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FURTHER CONSTRAINTS ON ELECTRON ACCELERATION IN SOLAR NOISE STORMS

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Abstract. We reexamine the energetics of nonthermal-electron acceleration in solar noise storms. A new result is obtained for the minimum nonthermal-electron number density required to produce a Langmuir-wave population of sufficient intensity to power the noise-storm emission. We combine this constraint with the stochastic electron acceleration formalism developed by Subramanian and Becker (2005) to derive a rigorous estimate for the efficiency of the overall noise-storm emission process, beginning with nonthermal-electron acceleration and culminating in the observed radiation. We also calculate separate efficiencies for the electron acceleration–Langmuir wave generation stage and the Langmuir wave–noise-storm production stage. In addition, we obtain a new theoretical estimate for the energy density of the Langmuir waves in noise-storm continuum sources.

1. Introduction

Noise-storm radio emission from the solar corona is typically manifested as long-lived, broadband noise (continuum) radiation at metric/decimetric wavelengths, with occasional short-lived (0.1 – 1 second), intense, narrowband bursts. The storms are typically observed in the 50 – 500 MHz range and are brightest around 150 – 200 MHz. At 164 MHz, the peak intensity of noise-storm continua is ≈ 20 SFU, while that of the short bursts is ≈ 100 SFU (Kerdraon, 1973, 1979; Malik and Mercier, 1996).

1.1. Estimate of Total Power

Aside from solar flares, noise storms are the only known sites of electron acceleration in the corona. Estimates of the power budget involved in the electron acceleration process are few and sketchy, and this will be the focus of our paper. We first turn our attention to the power emitted in noise-storm radiation. Elgarøy

(1977) used the oft-quoted multi-wavelength observational study of 28 noise storms conducted by Smerd (1964) to arrive at a figure of $F_{\rm out} \approx 3 \times 10^{-12}\,{\rm W\,m^{-2}}$ for the frequency-integrated energy flux measured at the Earth for a typical noise storm. He then estimated the total noise-storm power, $L_{\rm out}$, by writing

$$L_{\rm out} = F_{\rm out}\, R^2\, \Omega\, e^{-\tau}, \tag{1}$$

where R is the Sun–Earth distance, Ω is the solid angle through which the radiation is beamed, and τ is the optical depth of the source. By setting $\tau = 2$ and $\Omega = 0.6$ steading, Elgarøy obtained the canonical result

$$L_{\rm out} \sim 10^{17}\,{\rm erg\,s^{-1}}. \tag{2}$$

Solar noise-storm radiation is believed to form via a two-stage process. First, thermal electrons are accelerated to form a nonthermal-electron population and second, the Langmuir waves produced by the nonthermal-electrons are converted into observable radio emission via coalescence with a suitable population of low-frequency waves. Melrose (1975), Robinson (1978), and Wentzel (1985) have considered the issue of Langmuir-wave generation by a population of electrons. While any electron population will generate some level of Langmuir waves via spontaneous emission, the production of bright noise storms requires an appreciable power in the Langmuir waves.

1.2. Review of Previous Results

In a previous paper, we have analyzed the evolution of the Green's function ($f_{\rm G}$) describing the nonthermal-electron distribution resulting from the stochastic acceleration of particles injected with a mono-energetic distribution (Subramanian and Becker, 2005, hereafter paper 1). In a steady-state situation, the Green's function is governed by a linear, partial-differential Fokker–Planck transport equation with momentum diffusion coefficient $\mathcal{D}$ given by

$$\mathcal{D} = D_0\, p^2, \tag{3}$$

where D_0 is a constant with the units of inverse time. This specific form for $\mathcal{D}$ is motivated by a number of particle transport scenarios such as the acceleration of electrons by large-scale, compressible-magnetohydrodynamical (MHD) turbulence (Ptuskin, 1988; Chandran and Maron, 2003), the energization of electrons by cascading fast-mode waves in flares (Miller, LaRosa, and Moore, 1996), and the acceleration of electrons due to lower-hybrid turbulence (Luo, Wei, and Feng 2003).

The resulting Green's function for the nonthermal-electron distribution is found to be (see paper 1; also Subramanian, Becker, and Kazanas, 1999)

$$f_G(p, p_0) = A_0 \begin{cases} (p/p_0)^{\alpha_1}, & p \le p_0, \\ (p/p_0)^{\alpha_2}, & p \ge p_0, \end{cases} \tag{4}$$

where p is the electron momentum, p_0 is the momentum of the injected mono-energetic electrons, and the exponents α_1 and α_2 are given by

$$\alpha_1 \equiv -\frac{3}{2} + \left(\frac{9}{4} + \frac{1}{D_0 \tau}\right)^{1/2}, \qquad \alpha_2 \equiv -\frac{3}{2} - \left(\frac{9}{4} + \frac{1}{D_0 \tau}\right)^{1/2}. \tag{5}$$

The quantity τ in these expressions represents the mean residence time for electrons in the acceleration region and the normalization parameter A_0 is computed using

$$A_0 \equiv \frac{\dot{N}_0}{2 D_0 p_0^3} \left(\frac{9}{4} + \frac{1}{D_0 \tau}\right)^{-1/2}, \tag{6}$$

where the constant $\dot{N}_0$ denotes the number of electrons injected per unit time per unit volume into the acceleration region. The value of the total electron number density associated with the Green's function distribution is given by

$$n_G \equiv \int_0^\infty p^2 f_G dp = \dot{N}_0 \tau, \tag{7}$$

as expected in this steady-state situation. Since the nonthermal-electrons are non-relativistic with kinetic energy $p^2/(2m_e)$, it follows that we must require $\alpha_2 < -5$ in order to avoid an infinite energy density, and therefore $D_0 \tau < 10^{-1}$.

