That is, a radio $s \in R$ may be assigned channel $c \in C$ only if the interference received from any other single radio r on channel c is at or below the pre-calculated $\mathit{max\_interference}_s^c$ threshold (note there is no interference between radios assigned to the same unit). Following [10], [12], [19], [26], [27], we consider the cumulative effect of all interference sources. That is, a radio may be unable to use a channel because the total sum of interference exceeds the threshold $\mathit{max\_interference}_s^c$, even if the interference received from any single radio is less than the threshold. Summing along all arcs yields:

$$\sum_{r:(r,s)\in W} \mathit{interference}_{rs}^c X_r^c X_s^c \leq \mathit{max\_interference}_s^c \quad \forall s \in R, c \in C. \quad (7)$$

To linearize these constraints, we introduce the variable $Z_{rs}^c \in \{0,1\}$ where:

$$Z_{rs}^c = \begin{cases} 1, & \text{if } X_r^c = X_s^c = 1 \\ 0, & \text{otherwise} \end{cases} \quad \forall r, s \in R, c \in C \quad (8)$$

which is enforced via:

$$Z_{rs}^c \geq X_r^c + X_s^c - 1 \quad \forall r, s \in R, c \in C \quad (9)$$
$$Z_{rs}^c \leq X_{r,c} \quad \forall r, s \in R, c \in C \quad (10)$$
$$Z_{rs}^c \leq X_{s,c} \quad \forall r, s \in R, c \in C. \quad (11)$$

We thus obtain our cumulative co-channel interference constraints:

$$\sum_{r:(r,s)\in W} \mathit{interference}_{rs}^c Z_{rs}^c \leq \mathit{max\_interference}_s^c \quad \forall s \in R, c \in C. \quad (12)$$

The goal of MO-CAP is to minimize the total number of required channels, so its objective function is:

$$\min \sum_{c \in C} Y^c. \quad (13)$$

The objective of MI-CAP is to minimize total interference, so its objective function is:

$$\min \sum_{r:(r,s)\in W} \sum_{c \in C} \mathit{interference}_{rs}^c Z_{rs}^c. \quad (14)$$

MO-CAP and MI-CAP are pure 0-1 integer programs. MO-CAP comprises constraints (2)-(3), (5), (9)-(12) and objective function (13); MI-CAP differs in its objective function (14). To consider these problems at multiple time steps, one may add an additional index, say $t \in T$, to each variable and input parameter. Also, the objective functions (13) and (14) could easily be modified to penalize channel changes (see, e.g., [28]).

B. Realistic Datasets

To illustrate the computational difficulties of solving the cumulative co-channel interference CAP, we use realistic datasets depicting particular time-steps within high-fidelity simulations of Marine Corps combat operations. We consider three tactical Marine Air-Ground Task Force (MAGTF) scenarios, each with different network topologies. The first scenario, based on Major Combat Operation 1 [29] involves a *Marine Expeditionary Unit* (*MEU*) conducting an amphibious assault on an island. The second scenario, based on combat operations in Helmand Province, Afghanistan circa January 2010, is a *Marine Expeditionary Brigade* (*MEB*) conducting irregular warfare (IW) operations in a desert environment. Our final scenario, based on Integrated Security Construct B [30], is a *Marine Expeditionary Force* (*MEF*) conducting a major amphibious assault. A summary of these scenarios and their associated number of Marines, units, and radios is displayed in Table I. Each unit requires a channel assignment to support its individual MANET, and a single MANET may support up to 30 radios. See [4] for further details on our scenarios.

TABLE I. SIZE OF SCENARIOS BY NUMBER OF MARINES, UNITS, AND RADIOS, AND ASSOCIATED MO-CAP RELATIVE OPTIMALITY GAP AND APPROXIMATE SOLUTION TIMES USING CPLEX SOLVER (VERSION 12.6).

Scenario	Marines	Units	Radios	Relative Optimality Gap	Solution Time
MEU	2000	6	131	0%	$<$ 2 sec
MEB	15,000	24	641	0%	24 hours
MEF	60,000	118	1887	77%	$>$ 60 hours

Several characteristics of tactical military data communications make the CAP even more difficult to solve than for typical civilian applications. For example, in radio or television broadcast there are relatively few transmission towers and many nodes functioning only as receivers, whereas in our scenarios each node functions as both a transmitter and receiver. Also, our nodes may be on the move. They cannot benefit from specially-tuned transmission antennae, and instead use omnidirectional antennae that reduce their ability to project power in desired directions and increase their susceptibility to interference. Though mobile phone applications consider mobility, our node formations are denser relative to the transmission power of each radio. Our radios transmit from five to 50 watts, whereas most mobile phone handsets are limited to three watts [31] and cellular transmission towers to an effective five to ten watts [32]. Further, our radios occupy large bandwidths (each channel occupies 1.2 to 5 MHz). These factors decrease the ability to reuse channels, even if the associated CAP is solved to optimality.

