

## NOISE-STORM CONTINUA: POWER ESTIMATES FOR ELECTRON ACCELERATION

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**Abstract.** We use a generic stochastic acceleration formalism to examine the power  $L_{\text{in}}$  ( $\text{erg s}^{-1}$ ) input to non-thermal electrons that cause noise-storm continuum emission. The analytical approach includes the derivation of the Green's function for a general second-order Fermi process, and its application to obtain the particular solution for the non-thermal electron distribution resulting from the acceleration of a Maxwellian source in the corona. We compare  $L_{\text{in}}$  with the power  $L_{\text{out}}$  observed in noise-storm radiation. Using typical values for the various parameters, we find that  $L_{\text{in}} \sim 10^{23} - 10^{26} \text{ erg s}^{-1}$ , yielding an efficiency estimate  $\eta \equiv L_{\text{out}}/L_{\text{in}}$  in the range  $10^{-10} \lesssim \eta \lesssim 10^{-6}$  for this non-thermal acceleration/radiation process. These results reflect the efficiency of the overall process, starting from electron acceleration and culminating in the observed noise-storm emission.

### 1. Introduction

Solar noise storms are a very well-studied phenomenon. They occur mostly at meter wavelengths, and comprise a long-lasting (1 hr to several days) broadband ( $\delta f/f \sim 100\%$ ) component together with narrow-band ( $\delta f/f \sim$  a few percent) spiky bursts that last from 0.1–1 s. Elgarøy (1997) gives a thorough observational review of noise-storms. Malik and Mercier (1996) present a comprehensive study of noise-storms observed with the Nancay Radioheliograph (NRH). Klein (1998) presents a recent review of the role of suprathermal electrons in the solar corona that includes a broad discussion of the noise-storm phenomenon.

In this study, we confine our attention primarily to type I noise-storm *continua*, rather than the sporadic type I bursts, because we are interested in examining the basic energetics of the electron acceleration processes responsible for producing the quasi-continuous radio emission.

Most theories of type I phenomena invoke non-thermal electrons as a crucial ingredient in producing the observed radiation. It is recognized that the non-thermal electrons that are involved in generating the noise-storm continua are probably accelerated in closed coronal loops above active regions. Noise-storm continua

are sometimes accompanied by coincident (thermal) soft X-ray brightenings (e.g., Raulin and Klein, 1994; Krucker *et al.*, 1995), which are also signatures of coronal magnetic field evolution. When this occurs, it is clear that the electrons in the tail of the thermal distribution that produce the accompanying soft X-radiation cannot also produce the observed radio noise-storm emission for more than a few minutes. This is inconsistent with the fact that noise-storms are observed to persist for several hours to days (e.g., Raulin and Klein, 1994; Malik and Mercier, 1996 also present similar arguments). Since the X-ray emitting regions are typically situated at least one scale height below the layer in the corona where the noise-storms originate, the two associated electron distributions cannot be co-spatial. Crosby *et al.* (1996) find that deka-keV X-ray emission is often observed towards the beginning of noise-storms. The electrons producing this X-ray emission are energetic enough to power the noise-storm simultaneously, and they could be transported to the noise-storm emitting region either by turbulent diffusion or by direct transport along connecting magnetic field lines. However, the duration of the noise-storm is much longer than the X-ray emission, and consequently continual electron acceleration is required. The acceleration is probably triggered by the same processes that give rise to the X-ray brightenings accompanying the onset of the noise-storms.

Clearly an underlying acceleration mechanism is required in order to produce the non-thermal electron distribution implied by the noise-storm emission. However, very little attention has been focused on this problem in the previous literature. The majority of the current theories simply assume that non-thermal electrons are present, and focus most of their attention on examining the wave–wave interaction processes through which observable radio emission is ultimately produced. It is fairly well established that noise-storms are intimately connected with the temporal variation of the magnetic fields via the process of coronal evolution (e.g., Brueckner, 1983; Stewart, Brueckner, and Dere, 1986; Raulin and Klein, 1994; Willson, Kile, and Rothberg, 1997; Bentley *et al.*, 2000). However, the precise physical processes that link the magnetic field evolution to the presence of non-thermal electrons is unclear. Spicer *et al.* (1981) adopted a specific driver model in order to examine the basic energetics of the process leading to type I bursts. Benz and Wentzel (1981) developed a similar treatment that is also valid for type I bursts. The basic driver in both of these pictures is the process of coronal evolution, which causes magnetic fields to emerge into the corona and drive microinstabilities that spawn low-frequency turbulence. This turbulence in turn resonates with the electrons and stochastically accelerates them to form a non-thermal tail.