In paper 1, we applied the formalism described above to model the transport in momentum space of electrons injected from the high-energy portion of the Maxwellian distribution in the corona. We found that stochastic acceleration of the electrons dominates over losses due to collisions and Langmuir damping for particles with momenta $p > p_c$, where

$$p_c \equiv 5.35 \times 10^{-22} \left(\frac{\Lambda n_e}{D_0}\right)^{1/3} \tag{8}$$

denotes the "critical momentum" in cgs units, Λ is the Coulomb logarithm, and n_e represents the total electron number density in the corona. Electrons with $p > p_c$ experience net acceleration on average and those with $p < p_c$ are decelerated on average. It is expected that a "gap" distribution will form as a result of the collisional and Langmuir losses experienced by electrons with $p < p_c$.

Based on the analysis of a generic second-order Fermi (stochastic) acceleration mechanism, we demonstrated in paper 1 that if all of the electrons in the Maxwellian distribution with $p > p_c$ are subject to acceleration, then the nonthermal-electron fraction is given by

$$\frac{n_*}{n_e} = \frac{2\xi_c^{(3+\alpha_1)/2}\Gamma(-\alpha_1/2, \xi_c)}{\sqrt{\pi}(3+\alpha_1)(\alpha_2-\alpha_1)D_0\tau} + 2e^{-\xi_c}\left(\frac{\xi_c}{\pi}\right)^{1/2} + \mathrm{Erfc}\left(\xi_c^{1/2}\right), \tag{9}$$

where n_* denotes the nonthermal-electron number density, α_1 and α_2 are the power-law indices given by Equations (5), and the dimensionless critical electron energy, ξ_c, is defined by

$$\xi_c \equiv \frac{p_c^2}{2m_e k_B T_e}, \tag{10}$$

with T_e denoting the temperature of the thermal electrons, m_e the electron mass, and k_B Boltzmann's constant.

The quantities α_1 and $D_0\tau$ can be expressed in terms of the high-energy power-law index α_2 by using Equations (5) to write

$$\alpha_1 = -3 - \alpha_2, \qquad D_0\tau = \frac{1}{\alpha_2(3+\alpha_2)}. \tag{11}$$

These relations can be combined with Equation (9) to obtain the alternative result for the nonthermal-electron fraction

$$\frac{n_*}{n_e} = -\frac{2\xi_c^{-\alpha_2/2}(3+\alpha_2)\Gamma\left(\frac{3+\alpha_2}{2}, \xi_c\right)}{\sqrt{\pi}\,(3+2\alpha_2)} + 2e^{-\xi_c}\left(\frac{\xi_c}{\pi}\right)^{1/2} + \mathrm{Erfc}\left(\xi_c^{1/2}\right), \tag{12}$$

which is convenient because the right-hand side is now an explicit function of α_2 and ξ_c. In paper 1, both α_2 and n_*/n_e were treated as free parameters; however, in the present paper, the second quantity is computed self-consistently using Equation (12). Hence, the only remaining free parameter is α_2.

2. Langmuir Wave Generation and the "Gap" Electron Distribution

For the sake of simplicity, we concentrate here on isotropic electron distributions, in which case the most suitable candidate for producing a sufficiently intense population of Langmuir waves is a gap electron distribution (Melrose, 1975). Such a gap distribution is often idealized as a narrow "hump" of nonthermal-electrons located far above the thermal distribution in the energy space. Melrose (1975) elaborates on the conditions for the formation of a gap distribution and on the parameters of such a distribution required to ensure a sufficiently intense population of Langmuir waves. The gap is the result of collisional and Langmuir losses experienced by particles in the energy range between the typical thermal energy and highly-superthermal energies. In a steady state, the intensity of the Langmuir waves is governed by the balance between emission and absorption of the waves by the thermal and nonthermal-electrons. A gap electron distribution with the right parameters allows superthermal electrons to emit Langmuir waves without reabsorbing them, resulting in a high wave intensity.

We now turn our attention to the specific parameters of a gap electron distribution that will enable it to produce a high-intensity Langmuir wave population, which, in turn, is needed to produce bright noise-storm emission. Consider a gap distribution where the nonthermal-electrons are peaked around the critical momentum (p_c) given by Equation (8). In this case, Melrose (1975) points out that in order to form an intense Langmuir wave distribution, the number density of the nonthermal-electrons must be high enough for them to dominate over the thermal electrons in emitting and absorbing waves with phase velocities $v_\phi \approx v_c$, where $v_c \equiv p_c/m_e$ is the "critical velocity." The phase velocity of the Langmuir waves is related to the wavenumber (k) via

$$v_\phi = \frac{\omega_p}{k}, \tag{13}$$

where

$$\omega_p = \left(\frac{4\pi e^2 n_e}{m_e}\right)^{1/2} \tag{14}$$

denotes the electron plasma frequency, and e is the electron charge. The formation of a high-intensity (or, equivalently, a high-brightness-temperature) Langmuir wave population requires that the wave damping due to the nonthermal-electrons must dominate over that due to the thermal electrons for waves with wavenumber $k \approx k_c$, where

$$k_c \equiv \frac{\omega_p}{v_c} = \frac{\omega_p m_e}{p_c}. \tag{15}$$

In the following analysis, we will concentrate on the constraints imposed by this criterion on the physical mechanism that accelerates the nonthermal-electrons with $p \geq p_c$.