C. Importance of Cumulative Interference

To avoid the computational difficulties of modeling cumulative interference, the vast majority of work on the CAP assumes only pairwise interference [10]–[13]. In the following numerical examples, we demonstrate why this is an unrealistic assumption in our scenarios.

First, for each of the roughly 1800 radios in the MEF scenario, we sum the total interference received from all other radios not assigned to the same unit. We then calculate the total percentage of interference that is captured by the single largest source of interference. Ideally this is a large percentage, indicating that we can use pairwise constraints to reasonably represent co-channel interference. Fig. 2 presents the results for each radio, where the vertical axis displays the percentage of total interference. On average, the single largest source of interference (blue line) accounts for 73.4% of total interference received by each radio. However, for about 34% of radios, this single source accounts for only half or less of total received interference. Hence, pairwise interference constraints

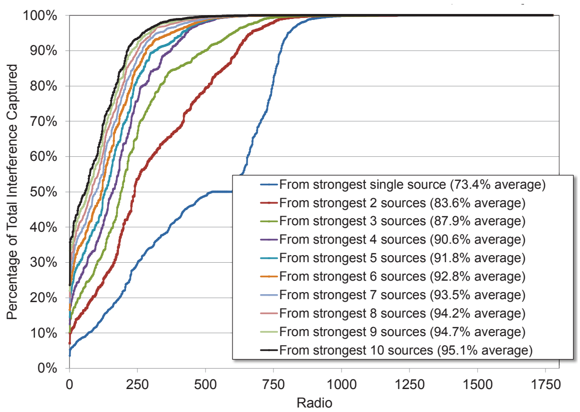

Fig. 2. The percentage of total interference captured by considering the strongest sources of interference (for one to ten sources), for each radio in the MEF scenario.

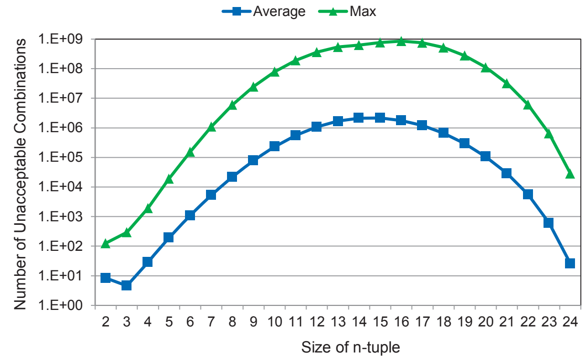

Fig. 3. Average and maximum number of unacceptable combinations of radios assigned to the same channel within the MEF scenario, by size of *n*-tuple.

(considering only the single strongest source of interference) would fail to capture a large portion of the total interference received by most nodes. By considering the strongest ten sources (black line in Fig. 2), on average 95.1% of interference is captured, and for less than four percent of radios would these ten sources capture less than 50% of total interference.

However, this analysis does not consider the maximum allowable interference $max\_interference_s^c$. That is, capturing a large percentage of interference isn't necessary if this interference is well below a radio's threshold and would not affect the ability of the radio to operate (i.e., violate the interference constraint). Over 1200 radios in the MEF scenario receive unacceptable interference from only one unit. These unacceptable combinations can be modeled as pairwise constraints (6), but this doesn't consider combinations greater than two. To examine this, we consider the total number of unacceptable combinations of radios by the size of combination or *n-tuple*, e.g., pairs, triples, etc. The results, averaged over all radios in the MEF scenario, are presented in Fig. 3. On average for each radio, there are about ten pairs of units that provide unacceptable interference. Not counting these pairs, on average for each radio there are six triplets of units that provide unacceptable interference. Not counting these pairs or triplets, there are on average 51 quadruplets, etc. The maximums for each *n*-tuple are much higher. Clearly, only using pairwise interference constraints – as in most of the CAP literature – does not realistically model our interference environment. Palpant et al. [12] represent *all* unacceptable combinations using pairwise constraints; this greatly over-constrains the problem and results in inefficient solutions.