Our goal here is to develop a model-independent approach to the estimation of the noise-storm energetics that avoids focusing on a specific mechanism for accelerating the electrons. Instead, we prescribe a generic second-order (stochastic) Fermi mechanism. Electrons from the tail of the thermal distribution are injected into the acceleration process, forming a non-thermal distribution. Only electrons above a critical energy in the tail of the thermal distribution are subjected to net acceleration; the rest remain thermal due to collisions. The basic parameters of the

acceleration mechanism are constrained as follows. First we borrow from the literature estimates of the non-thermal electron fraction needed to produce the observed noise-storm continua. By combining this estimate with approximate expressions for the dominant loss time scales influencing the electrons, we derive an estimate for the power that drives the electron acceleration process. This yields an approximate determination of the efficiency of the process starting from non-thermal electron acceleration and culminating in the observed noise-storm emission. The efficiency so obtained is a relatively well-defined quantity that provides a general, model-independent constraint on the acceleration/radiation mechanisms, which in turn serves as a useful guide in the subsequent development of more detailed models. Furthermore, we expect that a good understanding of the efficiency of electron acceleration in the context of the solar corona will also provide useful insights into similar phenomena occurring in other astrophysical environments.

## 2. Electron Acceleration

### 2.1. WHY ARE NON-THERMAL ELECTRONS IMPORTANT?

There is a considerable observational evidence for the presence of non-thermal electrons in non-flaring regions of the solar corona (e.g., Klein, 1998). The high brightness temperatures and significant positive spectral slopes in multi-frequency observations of noise-storms strongly suggest an underlying non-thermal electron population (e.g., Thejappa and Kundu, 1991; Sundaram and Subramanian, 2004). An anisotropy in velocity or physical space causes these non-thermal electrons to spontaneously emit Langmuir/upper hybrid waves (e.g., Robinson, 1978; Wentzel, 1985), which can coalesce with a suitable low-frequency wave population to produce the observed electromagnetic emission.

Melrose (1980) argues that low-frequency turbulence (including ion-acoustic, lower-hybrid, or a variety of other waves) will be generated as a natural product of coronal heating. Thejappa (1991) considers lower-hybrid waves excited by protons (as in Wentzel, 1986) or by a series of weak shocks (as in Spicer *et al.*, 1981) as candidates for the low-frequency wave population. Benz and Wentzel (1981) investigate the possibility that the turbulence is composed of ion-acoustic waves excited by evolving magnetic fields in the corona.

While most of the theories for noise-storm continua do not specify the source of the non-thermal electrons powering the observed emission, Benz and Wentzel (1981) propose that these electrons are leftover particles from the population that produced the previous type I bursts. However, Krucker *et al.* (1995) note that the type I continuum and bursts are spatially separated. Furthermore, Malik and Mercier (1996) pointed out that solar noise-storms do not often burst in the beginning, and the continuum exists alone. These observational results contradict Benz and Wentzel's (1981) picture for the production of the non-thermal electrons. On the other hand,

the model proposed by Spicer, Benz, and Huba (1981) for type I bursts considers the electrons to be accelerated via the modified two-stream instability, which in turn is spawned by random weak shocks caused by the emerging magnetic flux. However, this suggestion is contradicted by the work of Krucker *et al.* (1995).

Although there is currently no theoretical consensus regarding the fundamental mechanism powering type I phenomena, there is no question that non-thermal electrons are responsible for the observed emission. This situation leads us to suggest that a careful examination of the energy budget and the associated constraints on the efficiency of the acceleration process may provide the best route towards enhanced physical understanding.