Based on Equation (20) from Robinson (1978), we write the collisional damping rate for the Langmuir waves in cgs units as

$$\gamma_0 = 30\frac{n_{th}}{T_e^{3/2}} \propto \mathrm{s}^{-1}, \tag{16}$$

where n_{th} is the number density of the thermal electrons, and the value of γ_0 is independent of the wavenumber (k). Note that the thermal-electron number density (n_{th}) is essentially equal to the total electron number density (n_e) appearing in Equation (8) because only the small population of electrons with $p > p_c$ in the tail of the Maxwellian experiences acceleration. The wave damping rate due to interactions with the nonthermal-electrons in the gap distribution is given by Equation (18) from Robinson (1978), which states that

$$\gamma_{gap}(k) = \frac{8\pi^2 n_* e^2 \omega_l^2(k)}{k^3 c^2 p_c}, \tag{17}$$

where n_* ($\ll n_{\rm th}$) is the number density of the nonthermal-electrons and the Langmuir frequency ($\omega_{\rm l}(k)$) is given by the dispersion relation (*e.g.*, Robinson, 1978)

$$\omega_{\rm l}^2(k) = \omega_{\rm p}^2 + 3k^2 v_{\rm th}^2 \tag{18}$$

for electrons with characteristic thermal velocity $v_{\rm th} \equiv (k_{\rm B} T_{\rm e}/m_{\rm e})^{1/2}$.

The requirement that the nonthermal-electrons dominate over the thermal particles in damping the Langmuir waves with $k \approx k_{\rm c}$ can be stated quantitatively as

$$\gamma_{\rm gap}(k_{\rm c}) > \gamma_0, \tag{19}$$

or equivalently,

$$\frac{n_*}{n_{\rm e}} > 1.5 \times 10^{39} \frac{k_{\rm c}^3 p_{\rm c}}{T_{\rm e}^{3/2} \omega_{\rm l}^2(k_{\rm c})}, \tag{20}$$

in cgs units where we have employed Equations (16) and (17) along with the fact that $n_{\rm th} \approx n_{\rm e}$ to obtain the final result. Equation (20) represents an interesting constraint on the nonthermal-electron fraction ($n_*/n_{\rm e}$) in the solar corona during the production of the noise-storm emission. The right-hand side of Equation (20) can be stated in terms of $p_{\rm c}$, $T_{\rm e}$, and $\omega_{\rm p}$ by employing Equations (15) and (18), which yields

$$\frac{n_*}{n_{\rm e}} > 1.1 \times 10^{-42} \frac{\omega_{\rm p}}{T_{\rm e}^{3/2}(p_{\rm c}^2 + 3k_{\rm B} T_{\rm e} m_{\rm e})}. \tag{21}$$

Satisfaction of this inequality ensures that the intensity of the Langmuir waves generated by the gap distribution is sufficient to power the observed noise-storm emission.

3. Acceleration of Nonthermal Electrons

We can use Equation (10) to re-express the right-hand side of Equation (21) in terms of $\xi_{\rm c}$, $\omega_{\rm p}$, and $T_{\rm e}$, which yields

$$\frac{n_*}{n_{\rm e}} > \frac{8.9\omega_{\rm p}}{T_{\rm e}^{5/2}(2\xi_{\rm c} + 3)} \tag{22}$$

in cgs units. The nonthermal density fraction must satisfy this inequality in order to guarantee a sufficiently intense Langmuir wave distribution. By substituting for the nonthermal-electron fraction ($n_*/n_{\rm e}$) in Equation (22) using Equation (12), we can show that the dimensionless critical energy ($\xi_{\rm c}$) must satisfy the corresponding inequality

$$\xi_{\rm c} < \xi_{\rm max}, \tag{23}$$

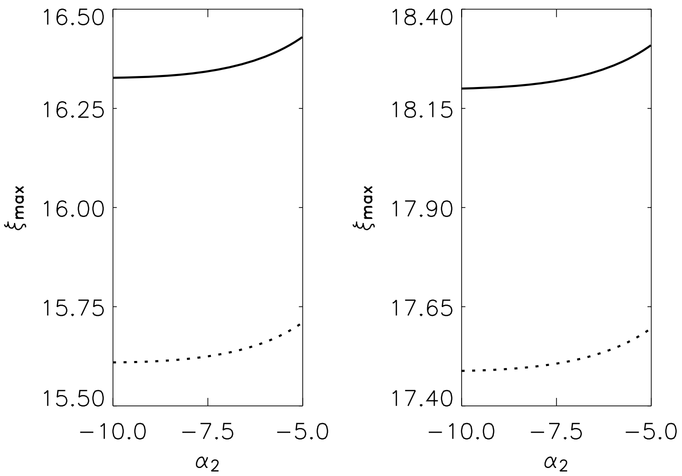

Figure 1. Maximum value of the dimensionless critical energy ($\xi_{\rm max}$) plotted as a function of α_2 using Equation (24). *Left panel*: $T_{\rm e} = 10^6$ K. *Right panel*: $T_{\rm e} = 2 \times 10^6$ K. *Solid line*: $\nu = 169$ MHz. *Dotted line*: $\nu = 327$ MHz.

