

STATICS LESSON PLAN
(Approximate lecture time: 3 hours)

FORCE [FERM 10:1]

$$F = \Sigma F_i = \Sigma F_{x,i} \mathbf{i} + \Sigma F_{y,i} \mathbf{j} + \Sigma F_{z,i} \mathbf{k}$$

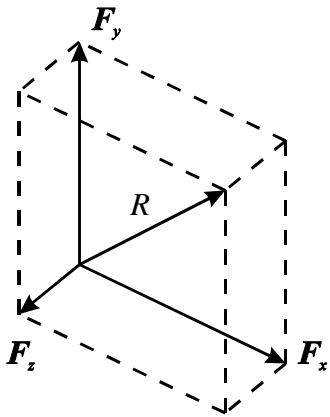
$$F_x = R \cos \theta_x; \quad F_y = R \cos \theta_y; \quad F_z = R \cos \theta_z$$

Concurrent forces

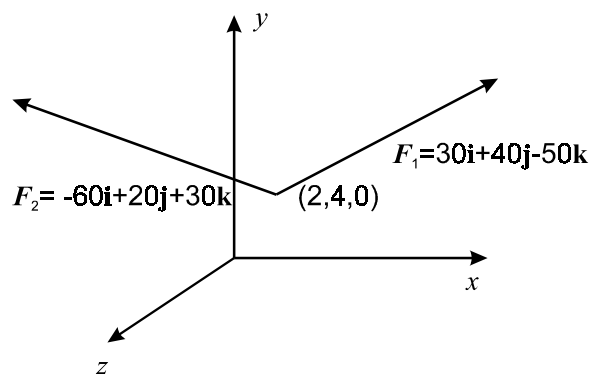
When all the forces act at a single point, the system is concurrent. With concurrent forces there can be no moments (couples), so all that matters is the sum of the forces in each coordinate.

Resultant

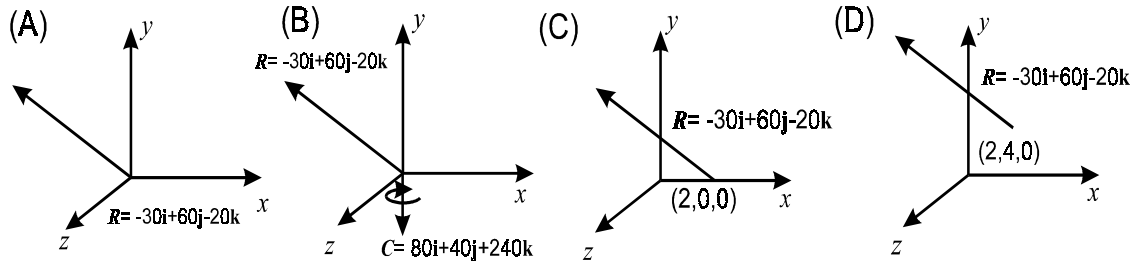
The resultant is the vector sum of all the forces acting at a concurrent point.



Examples: Force

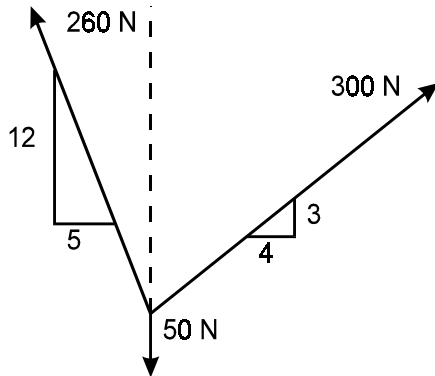


Which of the following systems is equivalent to the system shown?



The vector sum of F_1 and F_2 is $-30\mathbf{i} + 60\mathbf{j} - 20\mathbf{k}$ applied to the original point of application.

Therefore (D) is correct.



