Reif 2.6

$$(x^2-y)\,dx + x\,dy = dF$$

a) $\frac{\partial}{\partial y}(x^2-y) = -1$

$\frac{\partial}{\partial x}(x) = 1$

The differential is not exact

b) $$\int_{(1,1)}^{(1,2)} dF + \int_{(1,2)}^{(2,2)} dF$$

$$= \int_1^2 dy + \int_1^2 (x^2-2)\,dx = y\Big|_1^2 + \frac{x^3}{3} - 2x\Big|_1^2$$

$$= \cancel{2} - 1 + \frac{8}{3} - \cancel{4} - \frac{1}{3} + \cancel{2} = \frac{4}{3}$$

$$\int_{(1,1)}^{(2,1)} dF + \int_{(2,1)}^{(2,2)} dF = \int_1^2 (x^2-1)\,dx + \int_1^2 2\,dy$$

$$= \frac{x^3}{3} - x\Big|_1^2 + 2y\Big|_1^2 = \frac{8}{3} - \cancel{2} - \frac{1}{3} + 1 + \cancel{4} - \cancel{2}$$

$$= \frac{10}{3}$$

$$\int_{(1,1)}^{(2,2)} dF = \int_1^2 (x^2 - \cancel{x} + \cancel{x})\,dx = \frac{x^3}{3}\Big|_1^2 = \frac{8}{3} - \frac{1}{3}$$

$$= \frac{7}{3}$$

c) $$dG = \frac{dF}{x^2} = \left(1 - \frac{y}{x^2}\right)dx + \frac{dy}{x}$$

$$\frac{\partial}{\partial y}\left(1 - \frac{y}{x^2}\right) = -\frac{1}{x^2}, \quad \frac{\partial}{\partial x}\left(\frac{1}{x}\right) = -\frac{1}{x^2} \Rightarrow dG \text{ is exact}$$

Integration along the same paths as before yields

$$\int_{(1,1)}^{(1,2)} dF + \int_{(1,2)}^{(2,2)} dF = \int_1^2 dy + \int_1^2 \left(1 - \frac{2}{x^2}\right) dx$$

$$= y \Big|_1^2 + x + \frac{2}{x} \Big|_1^2 = 2 - 1 + \cancel{2} - \cancel{1} + \cancel{1} - \cancel{2}$$

$$= 1$$

$$\int_{(1,1)}^{(2,1)} dF + \int_{(2,1)}^{(2,2)} dF = \int_1^2 \left(1 - \frac{1}{x^2}\right) dx + \int_1^2 \frac{dy}{2}$$

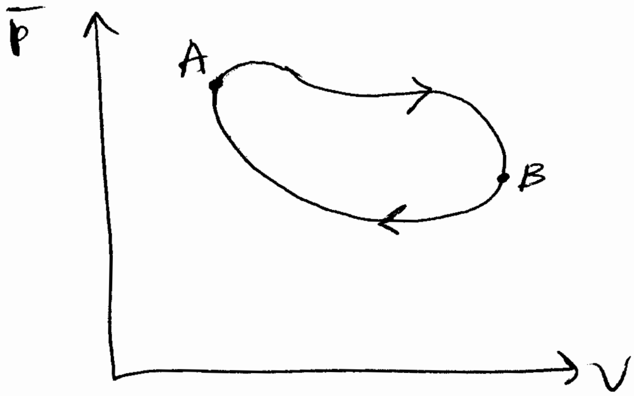
$$= x + \frac{1}{x} \Big|_1^2 + \frac{y}{2} \Big|_1^2 = 2 + \cancel{\tfrac{1}{2}} - 1 - \cancel{1} + \cancel{1} - \cancel{\tfrac{1}{2}}$$

$$= 1$$

$$\int_{(1,1)}^{(2,2)} dF = \int_1^2 \left(1 - \cancel{\frac{x}{x^2}} + \cancel{\frac{1}{x}}\right) dx = x \Big|_1^2 = 2 - 1 = 1$$

All of these results for the integral agree.

Reif 2.8



We have from the first law

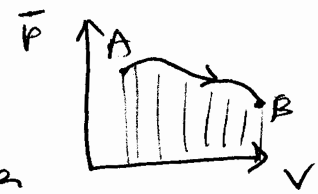
$$dE = dQ - p\,dV$$

The mechanical work done by the system is given by

$$dW = p\,dV$$

From point A to point B we have

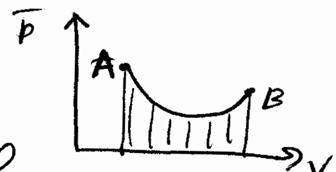
$$W_{AB} = \int_A^B p\,dV$$



which is the area under the upper curve.

From point B to point A we have

$$-W_{BA} = \int_B^A p\,dV$$



which is the area under the lower curve.

The net work done by the system is therefore

$W = W_{AB} + W_{BA}$ = area <u>within</u> figure <u>between</u> the two curves.