III. Solution Methods and Challenges

We next describe the most common CAP solution methods, and the associated computational challenges presented by our realistic datasets and cumulative co-channel interference constraints (12).

A. Integer Optimization

The most common method of solving the MO-CAP is using exact *integer optimization* methods with variations of *combinatorial tree search*, including *branch-and-bound*, *branch-and-cut*, and *implicit enumeration* [11], [26], [27], [33], [34] coupled with heuristics that allow bounding of the optimal solution. In general, these methods navigate the solution tree by selecting variables to fix, solving the associated sub-problem, and using the result to update upper and lower bounds in order to *fathom* (i.e., cut off) suboptimal portions of the tree. Solving sub-problems is generally done via *linear programming* (*LP*) *relaxation*, i.e., relaxing the integer constraints of the decision variables and solving using a variation of the *simplex method* or other LP method [35]–[37].

The most obvious computational challenge affecting all solution techniques is the sheer size of the problem. The problem grows exponentially in both the number of units and channels [4], [17]. Of course, a large portion of these solutions may be either infeasible (i.e., violating the interference constraints), or inefficient (i.e., MO-CAP using far more channels than necessary). In general, combinatorial tree search methods are very good at fathoming these sections of the tree, but our realistic datasets and constraints provide several challenging computational hurdles.

First, commercial solvers may be sensitive to vast differences in input parameters, such as our interference values which range from extremely small (for wireless arcs that experience high propagation losses) to quite large (for wireless arcs between radios that are next to each other). These values vary by 24 orders of magnitude, and are generally quite small. Per CPLEX documentation, the solver may have difficulty when objective function and constraint coefficients vary by six or more orders of magnitude [38]. Further, many solvers, including CPLEX, are limited to double floating-point precision [39], and thus are unable to tell the difference between numbers smaller than 1.0×10^{-15} or 1.0×10^{-16} [40]. Also, non-integral data will result in highly fractionalized LP solutions, as the solver will attempt to "pack" the most units (including fractions of units) onto the same channel; these fractional solutions must then undergo a computationally-costly *repair* process to become integer-feasible.

Another computational problem (also identified by [12]) is that of *symmetry*, which occurs when channel assignments may be changed without altering the objective value [41]. There are performance differences between channels on different frequencies, i.e., lower frequency channels will, *ceteris paribus*, propagate farther than higher frequencies and thus may provide more interference. However, these differences may be very slight or even indistinguishable between proximate channels. When conducting a tree search over problems exhibiting near symmetry, solvers may waste computational time considering many different solutions that provide essentially identical utility [41]. The very near symmetry of our dataset (as opposed to exact symmetry) is especially difficult for solvers to detect and mitigate [42]–[44].

Perhaps the most difficult challenge to commercial solvers are the cumulative interference constraints (12). Most commercial LP solvers leverage the *sparse* nature of a problem by considering only subsets of variables at a time. Consider a pairwise interference constraint, i.e.,

$$\textit{interference}^c_{rs} Z^c_{rs} \leq \textit{max\_interference}^c_s \quad \forall r, s \in R, c \in C. \quad (15)$$

The system of linear equations formed by these constraints would be very sparse, i.e., each row may contain only one nonzero coefficient (representing a pairwise constraint); all other column entries would be zero. However, in our cumulative interference constraints (12), a row may contain hundreds of nonzero coefficients, and thus the overall constraint matrix is much more dense.

The vast majority of exact optimization work on the CAP consider only pairwise interference constraints [11]. Dunkin et al. [10] describe the computational challenge of cumulative interference, and suggest methods of using binary and tertiary constraints. Daniels et al. [45] formulate an integer MO-CAP that considers cumulative interference, and establish the *NP-hardness* of the problem. Their heuristic provides solution values within 3% of those provided by CPLEX, but the impact of cumulative interference in their problem sets is far less than in ours. Fischetti et al. [26] use *pre-processing* and branch-and-cut to solve their CAP, and tune their *Big M* value to improve convergence performance. They solve a number of real-world problem instances in a reasonable amount of time, but their problem sizes are much smaller than those described here and consider few sources of interference.