The magnitude of the resultant in the diagram shown is most nearly
 (A) 140 N (B) 191 N (C) 396 N (D) 470 N

This is only a two-dimensional problem, but whether two-dimensional or three-dimensional, the magnitude is only an application of the Pythagorean theorem. Also, our good friends the 3-4-5 triangle and the 5-12-13 triangle are featured in this example. These two triangles are popular with writers of exam problems because they make solutions turn out in whole numbers. The sine or cosine of the angles is a simple ratio of the side to the hypotenuse. The problem can also be solved with some trigonometry, but this way is much easier. On the FE exam, NCEES will most likely give the angle(s) if the triangle is not of the 3-4-5 or 5-12-13 variety.

$$\sqrt{\left(\left(\frac{260 \times -5}{13}\right) + \left(\frac{300 \times 4}{5}\right)\right)^2 + \left(-50 + \left(\frac{260 \times 12}{13}\right) + \left(\frac{300 \times 3}{5}\right)\right)^2} = 396 \text{ N}$$

Therefore (C) is correct.

Moments (Couples) [FERM 10:2]

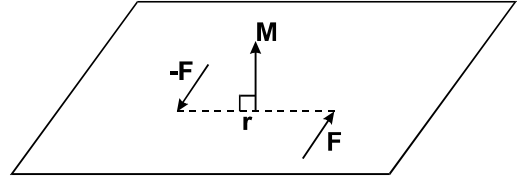
Two forces of equal magnitude in opposite directions but not in line are a couple. A couple can be thought of as two concurrent systems tied together by a lever arm.

The moment of the couple is the cross product of one force in a couple with a vector perpendicular in between the couple.

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

$$|\mathbf{M}| = r F$$

$$\mathbf{M}_{\text{total}} = \Sigma(\mathbf{r} \times \mathbf{F}) = \mathbf{r} \times \Sigma(\mathbf{F})$$



Equilibrium [FERM 10:3]

The sum of the forces and moments is zero.

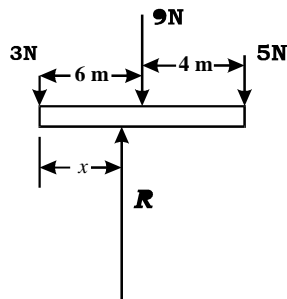
$$\Sigma \mathbf{F} = 0 \quad \Sigma \mathbf{M} = 0$$

Two-dimensional equilibrium

When we solve two-dimensional equilibrium problems, the force equation and moment equations will give two simultaneous equations, so we can solve for up to two unknowns.

Example: Equilibrium

For the system shown to be at equilibrium, what is the resultant \mathbf{R} ? What is the distance x ?



$$\Sigma \mathbf{F} = 0 = R - 3 \text{ N} - 9 \text{ N} - 5 \text{ N}$$

$$R = 17 \text{ N}$$

$$\Sigma \mathbf{M} = 0 = 3x + 9(x - 6) + 5(x - 10)$$

$$x = 104/17 = 6.1 \text{ m}$$

Three-dimensional equilibrium

Most likely, three-dimensional equilibrium problems will be limited to observation and basic understanding. The systems requiring simultaneous equation solution will likely be confined to a plane.

Example: Equilibrium

A bar is anchored at the middle with a universal, frictionless joint. A force of $\mathbf{F} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ is applied one end of the bar. What force must be applied to the other end of the bar to keep the system in equilibrium?

- (A) $3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ (B) $5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ (C) $3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ (D) $3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$

The other force is at the other side of the joint, so the force must be in the same direction. $3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$
Therefore (A) is correct.

CENTROIDS AND MOMENTS OF AREA [FERM 13:1]

The moment of area is used in finding the centroid. Strictly speaking, the moment of area is an integral of an infinite number of infinitesimal chunks of area. For the exam, the chunks of the object will have shapes that have a known area and center.

$$M_{ay} = \sum x_n a_n, \text{ with respect to the } y\text{-axis}$$

$$M_{ax} = \sum y_n a_n, \text{ with respect to the } x\text{-axis}$$

a_n is the area of the n^{th} chunk of the object

x_n and y_n are the coordinates of the center of the n^{th} chunk

Centroids [FERM 13:1]

The centroid is also called the center of mass. Strictly speaking, the centroid is an integral of an infinite number of infinitesimal chunks of an object times the density of the chunks. The integral takes into account that the object can have density that is not homogeneous. For the exam, we can ignore cases where the density is not homogeneous, and instead stick with the simple summation.

