Paper Clip Geometry

Suppose there are five postulates about strings and paper clips.

Postulate A: There is at least one piece of string.

Postulate B: There are exactly three paper clips on every piece of string.

Postulate C: Not all paper clips are on the same string.

Postulate D: There is exactly one string through any two paper clips.

Postulate E: Any two strings have at least one paper clip in common.
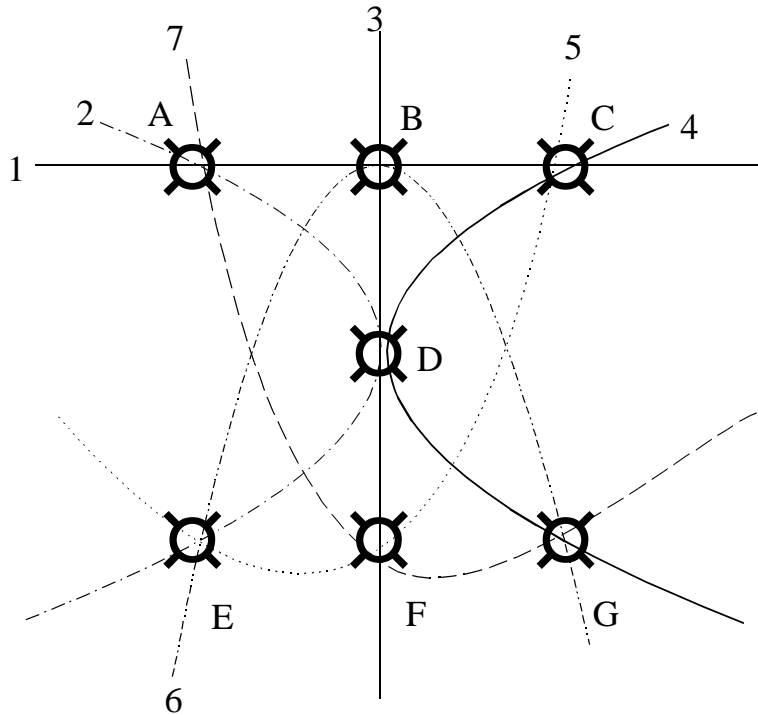
1.   Make a single sketch that illustrates all of these postulates.

2.   Using the above postulates, prove that there are at least five paper clips.

3.   What are the minimum numbers of strings and paper clips needed to fulfill all of the conditions of the postulates?

4.   How does eliminating Postulate D affect your answer to #3 above. Explain your thinking.

Answers:

1.

2.   There is at least one piece of string (1) by Postulate A. By Postulate B, there are at least three paper clips (A, B, C). Postulate C implies there is at least one paper clip (D) that is not on the first string (1). By Postulate D, there is a second string (2) passing through one of the paper clips on the first string (A) and the fourth paper clip (D). By Postulate B, the second string (2) must have 3 paper clips on it (A, D, E). Therefore, there are at least five paper clips (A, B, C, D, E).

3.   Seven strings and seven paper clips.

4.   Four paper clips, one string; Without Postulate D, there is no requirement for a paper clip to have a string through it.