where $\xi_{\rm max}$ is defined by the relation

$$\frac{4.4\omega_{\rm p}}{T_{\rm e}^{5/2}(\xi_{\rm max}+1.5)} = -\frac{2\xi_{\rm max}^{-\alpha_2/2}(3+\alpha_2)\Gamma\left(\frac{3+\alpha_2}{2}, \xi_{\rm max}\right)}{\sqrt{\pi}\,(3+2\alpha_2)} + 2e^{-\xi_{\rm max}}\left(\frac{\xi_{\rm max}}{\pi}\right)^{1/2} + \mathrm{Erfc}\left(\xi_{\rm max}^{1/2}\right). \quad (24)$$

The quantity $\xi_{\rm max}$, which is a function of $T_{\rm e}$, $\omega_{\rm p}$, and α_2, represents the *maximum value* of $\xi_{\rm c}$ such that the intensity of the associated Langmuir wave distribution is sufficient to power the observed noise-storm emission.

In Figure 1, we plot $\xi_{\rm max}$ as a function of α_2 for observing frequencies ($\nu \equiv \omega_{\rm p}/(2\pi)$) of 169 and 327 MHz and for coronal temperatures $T_{\rm e} = 10^6$ and 2×10^6 K. Figure 1 indicates that $\xi_{\rm max}$ has a moderate dependence on the observing frequency (ν) and the coronal temperature ($T_{\rm e}$), but it is quite insensitive to the value of α_2.

In Figure 2, we plot the nonthermal-electron fraction ($n_*/n_{\rm e}$) evaluated using Equation (12) with $\xi_{\rm c} = \xi_{\rm max}$ as a function of the high-energy power-law index (α_2) based on the same values for the observing frequency (ν) and coronal temperature ($T_{\rm e}$) used in Figure 1. Since we have set $\xi_{\rm c} = \xi_{\rm max}$, it follows that the curves in Figure 2 represent the *minimum* values for $n_*/n_{\rm e}$ such that the Langmuir wave distribution has sufficient intensity to produce the observed noise-storm emission. This ratio exhibits marked dependences on ν and $T_{\rm e}$, but it is clearly a very weak function of α_2. We note that for a coronal electron temperature ($T_{\rm e}$) of 10^6 K and an observing frequency (ν) of 169 MHz, the nonthermal-electron fraction ($n_*/n_{\rm e}$) indicated by Figure 2 is close to the value of 2.2×10^{-7}, which Thejappa (1991) demonstrated to be the threshold value for the transition between the production of steady noise-storm emission and the development of intense type I bursts. Based

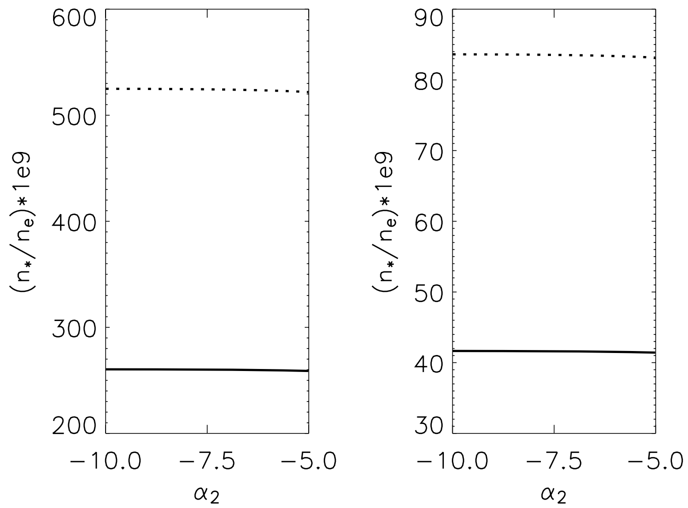

Figure 2. Minimum nonthermal-electron fraction (n_*/n_e) plotted as a function of α_2 using Equation (12) with $\xi_c = \xi_{max}$. *Left panel*: $T_e = 10^6$ K. *Right panel*: $T_e = 2 \times 10^6$ K. *Solid line*: $\nu = 169$ MHz. *Dotted line*: $\nu = 327$ MHz.

on Thejappa's result, the value $n_*/n_e = 2.2 \times 10^{-7}$ was adopted as an *ad hoc* input parameter in paper 1; however in the present paper, this ratio is calculated self-consistently based on the physics of the electron acceleration–Langmuir wave generation processes. We provide further discussion of the relationship between our model and that of Thejappa in Section 6.