Centroid of an area

$$x_{ac} = M_{ax}/A \qquad y_{ac} = M_{ay}/A \qquad z_{ac} = M_{az}/A$$

With respect to the coordinate system origin,

$$A = \sum a_n = \text{total area of the object}$$

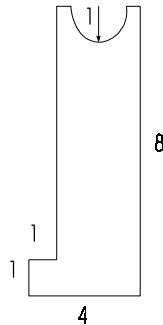
Centroid of an line

$$x_{lc} = \frac{\sum x_n l_n}{L} \quad y_{lc} = \frac{\sum y_n l_n}{L} \quad z_{lc} = \frac{\sum z_n l_n}{L} \quad \text{Where } L = \sum l_n$$

Centroid of a volume

$$x_{vc} = \frac{\sum x_n v_n}{V} \quad y_{vc} = \frac{\sum y_n v_n}{V} \quad z_{vc} = \frac{\sum z_n v_n}{V} \quad \text{Where } V = \sum v_n$$

Example: Centroid of an area



The area can be considered a 1×1 square plus a 4×8 rectangle minus a semicircle with radius $a = 1$.

From the table in the NCEES Handbook, y_c for a square or rectangle is one-half the height.

For a semicircle, the distance from the base to the centroid is:

$$4a/(3\pi) = 4(1)/(3\pi) = 0.424$$

The base of the semicircle is 8 units above the x -axis, so for the semicircle:

$$y_c = 8 - 0.424 = 7.576$$

The area of the semicircle is $\pi a^2/2 = 1.57$

Since the semicircle is a negative area, it gets a negative sign in the summation.

The total y_c is the sum of the individual areas times the individual y_c divided by the total area.

$$y_{ac} = \frac{\sum A_i y_i}{\sum A_i} = \frac{(1.0)(0.5) + (32)(4.0) + (-1.57)(7.576)}{1.0 + 32.0 - 1.57} = 4.98$$

MOMENT OF INERTIA [FERM 13:2]

The moment of inertia is also known as the second moment of the area.

$$I_y = \int x^2 f(x) dx = \int x^2 dA$$
$$I_x = \int y^2 g(y) dy = \int y^2 dA$$

Transfer or parallel axis theorem [FERM 13:2]

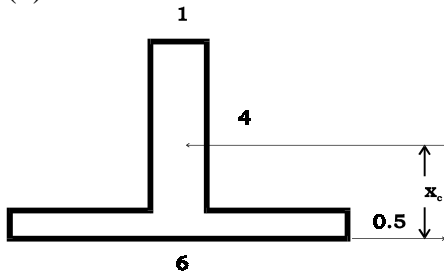
If the moment of inertia about an axis is known, the area is known, and the distance from the centroid is known, then the moment of inertia about another axis can be found.

$$I_x' = I_{xc} + d^2 A$$
$$I_y' = I_{yc} + d^2 A$$

Examples: Moment of Inertia

(1) Find the moment of inertia for the shape about the x -axis. (All dimensions show are in mm.)

(2) Find the moment of inertia about the horizontal centroid of the total shape.



(1) Consider the shape as two adjacent rectangles. The bottom rectangle has a moment of inertia taken about its edge (the x -axis) of I_x (lower) = $bh^3/3 = 0.25 \text{ mm}^4$. The moment of inertia for the upper rectangle taken about the axis of its centroid is I_{xc} (upper) = $bh^3/12 = 5.33 \text{ mm}^4$. Use the transfer theorem to move the upper rectangle to the x -axis.

