

A system contains N_1 molecules of type 1 and N_2 molecules of type 2 in volume V .

(a) How does Ω depend on V ?

We have

$$\Omega_1 = C_1 E_1^{\frac{3N_1}{2}} V^{N_1}$$

$$\Omega_2 = C_2 E_2^{\frac{3N_2}{2}} V^{N_2}$$

$$\therefore \Omega = \Omega_1 \Omega_2 = C_1 C_2 E_1^{\frac{3N_1}{2}} E_2^{\frac{3N_2}{2}} V^{N_1} V^{N_2} \propto V^{N_1+N_2}$$

(b) The entropy is given by

$$S = k \ln \Omega$$

$$= k \ln(C_1 C_2) + k \ln(E_1^{\frac{3N_1}{2}} E_2^{\frac{3N_2}{2}})$$

$$+ (N_1 + N_2) k \ln V$$

The equation of state is obtained using

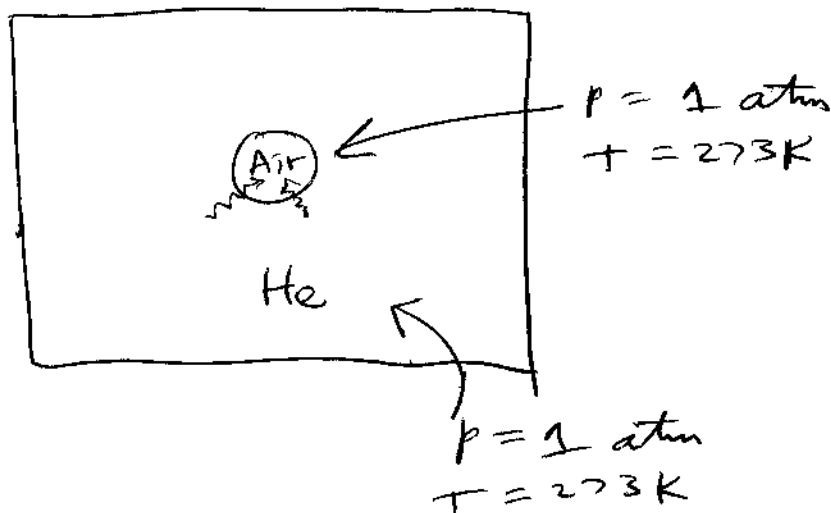
$$\left(\frac{\partial S}{\partial V} \right)_E = \frac{p}{T}$$

$$\Rightarrow \frac{p}{T} = \frac{(N_1 + N_2) k}{V} \Rightarrow$$

$$p = \frac{(N_1 + N_2) k T}{V}$$

* The separate contributions to the pressure are called "partial pressures".

Before:



The helium can leak into the bulb, but the air cannot escape from it. Therefore we have at a later time we have:

$$p_{\text{bulb}} = 1 \text{ atm} + 1 \text{ atm}$$

\uparrow
air
partial pressure

\uparrow
Helium partial
pressure

$$\Rightarrow \boxed{p_{\text{bulb}} = 2 \text{ atm}}$$

This is the pressure of the bulb a few months later.