4. Nonthermal Electron Input Power

In paper 1, we demonstrated that the energy density of the accelerated particles is given by

$$\frac{U_*}{n_e k_B T_e} = \frac{2\xi_c^{(5+\alpha_1)/2}\Gamma\left(-\frac{\alpha_1}{2}, \xi_c\right)}{\sqrt{\pi}(5+\alpha_1)(\alpha_2-\alpha_1)D_0\tau} + \frac{2\sqrt{\pi\xi_c}(3+2\xi_c)e^{-\xi_c} + 3\pi\,\mathrm{Erfc}\left(\xi_c^{1/2}\right)}{2\pi(1-10D_0\tau)}. \tag{25}$$

By utilizing Equations (11) to eliminate α_1 and $D_0\tau$, we can obtain the alternative form

$$\frac{U_*}{n_e k_B T_e} = \alpha_2(3+\alpha_2)\left[\frac{2\xi_c^{(2-\alpha_2)/2}\Gamma\left(\frac{3+\alpha_2}{2}, \xi_c\right)}{\sqrt{\pi}(2-\alpha_2)(3+2\alpha_2)} + \frac{2\sqrt{\pi\xi_c}(3+2\xi_c)e^{-\xi_c} + 3\pi\,\mathrm{Erfc}\left(\xi_c^{1/2}\right)}{2\pi(\alpha_2^2+3\alpha_2-10)}\right], \tag{26}$$

where the right-hand side is now an explicit function of α_2 and ξ_c.

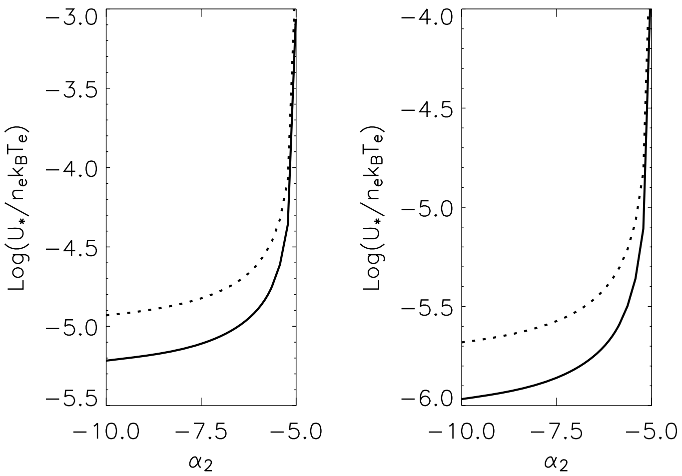

Figure 3. Minimum nonthermal energy density ratio ($U_*/(n_e k_B T_e)$) plotted as a function of α_2 using Equation (26) with $\xi_c = \xi_{max}$. *Left panel*: $T_e = 10^6$ K. *Right panel*: $T_e = 2 \times 10^6$ K. *Solid line*: $\nu = 169$ MHz. *Dotted line*: $\nu = 327$ MHz.

In Figure 3, we plot the energy density ratio ($U_*/(n_e k_B T_e)$) as a function of α_2 and T_e evaluated using Equation (26) with $\xi_c = \xi_{max}$. The curves therefore represent the *minimum* values for $U_*/(n_e k_B T_e)$ that are consistent with the required intensity of the Langmuir waves. In contrast with the nonthermal-electron fraction (n_*/n_e) plotted in Figure 2, we note that the energy-density ratio displays a marked dependence on the high-energy power-law index (α_2), diverging as $\alpha_2 \to -5$ as expected.

Based on Equations (23) and (29) from paper 1, we can express the total power required to drive the acceleration of the nonthermal-electrons as

$$L_{in} = 8 V D_0 U_*, \tag{27}$$

where U_* is given by Equation (26), V is the volume of the acceleration/emission region, and the diffusion constant (D_0) can be evaluated in terms of ξ_c using (see Equations (8) and (10))

$$D_0 = \frac{1.2 \Lambda n_e}{\xi_c^{3/2} T_e^{3/2}}. \tag{28}$$

By combining our various relations, the required input power (L_{in}) can be evaluated in terms of α_2, ξ_c, n_e, T_e, Λ, and V. In general, we set the Coulomb logarithm (Λ) = 29.1 (*e.g.*, Brown, 1972). We find that by setting ξ_c equal to the maximum value ξ_{max} given by Equation (24), we obtain the *minimum* value for the input power (L_{in}) consistent with the constraint that the intensity of the generated Langmuir waves be sufficient to power the observed noise-storm emission. Hence by setting $\xi_c = \xi_{max}$ and selecting specific values for the physical parameters n_e, T_e and V, we can use Equation (27) to compute the minimum input power (L_{in}) as a function of only one parameter, α_2.

5. Results

5.1. Efficiency of overall electron acceleration – noise-storm continuum radiation process

Once the upper limit for the dimensionless critical electron energy ($\xi_{\max}$) has been computed using Equation (24), we can use Equation (27) to obtain the minimum value for the input power ($L_{\rm in}$) consistent with the required intensity of the Langmuir waves. The efficiency of the overall noise-storm emission process, beginning with the acceleration of the nonthermal-electrons and ending with the production of the observed radiation, is characterized by the efficiency parameter (η) defined by

$$\eta \equiv \frac{L_{\rm out}}{L_{\rm in}}, \tag{29}$$

where $L_{\rm out}$ denotes the observed power of the noise-storm emission. Since we use the minimum possible value of $L_{\rm in}$, it follows that the we are in fact computing the *maximum value* of the efficiency (η). The crucial difference between the approach taken here and that used in paper 1 is that in the present treatment we set $\xi_{\rm c} = \xi_{\max}$, which allows us to obtain the final result for the overall efficiency (η) in terms of the single parameter α_2.