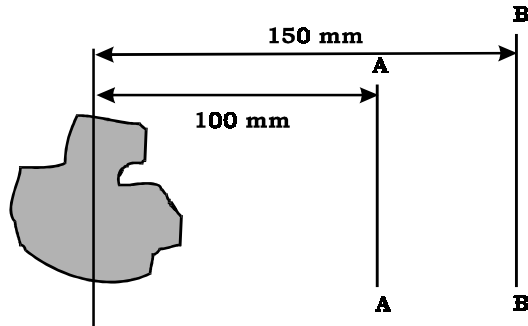
$$A = (4 \text{ mm})(1 \text{ mm}) = 4 \text{ mm}^2$$
$$I_x \text{ (upper)} = I_{xc} \text{ (upper)} + Ad^2 = 5.33 \text{ mm}^4 + (4 \text{ mm}^2)(0.5 + (4\text{mm}/2))^2 = 30.33 \text{ mm}^4$$
$$I_x \text{ total} = I_x \text{ (lower)} + I_x \text{ (upper)} = 30.33 \text{ mm}^4 + 0.25 \text{ mm}^4 = 30.58 \text{ mm}^4$$

(2) The centroid of the total shape is:

$$y_c = \frac{\sum A_i y_i}{\sum A_i} = \frac{(4.0)(2.5) + (3.0)(0.25)}{4.0 + 3.0} = 1.536 \text{ mm}$$

Use the transfer axis theorem to convert the x -axis to the centroid:

$$I_x = I_{cx} + Ad^2$$
$$I_{cx} = I_x - Ad^2 = 30.58 \text{ mm}^4 - (4 \text{ mm}^2 + 3 \text{ mm}^2)(1.536 \text{ mm})^2 = 14.07 \text{ mm}^4$$



The area of the shape is 6000 mm^2 . The moment of inertia about the A-A axis is $63.0 \times 10^6 \text{ mm}^4$. (Distances shown are measured from the centroid.) The moment of inertia about the B-B axis is most nearly:

- (A) $78 \times 10^6 \text{ mm}^4$ (B) $126 \times 10^6 \text{ mm}^4$ (C) $135 \times 10^6 \text{ mm}^4$ (D) $138 \times 10^6 \text{ mm}^4$

$$I_{A-A} = I_c + Ad^2 = I_c + (6000 \text{ mm}^2)(100 \text{ mm})^2 = 63.0 \times 10^6 \text{ mm}^4$$

$$I_c = 3 \times 10^6 \text{ mm}^4$$

$$I_{B-B} = I_c + Ad^2 = 3 \times 10^6 \text{ mm}^4 + (6000 \text{ mm}^2)(150 \text{ mm})^2 = 138 \times 10^6 \text{ mm}^4$$

Therefore (D) is correct.

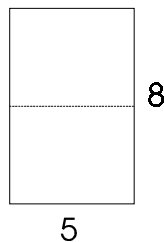
Radius of gyration [FERM 13:3]

The distance from the centroid of the area at which you could place the whole area, rotate it about that axis, and have the same moment of inertia is the radius of gyration.

$$r_x = \sqrt{\frac{I_x}{A}} \quad r_y = \sqrt{\frac{I_y}{A}} \quad r_z = \sqrt{\frac{I_z}{A}}$$

Example: Radius of gyration

What is the radius of gyration for the shape about the x_c axis?



From the table in the NCEES Handbook, $r_x = (h^2/12)^{1/2} = (8^2/12)^{1/2} = 2.31$

Product of inertia [FERM 13:3]

Equivalent to the moment of inertia taken about a coordinate system 45° from the x - y - z system:

$$I_{xy} = \int xy \, dA_{xy} \qquad I_{xz} = \int xz \, dA_{yz} \qquad I_{yz} = \int yz \, dA_{xz}$$

The transfer-axis theorem works much the same for the product of inertia as it does for moment of inertia. The NCEES Handbook gives only the transfer theorem for a coordinate 45° between the x -axis and y -axis.