Table I presents the results of solving MO-CAP using our datasets and the CPLEX optimizer (version 12.6) [39] using branch-and-cut. We use a Dell Precision T5500 desktop computer with twelve 3.47 GHz Xeon processors and 72 gigabytes (GB) of random access memory (RAM). It takes nearly 24 hours to obtain an exact solution to the MEB scenario. The MEF scenario solution time illustrates the computational difficulties of this problem. Initially, we are unable to find any solution to the MEF dataset, even after two weeks of computation. We change our approach and use a heuristic (developed by [4] and described in [17]) to find an initial feasible solution and provide it to a distributed implementation of CPLEX to solve using a 14-computer cluster. After approximately 60 hours of computation, the solver improves upon our initial solution of 46 channels to 35 channels; this inexact solution has a relative *optimality gap*, i.e., distance to a known (albeit integer infeasible) lower bound of 77%.

B. Constraint Optimization

Another applicable exact optimization method, first suggested for cumulative interference CAPs by [46], is based on *constraint satisfaction problems* (*CSPs*). CSPs determine if there exists a consistent assignment of Boolean variables that satisfies a system of logical constraints. Related *constraint optimization* problems aim to find a solution which minimizes penalties associated with violating these logical constraints [47]. Our CAP consists entirely of binary variables and can be formulated as a constraint optimization problem, where each cumulative interference constraint is represented in the *conjunctive normal form* usually used to express constraints to satisfaction (*SAT solvers*) [48]. Dunkin et al. [10] model their problem and solve using custom CSP code, but they consider only groups of seven or fewer transmitters for their dataset of 37 transmitters. The logical clauses associated with our datasets are much larger and may be beyond the ability of current constraint satisfaction solvers. However, constraint satisfaction techniques may be useful in solving sub-problems within a larger CAP solution framework. Palpant et al. [12] solve their cumulative interference CAP using a hybrid of constraint programming and heuristic methods, and provide comparable or better performance than heuristic-centric methods (specifically, [49] and [50]) in a competition using a dataset from a military application. Constraint satisfaction may also be used within a *Benders decomposition* framework (see, e.g., [51]–[53]).

C. Heuristics

Due to the computational difficulties of exactly solving the CAP, heuristics are often used to solve the problem [11], [27]. Heuristics that consider cumulative interference include *neighborhood search* [12], [54], *simulated annealing* [49], [55], *tabu search* [50], [55]–[57], *ant colony optimization* [58], *greedy heuristics* [4], [17], [28], [45], [48], [59], and a combination of greedy and exact methods [12], [60]. Heuristics are generally used to support *dynamic spectrum access* (*DSA*), a broad term that refers to dynamic (rather than fixed) allocation. In general, DSA technology assumes channels can be changed dynamically by each radio with little or no cost [61]. For our application there is a cost (namely, time) associated with changing channels, so these technologies are not directly applicable. See [61]–[63] for surveys of dynamic spectrum access technology.

While heuristics can often provide useful solutions in reasonable amounts of time, in general they do not provide *certificates of optimality* for any particular solution, i.e., the distance to the global optimum is unknown. We feel these bounds are important for understanding the goodness of a particular solution, especially since spectrum is so scarce. Based on input from a Marine Corps spectrum manager with deep knowledge of real-world conditions and the MEF scenario, the actual expected allocation of channels (i.e., the number likely to be assigned for use by the MEF from higher headquarters) is 14 [4]. Thus, even the best-known solution of 35 is still far from the actual number of allocated channels. If we can find better solutions faster, or state with a greater degree of certainty that the actual allocation of 14 is not sufficient, our spectrum manager could request additional spectrum, or then solve the MI-CAP to provide the least total interference with the allocated number of channels. Further research is needed to improve our ability to find provably good solutions.

IV. TOPICS FOR FUTURE RESEARCH

There are many facets of this problem that are ripe for new development; we provide the following suggestions for future research.

A. Preprocessing

New heuristics may be developed to reduce the size of the input and/or provide initial feasible solutions. The use of *clustering algorithms* and research related to *packing problems* seems to be a natural fit (e.g., [64]–[68]), as does smart preprocessing based on constraint satisfaction or constraint optimization solvers (e.g., [69]–[71]).

Smart preprocessing of the cumulative interference constraints (12) could reduce the numerical issues associated with the $interference_{rs}^{c}$ and $max\_interference_{s}^{c}$ values. Given a dataset, we can preprocess the interference constraints to create simplified and more computationally tractable packing constraints. For example, suppose two specific nodes r and s (not assigned to the same unit) are not both allowed to be assigned to channel c because to do so would violate the associated interference constraint. This may be represented as:

$$X_r^c + X_s^c \leq 1. \tag{16}$$

To generalize for larger *n*-tuples of units (triplets, quadruplets, etc.), let $S \subset R$ be a subset of radios (none assigned to the same unit) that cannot all be assigned to the same channel c. Then

$$\sum_{r \in S} X_r^c \leq |S| - 1. \tag{17}$$

Preprocessing all such unacceptable combinations may thus replace the cumulative co-channel interference constraints (12). For our datasets, preprocessing *n*-tuples of up to five or six nodes seems practical (see Fig. 3) and may provide reasonable, if not feasible, approximate solutions very quickly. This could then be combined with additional rounds of optimization designed to reduce or eliminate remaining infeasibility via a series of additional cuts. Such multi-step approaches are certainly worth investigating.