The maximum value of the overall efficiency (η) is plotted in Figure 4 as a function of α_2 for the same two values of the coronal electron temperature ($T_{\rm e}$) and observing frequency (ν) used in Figure 1. We set the output power and the emission region volume using $L_{\rm out} = 10^{17}\,{\rm erg\,s^{-1}}$ (see Equation (2)) and $V = 10^{30}\,{\rm cm}^3$, respectively. The total electron number density ($n_{\rm e}$) is calculated by setting $\omega_{\rm p} = 2\pi\nu$ and then employing Equation (14). In general, it is seen that the efficiency of the overall process is $\eta \lesssim 10^{-6}$.

5.2. Efficiency of first stage: electron acceleration-Langmuir wave generation

Having computed the efficiency of the overall emission process in Section 5.1, we next seek to evaluate the efficiency of the first stage of the scenario by calculating the ratio of the power in the Langmuir wave population ($L_{\rm L}$) to the power input to the electron acceleration process ($L_{\rm in}$). The former quantity can be calculated using (Melrose, 1975)

$$L_{\rm L} = \frac{V}{2}\frac{W_{\rm L}}{t_{\rm L}}, \tag{30}$$

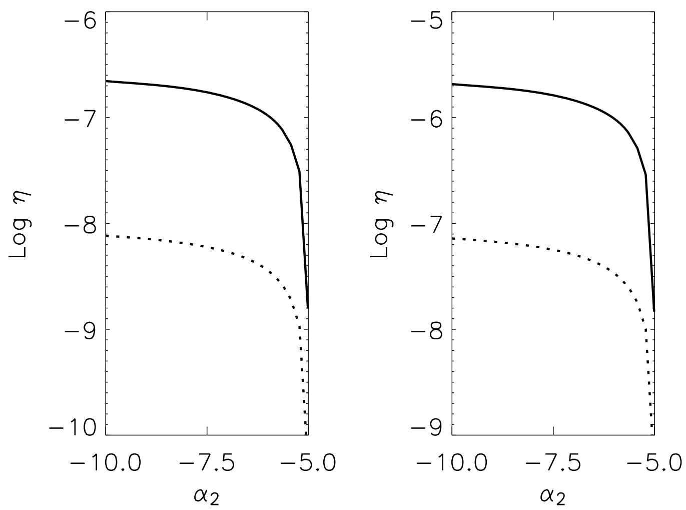

Figure 4. Maximum efficiency of the overall noise-storm emission process ($\eta \equiv L_{\rm out}/L_{\rm in}$) plotted as a function of α_2 using Equation (29) with $\xi_{\rm c} = \xi_{\rm max}$. *Left panel*: $T_{\rm e} = 10^6$ K. *Right panel*: $T_{\rm e} = 2 \times 10^6$ K. *Solid line*: $\nu = 169$ MHz. *Dotted line*: $\nu = 327$ MHz. The output power $L_{\rm out}$ is taken to be 10^{17} erg s^{-1}.

where V is the volume of the acceleration region, $W_{\rm L}$ is the energy density in the Langmuir waves, and $t_{\rm L}$ is the Langmuir wave generation timescale. We can compute $W_{\rm L}$ using Equation (20) from Melrose (1975), which gives

$$W_{\rm L} = \frac{n_* m_{\rm e} v_{\rm c}^2}{2} \left[\left(\frac{v_{\rm th}}{v_{\rm c}} \right)^3 \ln \left(\frac{v_{\rm c}}{v_{\rm th}} \right) \right], \tag{31}$$

where $v_{\rm th}$ ($\equiv (k_{\rm B} T_{\rm e}/m_{\rm e})^{1/2}$) is the characteristic velocity of the thermal electrons, $v_{\rm c}$ ($\equiv p_{\rm c}/m_{\rm e}$) is the critical velocity (the speed of the nonthermal-electrons in the gap distribution), and n_* is the nonthermal-electron number density. In our current application, we set $\xi_{\rm c} = \xi_{\rm max}$ and compute n_* using Equation (12). The critical velocity is likewise given by $v_{\rm c} = (2 k_{\rm B} T_{\rm e} \xi_{\rm max}/m_{\rm e})^{1/2}$, which follows from Equation (10).

As discussed in paper 1 and also by Melrose (1980), we assume that the Langmuir wave generation process is proceeding at marginal stability, in which case the Langmuir wave generation timescale ($t_{\rm L}$) is comparable to the Coulomb loss timescale ($t_{\rm loss}$). We are focusing here on Langmuir waves with wavenumber (k) $\approx k_{\rm c}$ (see Equation (15)), and therefore the relevant Coulomb loss timescale is associated with electrons with momentum (p) $\approx p_{\rm c}$. It follows that we can evaluate the Langmuir wave generation timescale ($t_{\rm L}$) $\approx t_{\rm loss}$ using Equation (4) of paper 1 with $\epsilon = p_{\rm c}^2/(2m_{\rm e})$, which can be written in terms of the dimensionless critical energy ($\xi_{\rm c}$) as

$$t_{\rm L} = 0.10 \, \frac{\xi_{\rm c}^{3/2} T_{\rm e}^{3/2}}{\Lambda n_{\rm e}} \tag{32}$$