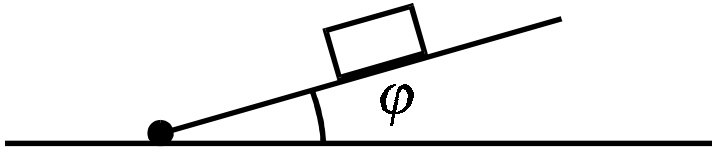
$$I_{x'y'} = I_{x_c y_c} + d_x d_y A \quad \text{where } d_x \equiv \text{distance between axes (x-direction)}$$
$$\qquad \qquad \qquad \text{where } d_y \equiv \text{distance between axes (y-direction)}$$

FRICTION [FERM 12:2]

Friction is independent of the surface area of contact between two objects. All that matters is the normal component of the force and the coefficient of friction. The coefficient of friction depends on the materials. Static friction is greater than dynamic friction.

$$F = \text{friction force} = \mu N$$
$$\mu = \text{coefficient of friction}$$
$$N = \text{force normal to the contact surface}$$
$$T = \text{tension}$$

Examples: Friction



A block on an inclined, hinged plane is at equilibrium. The coefficient of friction between the block and the plane is 0.25. The angle of the plane to the horizontal (ϕ) is increased. The block will begin to slide when:

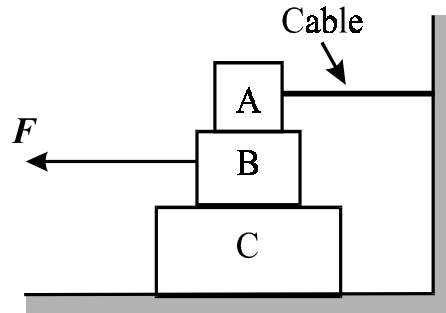
- (A) $\sin \phi = 1.0$ (B) $\tan \phi = 0.25$ (C) $\sin \phi = 0.25$ (D) $\cos \phi = 0.25$

The block will begin to move when $T \geq F$

$$T = mg \sin \phi = \mu N = \mu mg \cos \phi$$

$$\tan \phi = \mu = 0.25$$

Therefore (B) is correct.



Block A weighs 50 N, block B weighs 80 N, and block C weighs 100 N. The coefficient of friction on all surfaces is 0.30. The maximum force (F_{\max}) that can be applied to block B horizontally away from the wall without disturbing equilibrium is most nearly:
 (A) 15 N (B) 39 N (C) 54 N (D) 69 N

Between A and B the friction force is $F_{AB} = 0.30 \times 50 \text{ N} = 15 \text{ N}$.
 The force between B and C is $F_{BC} = 0.30 \times (50 \text{ N} + 80 \text{ N}) = 39 \text{ N}$.
 Equilibrium will be broken when $F \geq (15 \text{ N} + 39 \text{ N}) = 54 \text{ N}$.
 Therefore (C) is correct.

Note: In this example, the friction force between block C and the floor is greater than the force of friction between block B and block C; therefore block C will not slip.

Belt Friction

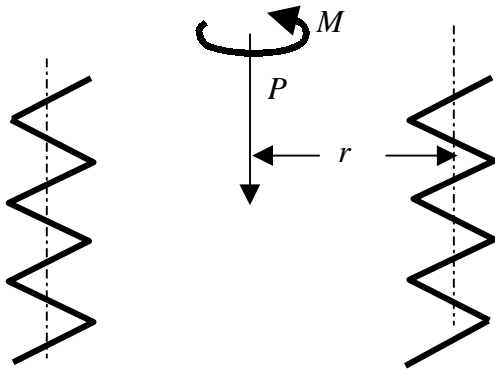
For a belt sliding over a stationary circular sheave, the force necessary to get the belt moving (F_1) must be greater than the force trying to keep the belt stationary (F_2). The relationship depends on the angle of contact on the pulley (θ in radians) and the coefficient of friction (μ). The relationship is:

$$F_1 = F_2 e^{\mu\theta}$$

Screw Thread [FERM 12:2]

Any screw-thread problems that appear on the exam will be solved by plugging them into the formula. However, it's worthwhile to review the variables.

