AP Calculus Students as Teachers

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Abstract

In AP Calculus class, we inflate and deflate balloons and use long lengths of plastic tubing to simulate falling ladders to understand related rates. We use Play-Doh to get a three dimensional, hands-on view of volumes of solids of revolution and of solids with known cross-sections. These student-centered activities are always well-received and undoubtedly aid in student comprehension of these concepts. However, meaningful manipulatives and interactive demonstrations are not the only creative ideas with potential pedagogical value. Peer teaching appears to be a potentially valuable tool to aid in the understanding of Calculus topics. This paper discusses the planning and execution of a research project in which AP Calculus students taught seven lessons to a Calculus class.

Key Terms: Peer teaching, Peer tutoring, AP Calculus, Student Led Instruction

Background

The idea of peer teaching is not a new one. Gerber & Kauffman (1998) describe the "monitorial system," developed by Joseph Lancaster over two hundred years ago, in which the basic learning of children was actually managed by a hierarchy of other students. More recently, Melaragno & Newmark (1969/1970) advocated peer tutoring to solve problems of underachievement in public schools.

As a teacher, I wanted to devise an activity for my AP Calculus class which would serve multiple purposes. First, it had to deepen my students' understanding of calculus. This could be accomplished by implementing a strategy to stretch students' thinking beyond the mere duplication of problem solutions which I provide them. Second, it had to be something about which my AP Calculus students could feel very positive. Third, it had to be a cooperative effort with each student taking the lead at some point. Fourth, I wanted my students to have something substantive that I could document and that could enhance their personal résumés.

As has probably occurred in most calculus classrooms, I have had students who occasionally were unable to cogently express a question in words; so, they asked, "can I show you on the board?" I noticed that they invariably had the attention of the entire class and that their classmates were eager to "get in on the act" by adding their approaches to problem solution and expressing anxiety at their shared difficulties. The idea of students teaching students sounded appealing; but, of course, it would need to be approached in the right light. Late in the Fall of 2009, I approached the calculus teacher to discuss the possibility of my students taking on the role of teacher in the calculus class several times in the January-April 2010 timeframe. The Calculus class covers only differential calculus and my AP Calculus class finishes differential calculus in December. My colleague was enthusiastic about the idea. I then presented the idea to my AP Calculus students and I received eager and excited responses from the whole class.

There were several potential benefits. The calculus class could benefit from having concepts explained to them and demonstrated to them by different teachers (their peers). Also, the appeal of listening to a fellow student teaching a lesson was an interesting factor. The AP Calculus students would need to be well-prepared for their lesson; they could gain an appreciation for the work and preparation required to present a lesson on one idea with a few examples. The AP Calculus students would also gain valuable public speaking experience. And, not insignificantly, the AP Calculus class end-of-year review sessions before the national AP exam may not need very much time on the topics which were covered in the student teaching sessions.

Method

With a fledgling idea and an eager class, we set out to make a plan and complete a pilot study. There were seven students in the AP Calculus class. Each chose a topic: Product Rule, Quotient Rule, Chain Rule, Chain Rule with Trigonometry, Implicit Differentiation, Related Rates I, and Related Rates II. All of these topics had already been taught by the calculus teacher. However, the calculus teacher assessed that her class could benefit from the re-introduction of these topics. Each student prepared a lesson. Pre-tests and post-tests were written. Practice sessions were run; constructive criticism was shared. Lessons were delivered and debriefings were conducted. It was lesson-study for students.

After the seven lessons had been completed, both the AP Calculus and calculus classes completed an anonymous survey about the value of the study. Everyone who was involved recommended that it be repeated.

I graded the pre-tests and post-tests and performed matched-pairs t-tests at the 95% confidence level for each of the seven pre-test and post-test sets. The largest p-value obtained was 0.04, with the exception of the lessons on related rates, which both had p-values slightly higher than .05. I considered the study to be a success because of the favorable reviews and the t-test results, which indicate that the interventions were statistically significant.