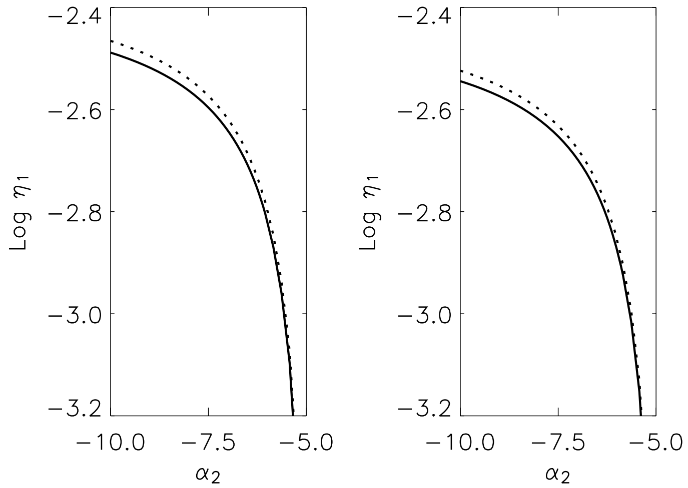

Figure 5. Minimum efficiency of the first stage of the noise-storm emission process (accelerated electrons–Langmuir wave production), $\eta_1 \equiv L_L/L_{in}$, plotted as a function of α_2 using Equation (33) with $\xi_c = \xi_{max}$. *Left panel*: $T_e = 10^6$ K. *Right panel*: $T_e = 2 \times 10^6$ K. *Solid line*: $\nu = 169$ MHz. *Dotted line*: $\nu = 327$ MHz.

in cgs units. We adopt the value $\Lambda = 29.1$ in our numerical calculations and set $\xi_c = \xi_{max}$ in order to treat the critical case of interest here. Detailed analysis establishes that the result obtained for L_L using Equations (30) – (32) is the *minimum* value consistent with the required Langmuir wave intensity.

The efficiency of the first stage of the noise-storm emission process is defined by

$$\eta_1 \equiv \frac{L_L}{L_{in}}, \tag{33}$$

which can be evaluated by combining Equations (27) and (30). In Figure 5, we plot η_1 as a function of α_2 with $\xi_c = \xi_{max}$ for the same two values of the coronal temperature (T_e) and observing the frequency (ν) used in Figure 1. We find that L_L decreases more rapidly than L_{in} as a function of ξ_c, and consequently the results obtained by setting $\xi_c = \xi_{max}$ represent the *minimum* value of η_1 consistent with the required Langmuir wave intensity. We find that this minimum value is quite insensitive to both the observing frequency and the coronal temperature.

Our knowledge of the overall efficiency (η) and the first-stage efficiency (η_1) can be combined to obtain an estimate of the efficiency of the *second stage* of the noise-storm generation process, comprising the conversion of the Langmuir waves into the observed noise-storm radiation. The efficiency of the second stage is defined by

$$\eta_2 \equiv \frac{L_{out}}{L_L}, \tag{34}$$

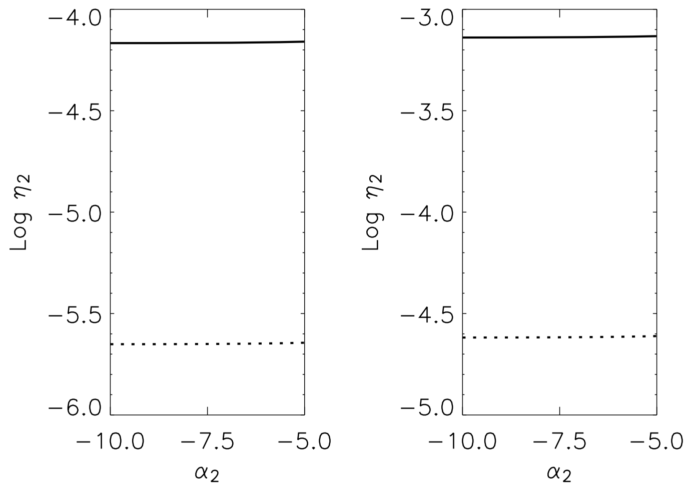

Figure 6. Maximum efficiency of the second stage of the noise-storm process (Langmuir waves – observable emission), $\eta_2 \equiv L_{out}/L_L$, plotted as a function of α_2 using Equation (34) with $\xi_c = \xi_{max}$. *Left panel*: $T_e = 10^6$ K. *Right panel*: $T_e = 2 \times 10^6$ K. *Solid line*: $\nu = 169$ MHz. *Dotted line*: $\nu = 327$ MHz.

which is related to η and η_1 via

$$\eta = \eta_1 \eta_2. \tag{35}$$

In Figure 6, we plot the second-stage efficiency (η_2) as a function of α_2 for the same two values of the coronal temperature (T_e) and observing frequency (ν) used in Figure 1, with $\xi_c = \xi_{max}$. The result obtained is the *maximum value* for η_2. In general, we find that $\eta_2 \lesssim 8 \times 10^{-4}$.

5.3. Energy in the Langmuir wave population

The energy density in the Langmuir wave population is an important ingredient in the noise-storm radiation process. Benz and Wentzel (1981), Jaeggi and Benz (1982), and Benz (1982) have derived upper limits for this quantity using radar measurements, observations of harmonic emission, and polarization data. The general consensus is that $W_L \lesssim 10^{-7} n_e k_B T_e$ in the noise-storm continuum sources.