- $M = Pr \tan(\alpha \pm \phi)$ where + is tightening and – is loosening
- M = moment applied
- P = load applied on the jack centerline
- r = mean screw thread radius
- α = pitch angle of thread
- μ = coefficient of friction = $\tan \phi$



The definition for pitch is the nominal distance between two adjacent thread roots or crests. The pitch angle is defined by:

$$\tan \alpha = \frac{\text{pitch}}{2\pi r} = \frac{\text{pitch}}{\text{circumference}}$$

DISTRIBUTED LOADS (EIT8 32:14)

The total force is the integral of the force over the distance.

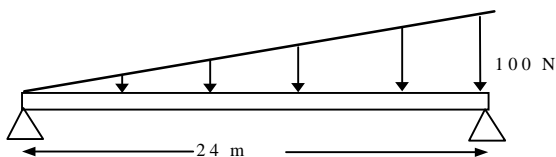
$$F_R = \int_a^b w(x) dx$$

The place the force acts is the center of pressure. The center of pressure is analogous to the centroid, so the table for the centroids in the NCEES Handbook can be used for finding the center of pressure.

$$x_R = \frac{\int_a^b xw(x) dx}{F_R}$$

Example: Distributed loads

What is the magnitude and location of the load on the beam?



$$F_R = \int_0^{24} \frac{100}{24} x dx = 1200 \text{ N}$$

$$x_R = \frac{\int_0^{24} x \frac{100}{24}(x) dx}{1200 \text{ N}} = \frac{100 \left[\frac{x^3}{3} \right]_0^{24}}{24 \times 1200} = 16 \text{ m}$$

An easier way to get the location of the load is to use the formula in the NCEES Handbook for the centroid of a triangle:

$$x_c = 2b/3 = (2)(24 \text{ m})/3 = 16 \text{ m}$$

STATICALLY DETERMINATE TRUSSES [FERM 11:1]

Plane trusses

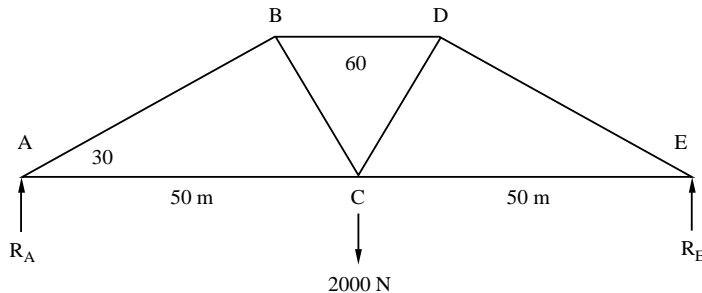
All trusses on the exam are single-plane, with frictionless pins, loads in the same planes, and at equilibrium. Symmetry can be a powerful shortcut.

Methods of joints

Write vertical and horizontal force equations for each joint.
Solve for unknowns.

Example: Method of joints

Use the methods of joints to calculate the force on section BD in the truss shown.



Calculate the moment around point A:

$$\Sigma M_A = (2000 \text{ N})(50 \text{ m}) - R_E(50 + 50 \text{ m}) = 0$$

Solving for R_E :

$$R_E = 1000 \text{ N}$$

Summing the forces on the truss in the y-direction:

$$\Sigma F_y = -2000 \text{ N} + R_E + R_A = 0$$

Solving for R_A :

$$R_A = 1000 \text{ N}$$

Note: By symmetry, we know $R_E = R_A$.