## 2.2. ENERGY LOSS MECHANISMS

It is expected that the electron momentum distribution,  $f$ , will have essentially a two-part structure, comprising a thermal component along with a non-thermal, high-energy (but still non-relativistic) electron “tail” that is responsible for producing the noise-storm emission. In view of the considerable uncertainty surrounding the precise physical mechanism that results in the formation of the non-thermal portion of the electron distribution, we shall work in terms of a generic, stochastic (second-order) Fermi process. The total electron number density,  $n_e$ , including both thermal and non-thermal particles, is related to the momentum distribution  $f$  via

$$n_e (\text{cm}^{-3}) = \int_0^\infty p^2 f dp, \quad (1)$$

where  $p$  is the electron momentum. The associated total electron energy density is given by

$$U_e (\text{erg cm}^{-3}) = \int_0^\infty \epsilon p^2 f dp = \frac{1}{2m_e} \int_0^\infty p^4 f dp, \quad (2)$$

where  $m_e$  is the electron mass and  $\epsilon = p^2/(2m_e)$  is the electron kinetic energy. In the present application, we are mainly interested in the non-thermal electrons, which are picked up from the thermal population and subsequently accelerated to high energies.

At marginal stability, the loss time scale due to the emission of Langmuir waves by electrons is similar to the Coulomb loss time scale (Melrose, 1980). The mean Coulomb energy loss rate for non-thermal electrons with energy  $\epsilon$  colliding with ambient electrons and protons in a fully-ionized hydrogen plasma is given in cgs units by (e.g., Brown, 1972b)

$$\left. \left\langle \frac{d\epsilon}{dt} \right\rangle \right|_{\text{loss}} = \frac{1.57 \times 10^{-23} \Lambda n_e}{\epsilon^{1/2}} \left( 1 + \frac{m_e}{m_p} \right), \quad (3)$$

where  $\Lambda$  is the Coulomb logarithm and  $m_p$  is the proton mass. The first and second terms in parentheses on the right-hand side of Equation (3) refer to electron–electron

and electron–proton collisions, respectively. The factor of  $m_e/m_p$  clearly indicates that the cooling rate due to electron–proton bremsstrahlung is negligible compared with the effect of electron–electron collisions. Ignoring the factor of  $m_e/m_p$  in Equation (3), we find that the characteristic time scale for Coulomb losses is given by

$$t_{\text{loss}} \equiv \epsilon \left\langle \frac{d\epsilon}{dt} \right\rangle^{-1} \Big|_{\text{loss}} = \frac{6.37 \times 10^{22} \epsilon^{3/2}}{\Lambda n_e} \quad (4)$$

in cgs units.

### 2.3. STOCHASTIC ACCELERATION OF ELECTRONS

For low electron energies,  $\epsilon \lesssim kT$ , collisions between electrons are expected to efficiently maintain a Maxwellian distribution. However, we shall demonstrate below that for electron energies exceeding the critical energy  $\epsilon_c$ , stochastic acceleration dominates over collision losses on average (see Equation (10)). In this situation, the high-energy tail of the electron distribution function is governed by a rather simple transport equation that describes the diffusion of electrons in momentum space due to collisions with magnetic scattering centers. The time evolution of the Green’s function for this process,  $f_G$ , is described by (e.g., Becker, 1992; Schlickeiser, 2002)

$$\frac{\partial f_G}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} \left( p^2 \mathcal{D} \frac{\partial f_G}{\partial p} \right) + \frac{\dot{N}_0 \delta(p - p_0)}{p_0^2} - \frac{f_G}{\tau}, \quad (5)$$

where  $\mathcal{D}$  is the (as yet unspecified) diffusion coefficient in momentum space and  $\tau$  is the mean residence time for electrons in the acceleration region. The source term in Equation (5) corresponds to the injection into the acceleration region of  $\dot{N}_0$  particles per unit volume per unit time, each with momentum  $p_0$ . Although we do not explicitly include losses due to the emission of Langmuir/upper hybrid waves by the accelerated electrons, it is expected that these waves will be generated as a natural consequence of the spatial anisotropy of the electron distribution (e.g., Thejappa, 1991). We will demonstrate below that our neglect of energy losses due to Coulomb collisions and the emission of Langmuir/upper hybrid waves is reasonable for the non-thermal electrons treated by Equation (5). Note that in writing Equation (5), we have ignored spatial transport so as to avoid unnecessary mathematical complexity.