By setting $\xi_c = \xi_{max}$ and using Equation (31), we can obtain the lower limit on the Langmuir wave energy density (W_L) predicted by our theoretical model. In Figure 7, we plot the ratio ($\beta_L \equiv W_L/(n_e k_B T_e)$) as a function of α_2 for the same two values of the coronal temperature (T_e) and observing frequency (ν) used in Figure 1. Since we have set $\xi_c = \xi_{max}$, it follows that this plot indicates the *minimum value* of β_L consistent with the required intensity of the Langmuir waves. We find that $8 \times 10^{-8} \gtrsim \beta_L \gtrsim 6 \times 10^{-9}$ is in agreement with the observational constraints derived by Benz and Wentzel (1981), Jaeggi and Benz (1982), and Benz (1982).

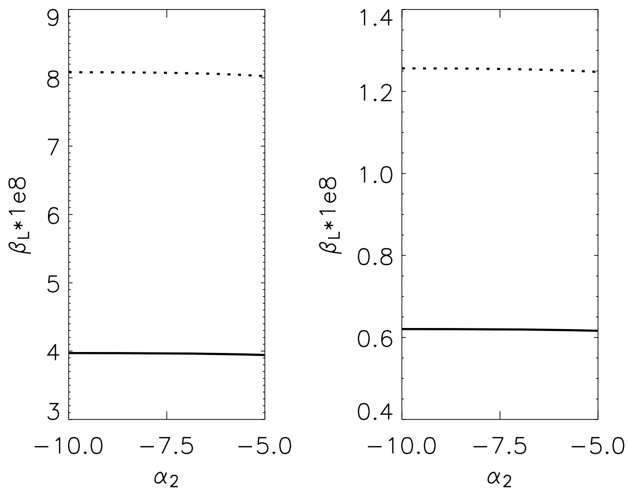

Figure 7. Minimum value of the Langmuir wave energy density ratio $(\beta_L) \equiv W_L/(n_e k_B T_e)$ plotted as a function of α_2 for $\xi_c = \xi_{max}$. *Left panel*: $T_e = 10^6$ K. *Right panel*: $T_e = 2 \times 10^6$ K. *Solid line*: $\nu = 169$ MHz. *Dotted line*: $\nu = 327$ MHz.

6. Summary and Discussion

In this paper, we have focused closely on the first stage of the noise-storm emission process, namely the generation of a sufficiently intense Langmuir wave distribution by the nonthermal-electrons. This requires that the nonthermal-electron density fraction (n_*/n_e) exceeds the minimum value given by the right-hand side of Equation (21). When this condition is satisfied, the damping of the Langmuir waves with $k \approx k_c$ is dominated by the nonthermal-electrons, which is an essential ingredient for the production of bright noise-storm emission (Melrose, 1975; Robinson, 1978; Thejappa, 1991).

The main contribution of this paper lies in the fact that we have self-consistently combined information about the minimum value of n_*/n_e with details of the electron acceleration process that were derived in paper 1. In doing so, we have obtained limiting values for the efficiency of the overall noise-storm emission process $(\eta \equiv L_{out}/L_{in})$ and also for the first-stage efficiency $(\eta_1 \equiv L_L/L_{in})$ that are described by the single parameter α_2, which is the high-energy power-law index of the nonthermal-electron distribution (see Figures 4 and 5). In particular, our theoretical results imply that the overall efficiency is bounded by the upper limit $\eta \lesssim 10^{-6}$. We have also estimated the second-stage efficiency $(\eta_2 \equiv L_{out}/L_L)$ describing the conversion of the Langmuir waves into observable noise-storm radiation and conclude that the upper limit for this quantity is $\eta_2 \lesssim 8 \times 10^{-4}$.

Our treatment also yields a lower limit for the dimensionless Langmuir wave energy density in noise-storm continuum sources $(\beta_L \equiv W_L/(n_e k_B T))$. We find that this lower limit lies in the range $8 \times 10^{-8} \gtrsim \beta_L \gtrsim 6 \times 10^{-9}$. This interesting new theoretical estimate for β_L, a central parameter in the noise-storm emission process, is shown to be consistent with the observational data. The efficiency estimates obtained here, therefore, represent an improved understanding of the detailed energy budget for the noise-storm generation process.

It is interesting to contrast our approach with that taken by Thejappa (1991). The model considered here includes an explicit treatment of the acceleration of an isotropic, nonthermal-electron distribution based on the associated Fokker–Planck transport equation analyzed previously in paper 1. Conversely, Thejappa's work focused on the emission of Langmuir waves by an anisotropic loss-cone distribution of electrons in the corona, and he did not consider the acceleration of the electrons in the turbulent plasma. It is therefore suggestive to note that the values we obtain for the nonthermal-electron fraction (n_*/n_e) are relatively close to Thejappa's results. In the interpretation of Thejappa, it is the *angular* anisotropy of the electrons in physical space (*i.e.*, the loss-cone distribution) that provides the basic source of free energy for the production of the Langmuir waves. However, our model utilizes the anisotropy in *energy* space (*i.e.*, the gap distribution) as the source of free energy.

In reality, one would expect both types of anisotropies to coexist, and, therefore, our results are in some sense complementary to Thejappa's. Hence, it is important to develop observational diagnostics for both models that can be used to reveal the true nature of the underlying nonthermal-electron distribution as a function of both energy and direction. It may be possible to develop the required observational tests by making a detailed analysis of the corresponding predictions for the time-dependent behavior of the associated radio signatures. We plan to pursue this question in a future work.

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