At point A, the member AC has no force in the y-direction, so the y-component of the member AB must be equal to R_A :

$$AB_y = R_A = 1000 \text{ N} = AB \sin 30^\circ$$

$$AB = 2000 \text{ N}$$

Writing the equilibrium equations for point B:

$$\begin{aligned} \Sigma F_x &= AB_x + BC_x - BD = 0 \\ &= AB \cos 30^\circ + BC \sin 30^\circ - BD = 0 \\ &= (2000 \text{ N})(0.866) + BC (0.5) - BD = 0 \\ \Sigma F_y &= AB_y - BC \cos 30^\circ = 0 \\ &= 1000 \text{ N} - BC (0.866) = 0 \end{aligned}$$

Solving for $BD = 2310 \text{ N}$

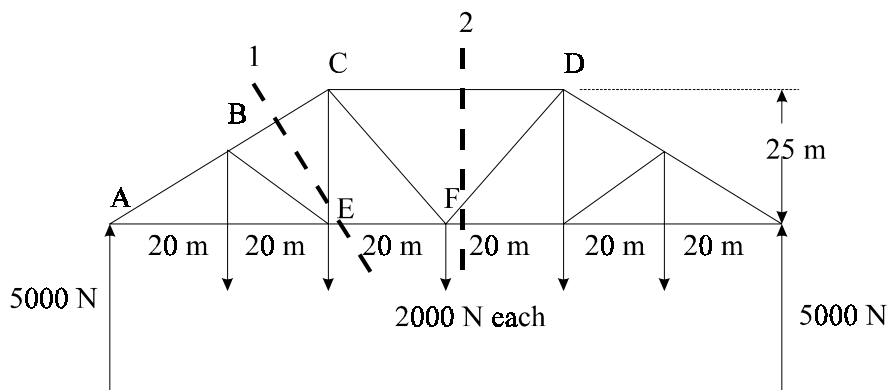
Method of sections [FERM 11:2]

Draw the free-body diagram for the portion of the truss cut through the member of interest.

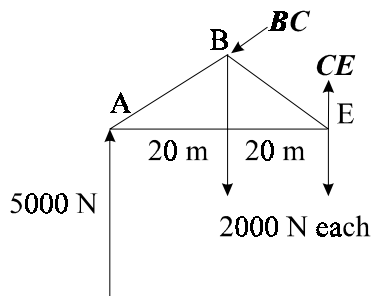
Treat the member of interest as an external force and calculate the force the remaining portion of the truss is exerting at that point.

Example: Method of sections

Find the forces on members CD and CE.



To find the force on CE, the truss is cut at 1.

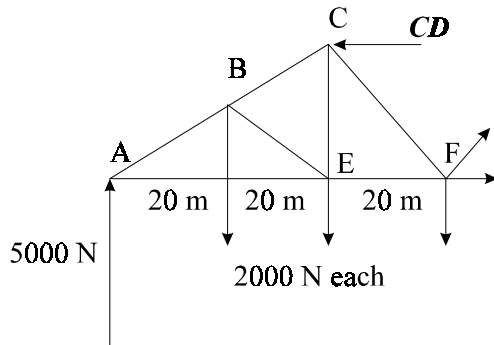


The vector **BC** exerts no moment about point A.

$$\Sigma M_A = (2000 \text{ N})(20 \text{ m}) + (2000 \text{ N})(40 \text{ m}) - (40 \text{ m})(CE)$$

$$CE = 3000 \text{ N}$$

To find the force on CD, the truss is cut at 2.



$$\Sigma M_F = (5000 \text{ N})(60 \text{ m}) - (25 \text{ m})(CD) - (2000 \text{ N})(20 \text{ m}) - (2000 \text{ N})(40 \text{ m})$$

$$CD = 7200 \text{ N}$$

Note: The vector **CD** is 25 m from point F, so the moment is 25 m times the magnitude of **CD**. Although it looks like we should find the distance along the CF member and resolve the CD vector into a component perpendicular to the CF member, these two operations are actually complements of each other—so doing this would have wasted time.

END OF STATICS LESSON