B. Parallel and Distributed Computation

Computing technology has advanced significantly since much of the work on CAPs in the late 2000s. Most computers and even smartphones have multiple cores, yet most algorithms specifically developed for solving CAPs are serial and do not take advantage of parallel and distributed computation. The problem has structure that seems to naturally lend itself to decomposition (e.g., into physical neighborhoods of radios, or by separate time steps), increasing the desirability of applying parallel and distributed techniques. The work of [69]–[72] could be extended to look at CAPs. Further, most integer solvers remain serial in nature, though new versions of both CPLEX and Gurobi enable distributed implementations. Custom coding could also leverage parallel and distributed computation, e.g., the Python `dispy` library [73].

C. Numerical Precision

We demonstrate some of the numerical problems associated with the generally very small and wide-ranging interference values in our realistic datasets. Gurobi is capable of using quadruple-precision floating point variables [74], [75]. Custom optimization code could be developed to handle even higher precision calculations. For example, the Python `mpmath` library enables arbitrary-precision floating point variables, limited in size only by available RAM [76].

D. Temporal Considerations

A seldom-researched challenge of the mobility-aware CAP is channel allocations changing over time, and not just at certain points in time, i.e., a *myopic* solution. Such a solution may needlessly flip-flop channel assignments, and may be particularly fragile to changes in physical network topologies. The movement of radios in a military environment is far from arbitrary [21], [77]; by leveraging available information on the future locations of radios and considering the effects of network perturbations (such as degraded signal quality), one can provide a more far-sighted and robust solution to reduce the number of required channel changes over time. This decreases the time used by operators to manually adjust radio configurations, and the time needed by the spectrum manager to de-conflict unexpected interference.

Changes over time make the challenges we consider that much more difficult, as now we must compute over multiple time steps. Most of the methods we reference in this paper are generally applied to *fixed* CAPs, where assignments are permanent or not expected to change quickly. *Dynamic* CAPs consider frequent channel changes, but most of these methods apply fixed CAP methodologies (usually heuristics), or employ schemes for borrowing channels between radios, without consideration of reducing reassignments over time. See [19] for a survey on dynamic CAPs.

An interesting idea worth further exploration is the use of *temporal* or *evolving graphs* to model network changes over time [78], [79]. Rather than just a series of snapshots, temporal graphs have structure in themselves that can be considered as a whole [28]. Casteigts et al. [80] provide an overarching framework of time-varying graphs in pursuit of general properties, and mention that very little work has been done in this area. In a seminal work, [81] describes the use of such graphs to consider time-varying MANETs. This work was further extended by [82] and [83]. Scellato et al. [84] present, apparently for the first time, a measure of temporal robustness for mobile networks, an area of research that surely is applicable to military problems.

Yu et al. [28] present a unique methodology using temporal graphs. They develop several heuristics to solve their multi-objective optimization problem to minimize the number of required channels (i.e., MO-CAP), while also considering cumulative co-channel interference and the cost of changing channels over time. To our knowledge, exact optimization techniques have not been applied to this particular problem, nor has a corresponding MI-CAP methodology been developed.

V. Conclusions

The increasing demands on scarce EM spectrum creates a need for highly efficient channel allocations. Many military EM systems, including independent tactical MANETs, single-channel radios, radars, and jammers, continue to require manual channel configuration and thus cannot directly benefit from technologies such as DSA. With the computational power now available in most desktop computers, we believe the time is right to develop new methods of solving the CAP considering both the effects of cumulative interference and the cost of manually configuring each radio (i.e., changing channels) over time. Armed with this ability, a spectrum manager is better able to estimate channel requirements before an operation, and efficiently utilize available channels and reduce manpower requirements during an operation.

We believe the most promising approach to this problem is a hybrid, combining and iterating between smart heuristics to preprocess the problem, and exact optimization methods using parallel and distributed computation to find new lower bounds and calculate optimality gaps. The future is ripe with research opportunities on this challenging and timely problem.

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