Although the specific form for  $\mathcal{D}$  as a function of  $p$  depends on the spectrum of the turbulent waves that accelerates the electrons (Smith, 1977), it is possible to make some fairly broad generalizations that help to simplify the analysis. In particular, we point out that a number of authors have independently suggested that  $\mathcal{D} \propto p^2$ . Examples include the treatment of particle acceleration by large-scale compressible magnetohydrodynamical (MHD) turbulence (Ptuskin, 1988; Chandran and Maron, 2003); analysis of the acceleration of electrons by cascading fast-mode waves in flares (Miller, LaRosa and Moore, 1996); and the energization of

electrons due to lower hybrid turbulence (Luo, Wei, and Feng, 2003). Hence we shall write

$$\mathcal{D} = D_0 p^2, \quad (6)$$

where  $D_0$  is a constant with the units of inverse time.

We can obtain an expression for the mean rate of change of the electron momentum  $p$  due to stochastic acceleration by focusing on the instantaneous evolution of a localized  $\delta$ -function distribution in momentum space. Following the procedure described by Subramanian, Becker, and Kazanas (1999), we find based on Equation (5) that the mean rate of change of the particle momentum due to stochastic acceleration is given by

$$\left\langle \frac{dp}{dt} \right\rangle \Big|_{\text{accel}} = \frac{1}{p^2} \frac{d}{dp} (p^2 \mathcal{D}) = 4 D_0 p. \quad (7)$$

Since  $\epsilon = p^2/(2m_e)$ , we conclude that the corresponding mean rate of change of the electron energy is

$$\left\langle \frac{d\epsilon}{dt} \right\rangle \Big|_{\text{accel}} = \frac{p}{m_e} \left\langle \frac{dp}{dt} \right\rangle \Big|_{\text{accel}} = 8 D_0 \epsilon. \quad (8)$$

The associated time scale for stochastic acceleration is therefore

$$t_{\text{accel}} \equiv \epsilon \left\langle \frac{d\epsilon}{dt} \right\rangle \Big|_{\text{accel}}^{-1} = \frac{1}{8 D_0}. \quad (9)$$

On average, acceleration dominates over losses for a particle with energy  $\epsilon$  if  $t_{\text{accel}} < t_{\text{loss}}$ . Comparison of Equations (4) and (9) establishes that acceleration is dominant if  $\epsilon > \epsilon_c$ , where the critical energy  $\epsilon_c$  is given in cgs units by

$$\epsilon_c \equiv 1.57 \times 10^{-16} \left( \frac{\Lambda n_e}{D_0} \right)^{2/3}. \quad (10)$$

It should be noted that due to the stochastic nature of the acceleration process, some particles with energy  $\epsilon > \epsilon_c$  will lose energy, although on average such particles will gain energy. Our assumption that the mean acceleration rate dominates over losses for the non-thermal particles is self-consistent provided the non-thermal particles all have  $\epsilon > \epsilon_c$ . The corresponding result for the critical momentum is

$$p_c \equiv (2m_e \epsilon_c)^{1/2} = 1.64 \times 10^{-21} \left( \frac{n_e}{D_0} \right)^{1/3}, \quad (11)$$

where we have set the Coulomb logarithm  $\Lambda = 29.1$  (see Brown, 1972a). Particles with  $p > p_c$  experience net acceleration on average. Note that the electron source term appearing in Equation (5) must have  $p_0 > p_c$  in order to validate our neglect of collisional losses in that Equation. When this condition is satisfied, the injected electrons are accelerated to form a non-thermal distribution. While we have considered only Coulomb losses in this calculation, we remind the reader that the loss

time scale due to the emission of Langmuir waves at marginal stability is similar to the Coulomb loss time scale (see Section 2.2).