However, I had made a few mistakes. First, there was no third-party master list which identified calculus students with their participant numbers. Several calculus students forgot their numbers from week to week and a new number had to be provided. This compromised the anonymity that I was trying to maintain. Second, I did not develop a grading rubric for the pre-tests and post-tests until I started grading them. And, third, I did not enforce a time limit on the pre-tests and post-tests.

I intended to conduct a similar study the following year, this time without the mistakes. Our school had a new calculus teacher, who, fortunately, was enthusiastic about the study. I again received enthusiastic responses from both the AP Calculus class and the calculus class. I also received several unsolicited notes from parents who were pleased that the study was being repeated; they had heard positive comments from the previous year.

In the Spring 2011, the plan was formalized into a research project including all the requisite approvals, consents, and assents. Neither the AP Calculus students nor the calculus students received any grade for their participation in this study. Participants again were identified only by a randomly assigned number.

The AP Calculus class had fourteen students; so, teams of two were randomly assigned to seven topics. These students then were charged with preparation of their lesson plans. I prepared pre-tests and post-tests for each lesson; the pre-test and post-test for each lesson were similar to each other and tested the same knowledge. I also prepared grading rubrics for each question. The calculus class has 18 students this year; I asked the calculus teacher to randomly assign each of the students a number between 1 and 18, inclusive. He kept a list of the names and numbers in case any one of his students forgot their number. This happened numerous times. The list was never shared with me and was destroyed at the end of the study.

Again this year, the calculus class and the AP Calculus class meet during the same time periods. For each lesson, the date was coordinated with the Calculus teacher. Usually, one lesson was completed per week for a seven week period. A day or two before each lesson, the student-teacher team presented an abbreviated version of their lesson in a practice session during the AP Calculus class. Classmates critiqued the lesson and offered pointers; the student-teacher team asked their colleagues specific questions about their presentations as well. The class also discussed what questions could reasonably be expected from the Calculus class. The AP Calculus class worked as a team, appreciating that the entire class was responsible for the success of each lesson.

At the beginning of each session, I thanked the calculus class for allowing us into their classroom and for helping me with my study. I referred to them as "resident scholars." I then had my AP Calculus students, the "visiting scholars," distribute the pre-test. The calculus teacher privately assisted any student who forgot her/his participant number. A maximum of ten minutes was allotted for completion of the pre-test. Then, the student-teacher teams taught the lesson for 20-30 minutes. Several AP calculus colleagues, one from each team, attended each session to provide backup in the event the student-teacher team encountered any difficulties. This never happened; but, it was a stress reliever for each of the student-teacher teams. At the end of the lesson, the post-test was administered. The calculus teacher allowed a maximum of ten minutes for the post-test and then collected them. While the Calculus class completed the post-test, the AP Calculus class returned to our classroom and discussed the lesson. First, the student-teacher team discussed the lesson and how they thought it was received. They noted any specific events which they considered to be important. Then, their classmates gave constructive criticism and praise, as appropriate. The student-teacher teams who had already presented their lessons appeared to be more relaxed and more willing to offer commentary. Most frequent criticisms included low talking volume, writing too small and standing in front of what they were writing on the board. Most frequent praises included being responsive to questions from the calculus students, being aware of the calculus class not understanding a step and repeating it to clarify, and adding a little humor to the lesson, usually by admitting that the topic of the day was difficult for the student-teacher when she/he first encountered it. As the study progressed, these debriefings became more insightful and encompassing. At first, the student-teachers focused on themselves and how well they did and how well they were received. As the weeks passed, they began shifting their focus to the calculus students, measuring the success of the lesson in terms of how well the calculus students appeared to "catch on," how many questions they asked, and how involved in the lesson they were. The AP Calculus students were beginning to think like teachers.

During the lessons, all the student-teachers focused on areas which had been challenging for them a few months earlier. One of the students who taught related rates directed attention to noticing whether $\frac{dy}{dt}$ and $\frac{dx}{dt}$ were each positive or negative. One of the students who taught the Chain Rule reminded the class several times to avoid shortcuts and to use u-substitutions and v-substitutions and even w-substitutions, if needed.

