Assignment # 2

Reif 1.1

Probability of ~~getting~~ 6 points or less on 3 die. The unique sequences yielding 6 points or less are:

1 1 1	2 1 1	
1 1 2	2 1 2	
1 1 3	2 1 3	
1 1 4	2 2 1	
1 2 1	2 2 2	2 3 1
1 2 2	3 1 1	
1 2 3	3 1 2	
1 3 1	3 2 1	
1 3 2	4 1 1	
1 4 1		

There are **20** sequences out of a total of $6^3 = 216$ sequences. Hence the probability of throwing 6 points or less with 3 die is

$$\boxed{p = \frac{20}{216} = 0.093}$$

Reif 1.2 Strange problem ... an "ace" in dice is a "1". We have

(a) $6 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^5 = 0.402$

(b) use binomial dist. with $p = \frac{1}{6}$ = prob of an ace and

$$P(n,N) = \left(\frac{1}{6}\right)^n \left(\frac{5}{6}\right)^{N-n} \frac{N!}{n!\,(N-n)!}$$

then $P(1,6) = \frac{1}{6}\left(\frac{5}{6}\right)^5 \frac{6!}{1!\,5!} = \left(\frac{5}{6}\right)^5 = 0.402$

Then $\sum_{n=1}^{6} P(n,6) = 1 - P(0,6) = 1 - \left(\frac{5}{6}\right)^6 = 0.665$

(c) $P(2,6) = \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^4 \frac{6!}{2!\,4!} = 0.201$ ← doesn't agree w/ book

Reif 1.4

(a) If N is even, then we can use the binomial ~~theorem~~ to write

$$P(n,N) = \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^{N-n} \frac{N!}{n!\,(N-n)!}$$

where n = number of steps to ~~the~~ right. If the drunk is to end up back where he started after N steps, then $n = \frac{N}{2}$

and
$$P = \left(\frac{1}{2}\right)^N \frac{N!}{\left[\left(\frac{N}{2}\right)!\right]^2}$$

(b) If N is odd, then $P = 0$.

Reif 1.5

a) The probability of remaining alive after N games of Russian roulette is obtained as follows. Let $p = \frac{1}{6}$ = prob. of death per turn

$q = \frac{5}{6}$ = prob. of no death per turn

n = # times death occurs

$$P(0,N) = \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{N-0} \frac{N!}{0!(N-0)!}$$

$$\boxed{P(0,N) = \left(\frac{5}{6}\right)^N}$$

b) The prob. of dying after N turns is given by

$$P = \underbrace{P(0, N-1)}_{\substack{\text{prob. of} \\ \text{surviving} \\ N-1 \text{ games}}} \cdot \underbrace{\frac{1}{6}}_{\substack{\text{prob. of} \\ \text{dying on} \\ N^{th} \text{ turn}}} = \left(\frac{5}{6}\right)^{N-1} \frac{1}{6}$$

$$\Rightarrow \boxed{P = \left(\frac{5}{6}\right)^{N-1} \frac{1}{6}}$$

c) What is the mean number of times a player pulls the trigger? This is given by

$$\bar{N} = \sum_{N=1}^{\infty} \underbrace{\left(\frac{5}{6}\right)^{N-1} \frac{1}{6}}_{\substack{\text{prob. of} \\ \text{dying on} \\ N^{th} \text{ turn}}} N$$

$$\bar{N} = \sum_{N=1}^{\infty} \frac{d}{dx} x^N \; \frac{1}{6} \Big|_{x=\frac{5}{6}}$$

$$= \frac{1}{6} \frac{d}{dx} \sum_{N=1}^{\infty} x^N \Big|_{x=\frac{5}{6}}$$

note that the function $\frac{1}{1-x}$ can be expanded using

$$f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} \frac{\cancel{n!}}{\cancel{n!}} x^n = \sum_{n=0}^{\infty} x^n$$

Hence we find that

$$\bar{N} = \frac{1}{6} \frac{d}{dx} \left(\frac{1}{1-x} - 1 \right) \Big|_{x=5/6} = \frac{1}{6} \frac{d}{dx} \left(\frac{\cancel{1}-\cancel{1}+x}{1-x} \right) \Big|_{x=5/6}$$

$$= \frac{1}{6} \frac{d}{dx} \left(\frac{x}{1-x} \right) \Big|_{x=5/6}$$

$$= \frac{1}{6} \frac{1-\cancel{x}-(-1)\cancel{x}}{(1-x)^2} \Big|_{x=5/6} = \frac{1}{6} \frac{1}{(1-\frac{5}{6})^2} = \frac{1}{6} \frac{1}{(\frac{1}{6})^2}$$

$$\Rightarrow \boxed{\bar{N} = 6}$$