#### 2.4. SOLUTION FOR THE GREEN'S FUNCTION

In a steady state, it is straightforward to show based on Equation (5) that when  $\mathcal{D} = D_0 p^2$  as assumed here, the solution for the Green's function is given by Subramanian, Becker, and Kazanas (1999)

$$f_G(p, p_0) = A_0 \begin{cases} (p/p_0)^{\alpha_1}, & p \leq p_0 \\ (p/p_0)^{\alpha_2}, & p \geq p_0, \end{cases} \quad (12)$$

where the exponents  $\alpha_1$  and  $\alpha_2$  are related to  $D_0$  and  $\tau$  via

$$\alpha_1 \equiv -\frac{3}{2} + \left(\frac{9}{4} + \frac{1}{D_0 \tau}\right)^{1/2}, \quad \alpha_2 \equiv -\frac{3}{2} - \left(\frac{9}{4} + \frac{1}{D_0 \tau}\right)^{1/2}, \quad (13)$$

and the normalization parameter  $A_0$  has the value

$$A_0 = \frac{\dot{N}_0}{2 D_0 p_0^3} \left(\frac{9}{4} + \frac{1}{D_0 \tau}\right)^{-1/2}. \quad (14)$$

Since the second-order Fermi acceleration process is stochastic in nature, the particles diffuse away from the injection momentum  $p_0$ , and the  $p \geq p_0$  and  $p \leq p_0$  branches of the Green's function appearing in Equation (12) describe the acceleration and deceleration of the source particles, respectively. However, on average, acceleration wins out over deceleration, as indicated by Equation (8) which demonstrates that the mean acceleration rate is positive. Substitution into the integral  $\int_0^\infty p^2 f_G dp$  using Equation (12) confirms that the number density of the Green's function is equal to  $\dot{N}_0 \tau$ , as expected in this steady-state situation. The values of  $D_0$ ,  $\tau$ , and  $\dot{N}_0$  will be constrained later using observational data.

The Green's function  $f_G$  describes the response to the injection of  $\dot{N}_0$  electrons per unit volume per unit time with momentum  $p_0$ , and therefore it is easy to show based on the linearity of the transport Equation (5) that the particular solution for a general source term  $j$  is obtained via the convolution (e.g., Becker, 2003)

$$f(p) = \int_0^\infty \frac{p_0^2 j(p_0)}{\dot{N}_0} f_G(p, p_0) dp_0, \quad (15)$$

where the quantity  $p_0^2 j(p_0) dp_0$  represents the number of electrons injected per second per  $\text{cm}^3$  with momenta between  $p_0$  and  $p_0 + dp_0$ . In the physical application of interest here, the injected particles are supplied by the high-energy ( $p_0 > p_c$ ) portion of the Maxwellian distribution in the corona. Assuming that the characteristic time scale for the thermal electrons to enter the acceleration region is equal to the

mean residence time  $\tau$ , it follows that the source term is given by

$$j(p_0) = \begin{cases} \frac{4\pi n_e \tau^{-1}}{(2\pi m_e k T)^{3/2}} e^{-p_0^2/2m_e k T}, & p_0 \geq p_c \\ 0, & p_0 < p_c. \end{cases} \quad (16)$$

## 2.5. PARTICULAR SOLUTION FOR MAXWELLIAN SOURCE

In the case of a Maxwellian source, which is our focus here, we can combine Equations (12) through (16) to compute the particular solution for the non-thermal electron distribution. The lower bound for the integration over  $p_0$  in Equation (15) is set equal to  $p_c$  since the source  $j(p_0)$  vanishes for  $p_0 < p_c$  according to Equation (16). The result obtained for the non-thermal electron distribution is therefore given by

$$f(p) = \frac{n_e \left\{ \xi^{\alpha_1/2} \Gamma\left(-\frac{\alpha_1}{2}, \xi\right) - \xi^{\alpha_2/2} \left[ \Gamma\left(-\frac{\alpha_2}{2}, \xi\right) - \Gamma\left(-\frac{\alpha_2}{2}, \xi_c\right) \right] \right\}}{\sqrt{2\pi} (m_e k T)^{3/2} (\alpha_1 - \alpha_2) D_0 \tau}, \quad (17)$$

where we have introduced the dimensionless electron energy  $\xi$  and the dimensionless critical energy  $\xi_c$ , defined by

$$\xi \equiv \frac{p^2}{2 m_e k T}, \quad \xi_c \equiv \frac{p_c^2}{2 m_e k T}. \quad (18)$$