In discussing grading rubrics, the AP Calculus students saw that simply giving a grade of zero to all incorrect responses on the pre-tests would result in a skewed picture of the abilities and thinking strategies of the Calculus students. It would also make the before-and-after comparisons questionable. So, a rubric for partial credit was devised for each question. For example, the student who incorrectly computes $\frac{d}{dx}cos(2x)$ as $-sin(2x)$ or $2sin(x)$ as or is exhibiting knowledge that the derivative of cosine is the sine (both examples) and negative (first example).

Results

The enthusiasm of the AP Calculus and calculus students was energizing and promising. However, the research question asks whether or not the project showed any correlation between student-led instruction and peer learning. A matched pairs t-test for the total (all fourteen questions) post-test scores vs. the total pre-test scores indicated that the interventions were effective, $t\ (17) = 8.115$, $p < .001$. See Table 1. A review of the individual paired t-tests for each of the seven lessons reveals some interesting results. Lessons on the Chain Rule, Chain Rule with Trigonometry, and, Implicit Differentiation showed statistically significant results, all $p < .05$. However, interesting anomalies were observed in: (1) Lesson 1, a Product Rule question, $t(17) = 1.844$, $p = .484$; (2) Lesson 2, a Quotient Rule question, $t(15) = 0.627$, $p = .541$; (3) Lesson 6, a Related Rates question, $t(15) = 1.000$, $p = .33$; and, (4) Lesson 7, both Related Rates questions, $t(15) = -0.432$, $p = .672$, and, $t(15) = 1.430$, $p = .173$.

Table 1
Matched pairs t-test for Post-Test Scores Minus Pre-Test Scores

Paired Samples Test

	Paired Differences							
				95% Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
PostALL - PreALL	15.556	8.133	1.917	19.600	11.511	8.115	17	.000

For the question in Lesson 1, the 95% confidence interval indicates that the mean increase in post-test score over the pre-test score is at most 1.12; it also indicates a possible mean loss of as much as 0.55. The p-value of 0.484 also indicates that the difference in scores is not statistically significant. This is not terribly surprising; a review of the scores shows that eight students answered the first question correctly; so, there was no way for them to improve.

For the question in Lesson 2, several students applied the quotient rule by switching the order of the numerator in

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$$

and incorrectly used

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{f(x)g'(x)-g(x)f'(x)}{[g(x)]^2}.$$

Few students incorrectly used a plus sign in the numerator. Most of them however, used the correct formula for the second question, $t(14) = 3.238$, $p = .006$. The same errors were committed in the post-tests, making a lack of significant change understandable.

For the question in Lesson 6, all students scored a zero on the pre-test question, which involved an inflating balloon. Similarly, Lesson 7, which covered more challenging related rates questions, showed little improvement. The AP Calculus students remarked that, of all the lessons they taught, related rates had been the one which was most difficult for them to learn (months earlier). Coupled with the results from the previous year, this result does beg further inquiry. It is possible that the students simply could not recall the needed formula, such as the volume of a sphere.

Conclusions

The study was an improvement over last year's pilot study but there are still some kinks to be worked out. Overall, however, it was a positive experience for both classes. The AP Calculus class felt that they reaped much more benefit from their participation than they had anticipated. Each one felt s/he had a much firmer grasp on his/her topic than before the lesson. The Calculus class believed that they benefitted from hearing explanations from their peers. For the most part, statistical results support that contention.

Overall, the results are promising. However, it appears students still need repetition even when the lessons are taught by colleague student-teachers. Evidence includes numerous examples of students incorrectly applying the product rule during the implicit differentiation lesson, and, incorrectly applying the chain rule during the related rates lesson. To address the problems noted, in future studies, the calculus students will be provided all formulas.

The level of enthusiasm for calculus that resulted from the study and the confidence which the AP Calculus students gain is sufficient impetus for me to want to continue this study every year, making improvements every time. Given the success of the project, I have added it to my AP Calculus syllabus.

Because this data collected is from only two classes at one school, no generalizations to the population of AP Calculus students or calculus students can be inferred; and, none is intended. However, larger studies could provide information which could be inferential in nature.

References

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