The number and energy densities associated with the particular solution  $f$  above the critical momentum  $p_c$  can be computed using

$$n_* (\text{cm}^{-3}) \equiv \int_{p_c}^{\infty} p^2 f(p) dp, \quad (19)$$

$$U_* (\text{erg cm}^{-3}) \equiv \int_{p_c}^{\infty} \epsilon p^2 f(p) dp. \quad (20)$$

By combining Equations (17), (19), and (20), we find that the exact solutions for  $n_*$  and  $U_*$  are given by

$$\begin{aligned} \frac{n_*}{n_e} &= \frac{2 \xi_c^{(3+\alpha_1)/2} \Gamma\left(-\frac{\alpha_1}{2}, \xi_c\right)}{\sqrt{\pi} (3 + \alpha_1)(\alpha_2 - \alpha_1) D_0 \tau} + \\ &+ 2 e^{-\xi_c} \left( \frac{\xi_c}{\pi} \right)^{1/2} + \text{Erfc}(\xi_c^{1/2}), \end{aligned} \quad (21)$$

$$\begin{aligned} \frac{U_*}{n_e k T} &= \frac{2 \xi_c^{(5+\alpha_1)/2} \Gamma\left(-\frac{\alpha_1}{2}, \xi_c\right)}{\sqrt{\pi} (5 + \alpha_1)(\alpha_2 - \alpha_1) D_0 \tau} + \\ &+ \frac{2\sqrt{\pi \xi_c} (3 + 2 \xi_c) e^{-\xi_c} + 3 \pi \text{Erfc}(\xi_c^{1/2})}{2 \pi (1 - 10 D_0 \tau)}. \end{aligned} \quad (22)$$

Based on Equations (8) and (20), we conclude that the rate of change of the energy density of the non-thermal electrons due to second-order Fermi acceleration is given by

$$\left. \frac{dU_*}{dt} \right|_{\text{accel}} = \int_{p_c}^{\infty} p^2 \left\langle \frac{d\epsilon}{dt} \right\rangle_{\text{accel}} f(p) dp = 8 D_0 U_*, \quad (23)$$

with  $U_*$  computed using Equation (22).

### 3. Estimate of Power Input to Electron Acceleration Process

Assuming that the non-thermal electrons spontaneously emit Langmuir waves, which then coalesce with lower hybrid waves to produce the observable electromagnetic radiation, Thejappa (1991) gives estimates of the fraction of non-thermal electrons,  $n_*/n_e$ , that are needed to produce a given noise-storm continuum brightness temperature  $T_b$  at a particular observing frequency. In this model, noise-storm radiation exhibits broadband continuum characteristics provided  $n_*/n_e$  is small enough so that collisional damping (due to ambient thermal electrons) dominates over negative damping due to non-thermal electrons. When  $n_*/n_e$  exceeds a certain threshold, the negative damping due to non-thermal electrons dominates, the brightness temperature increases steeply and type I bursts are produced. The threshold value of  $n_*/n_e$  at 169 MHz is  $n_*/n_e = 2.2 \times 10^{-7}$ , corresponding to  $T_b \sim 10^{10}$  K. Statistics of noise-storm continuum brightness temperatures are most abundant at 169 MHz, and Kerdraon and Mercier (1983) find that the brightest noise-storm continua at that frequency have  $T_b \sim 5 \times 10^9 - 10^{10}$  K. Accordingly, we consider the value  $n_*/n_e = 2.2 \times 10^{-7}$  in our calculations. However, some authors (e.g., Klein, 1995 and references therein) use a frequency-independent value of  $n_*/n_e = 10^{-5}$ , and consequently we also consider this value in our calculations (see Table I).

TABLE I  
Representative results.

$n_*/n_e$	$\alpha_2$	$\epsilon_c$	$D_0$	$L_{\text{in}}$	$L_{\text{out}}$	$\eta$
$2.2 \times 10^{-7}$	-5.1	1.43	0.185	$5.62 \times 10^{24}$	$10^{17}$ to $10^{18}$	$\sim 10^{-8}$ to $10^{-7}$
$2.2 \times 10^{-7}$	-6.0	1.42	0.186	$8.00 \times 10^{23}$	$10^{17}$ to $10^{18}$	$\sim 10^{-7}$ to $10^{-6}$
$1.0 \times 10^{-5}$	-5.1	1.09	0.270	$3.00 \times 10^{26}$	$10^{17}$ to $10^{18}$	$\sim 10^{-10}$ to $10^{-9}$
$1.0 \times 10^{-5}$	-6.0	1.08	0.280	$4.2 \times 10^{25}$	$10^{17}$ to $10^{18}$	$\sim 10^{-9}$ to $10^{-8}$

The critical energy  $\epsilon_c$  (Equation (10)) is expressed here in keV. The quantity  $D_0$  (Equation (27)) is in units of  $\text{s}^{-1}$ . The input power to the electrons  $L_{\text{in}}$  (Equation (29)) and the power observed in electromagnetic radiation  $L_{\text{out}}$  are expressed in units of  $\text{erg s}^{-1}$ .

It is convenient to treat the high-energy power law index,  $\alpha_2$ , as a free parameter in our model. According to Equation (20), in order to obtain a finite value for the non-thermal electron energy density  $U_*$ , we must have  $\alpha_2 < -5$ . Combining this constraint with Equation (13), we find that

$$D_0 \tau < \frac{1}{10}. \quad (24)$$

The same condition can also be derived by noting that the second term in Equation (22) for  $U_*$  diverges in the limit  $D_0 \tau \rightarrow 1/10$ . Using Equation (9) to substitute for  $D_0$  in terms of the stochastic acceleration time scale  $t_{\text{accel}}$  yields the equivalent result

$$\frac{\tau}{t_{\text{accel}}} < \frac{4}{5}. \quad (25)$$

Any steady-state physical configuration governed by the transport equation (5) must satisfy this condition. Once the value of  $\alpha_2$  is selected, then  $\alpha_1$  and the product  $D_0 \tau$  can be computed using Equations (13). By combining this information with observational estimates for the coronal temperature  $T$  and the ratio  $n_*/n_e$ , we can compute  $p_c$  using Equation (21), which is an implicit equation for  $\xi_c = p_c^2/(2m_e kT)$ . We remind the reader that  $p_c$  signifies the critical momentum above which stochastic acceleration dominates over losses on average (see Equation (11)). In our example calculations we use  $T = 10^6$  K for the coronal temperature and either  $n_*/n_e = 2.2 \times 10^{-7}$  or  $n_*/n_e = 10^{-5}$  for the fraction of non-thermal electrons.

Although noise-storm continua are observed between  $\sim 50$ – $300$  MHz, observations are most abundant at 169 MHz (Kerdraon and Mercier, 1983). In what follows, we will be using observations at 169 MHz to define the typical source size of noise-storm continua. We therefore utilize a value for the thermal electron density  $n_e$  that corresponds to a plasma frequency of 169 MHz (e.g., Krall and Trivelpiece, 1986),

$$n_e = 3.54 \times 10^8 \text{ cm}^{-3}. \quad (26)$$

With  $p_c$  already determined using Equation (21) along with selected values for  $n_*/n_e$  and  $T$  as explained above, we can now solve for the Fermi acceleration constant  $D_0$  by using Equation (11) to write

$$D_0 = 4.41 \times 10^{-63} n_e p_c^{-3}. \quad (27)$$

Finally, the stochastic energization rate per unit volume is obtained by combining Equations (22) and (23).

The typical size of a noise-storm continuum source at 169 MHz is  $3' \approx 10^{10}$  cm (Kerdraon and Mercier, 1983). The vertical extent of a noise-storm continuum source is not directly known, but the typical relative bandwidth of the observed

emission is  $\delta f/f \sim 100\%$ . If  $H = n_e (\nabla n_e)^{-1}$  is the scale height of density variation in the corona, the noise-storm continuum emission must emanate from a range of heights  $(\delta f/f) H/2$  (e.g., Melrose, 1980). Using  $H \sim 10^5$  km, this yields an estimate of

$$V \sim 10^{30} \text{ cm}^3 \quad (28)$$

for the volume of the acceleration region. This estimate assumes that the noise-storm emitting region is a column of height  $H/2$  and cross section  $3' \times 3'$ , which is consistent with the data at 169 MHz. However, the cross section of the emitting region may vary with frequency, and therefore with height in the corona, in which case the emission volume would not be a simple column. With this caveat in mind, we shall adopt the simple columnar picture here and use a value of  $n_e$  referenced to 169 MHz because noise-storm continua statistics are most abundant at this frequency.

The power input to the non-thermal electron acceleration process is computed using

$$L_{\text{in}} = V dU_*/dt \text{ erg s}^{-1}, \quad (29)$$

where  $dU_*/dt$  is given by Equations (22) and (23). Representative results are presented in Table I. We find that  $L_{\text{in}} \sim 10^{23-26} \text{ erg s}^{-1}$ . Expectedly,  $L_{\text{in}}$  is larger for the calculations where we have used the relatively larger value of  $n_*/n_e$ . From independent observational considerations, the power in noise-storm continua is estimated to be  $L_{\text{out}} \sim 10^{17-10^{18}} \text{ erg s}^{-1}$  (e.g., Elgarøy, 1977; Raulin and Klein, 1994). The associated efficiency,

$$\eta \equiv \frac{L_{\text{out}}}{L_{\text{in}}}, \quad (30)$$

is therefore estimated to be in the range  $10^{-10} \lesssim \eta \lesssim 10^{-6}$ .

#### 4. Summary and Discussion

We have stipulated a generic stochastic Fermi acceleration mechanism for generating the non-thermal electrons responsible for noise-storm continua. The mathematical approach is based on a rigorous derivation of the Green's function describing the acceleration of (initially) mono-energetic electrons, which achieve a power-law distribution at high energies (see Equation (12)). The Green's function is convolved with the high-energy portion of the electron Maxwellian in the corona to obtain the particular solution for the momentum distribution of the non-thermal electrons responsible for producing the observed noise-storm emission (Equation (17)). Integration of the particular solution in turn yields exact solutions for the number

and energy densities of the non-thermal electrons, as well as an expression for the rate of change of their energy density (Equations (21)–(23)).

Our work utilizes estimates for the ratio of the non-thermal to thermal electron densities  $n_*/n_e$  from Thejappa (1991) (implied by the typically observed values for the noise-storm continuum brightness temperature  $T_b$  at  $\sim 169$  MHz) and Klein (1995). Since we do not have reliable observational estimates for the residence time  $\tau$  of the electrons in the acceleration region, we instead parameterize the model using the high-energy power-law index  $\alpha_2$  of the electron distribution function. We must restrict ourselves to values of  $\alpha_2 < -5$  in order to obtain a finite value for the non-thermal electron energy density. By combining an observational value for  $n_*/n_e$  with a selected value for  $\alpha_2$ , we can use Equation (21) to determine the critical momentum  $p_c$ , which separates the thermal and non-thermal portions of the electron distribution. The analytical solution for the energy density of the non-thermal electrons is then used to compute the energization rate due to second-order Fermi acceleration (see Equations (22) and (23)).

Typical observational values for the total electron number density  $n_e$  and the volume of the noise-storm emitting region  $V$  were used to arrive at the results presented in Table I. These results demonstrate that the power input to the electron acceleration process is around  $10^{23}$ – $10^{26}$  erg s $^{-1}$ , which is 6–10 orders of magnitude larger than the power that is ultimately observed in the noise-storm continuum radiation. The efficiency of the process, starting from the acceleration of the non-thermal electrons and culminating in the observable noise-storm continuum radiation, is therefore in the range  $10^{-10} \lesssim \eta \lesssim 10^{-6}$ . Using data from type III decametric storms and impulsive 2–10 keV electron events at 1 AU associated with type I noise-storms, Lin (1985) estimates that the energy release rate for these electrons is around  $10^{23}$  erg s $^{-1}$  (cf. also Jackson and Leblanc, 1991). Klein (1995) estimates that the energy supply to the energetic electrons is of the order of  $10^{23}$ – $10^{24}$  erg s $^{-1}$ . Prior to the calculations presented here, these were the only (indirect, and rather approximate) estimates of the power input to non-thermal electrons required to drive noise-storm radiation.

We believe that the general analysis of the energy budget presented in this paper, based on a generic second-order Fermi energization mechanism, will help to guide subsequent work on the detailed acceleration processes responsible for powering the noise-storm continua. Our results also provide an important quantitative data point for discussions of electron acceleration in other high-temperature astrophysical environments.

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