Polynomial Dancing

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Background

Following a twenty year career in the U.S. Navy, I transitioned from military life to civilian and then to high school teacher. This civilian world was quite different than my life of military operations, emergency planning and scientific research. Although I treasure my service time, it was somewhat of a relief to not be responsible for all aspects of the lives of my Sailors 24/7. I thought about how my approach to interacting with people was changing. I no longer wore insignia on my collar which told everyone I was in charge. However, I certainly assume that motivating adults to get their oceanographic analysis reports completed on time is a different world from motivating teenagers to complete their homework. Perhaps, the underlying principles are the same, but the approaches, at least for me, are quite different. I realize that I tend to see situations as black and white, off or on, go or no-go; however, in the past several years, I have allowed several shades of grey to move into my thinking. So, while I am sure that several of the approaches which I use in teaching my classes are productive, I am open to the idea that some students may benefit from something different. I do not always know what that something new ought to be, though.

In my military life, each time I transferred to a new duty station, I had to establish myself. I did not arrive one day, announce that I was the new boss, and expect everyone’s immediate trust and confidence. I needed to earn that; and, my subordinates needed me to earn that. Now, each September, I step into new classroom of students who do not know me, do not know what is expected, do not know what is tolerated and what is not, etc. Reputation, work ethic, and approachability are not qualities which one can established once and then assume that everyone knows. I sometimes think of the exasperated face of Sidney Poitier in "To Sir with Love" as the second year begins and a new batch of undisciplined students shuffle into his class. Like him, I think, "here we go again from ground zero." But, really, the trip is the reason I went into teaching, to see the growth and development of young mathematicians.

I have stood in front of my high school math classes many times and asked what I thought was a simple mathematical question, one which I thought required a modicum of thought, but not one which I thought would be mentally exhausting. I noticed that some students would think while tracing numbers in the air with their forefingers, as if the air was a white board and their finger was the marker; some would stare into space; some would shuffle restlessly; some would reach for their calculators. These *calculator-reachers*, as often as not, arrived at an incorrect, and frequently implausible, answer. Yet, they would blurt it out as the definitive response. I began to wonder about the effects that calculators have on a student's mathematical reasoning and their understanding of reasonableness. I began to make notes to myself about my concerns, observations, and beliefs. It was due to this process of refection that I saw that the student who stares into space and the student who shuffles restlessly, probably fearing being called upon, are having difficulties with mathematical reasoning which are not necessarily related to calculators at all. I see that calculators were possibly a significant piece of the puzzle and that they certainly could play an important role in this development of mathematical reasoning, but they were not the centerpiece. I want to learn about how mathematical reasoning is developed. That became my research question.

Setting

All non-Algebra high school mathematics classes, Geometry, Trigonometry, PreCalculus, Calculus, and Statistics, require Algebra. So, I reasoned that the heart of development of mathematical reasoning in high school students would be found in Algebra classes. If it is not, then I believe the Algebra teachers would lead me to where the heart is. I set out to explore the views of Algebra teachers on how students learn to reason and problem solve and how the availability of calculators affects the students' development of mathematical reasoning. I decided to interview Algebra teachers who teach different levels of Algebra. Again, I thought it was important to keep my study confined to one branch of mathematics. Geometry, for example, requires abstract thinking and spatial awareness; trigonometry requires understanding of ratios. Both are beyond the scope of what I am currently seeking. I wanted these teachers' opinions on how mathematical reasoning is developed in Algebra students.

All my interviewees teach at the same school as I do. It is a small, independent school in Northern Virginia. It is a Pre-K through 12 school and the high school has approximately 225 students; class sizes are generally about 12. The maximum class size permitted is 18. My participants include two teachers who teach at the high school level, and one who teaches high school and 8th grade.

At this point, I believe it is helpful to depict clearly the levels of Algebra which are taught at this school and the "math tracks" students may take. These are shown in Table 1 below. The three teachers who were interviewed teach at different levels with a few overlaps. These are shown in Table 2 below.

Track	8th grade	Freshman	Sophomore	Junior	Senior
1	Algebra I, Part 1	Algebra I, Part 2	Informal Geometry	Algebra II	Algebra III
2	Algebra I	Geometry	Algebra II	PreCalculus	Calculus
3	Algebra I	Honors Geometry	Advanced Algebra	Honors PreCalculus	AP Calculus

Notes:

- For Track 1, Algebra I has been divided into two parts. These students cover the same material as the Algebra I students, but they take two years instead of one year.
- The material covered in Advanced Algebra is equal to the material covered in Algebra II plus Algebra III

Table 1: Common Tracks for High School Algebra Students

Teacher	Algebra I, Part 2	Algebra I	Algebra II	Advanced Algebra	Algebra III
Abby		X	X	X	X
Beth			X		
Connor	X	X			

Table 2: Algebra Teaching Assignments

Before I began my interviewing, I thought it important to examine my own beliefs on how mathematical reasoning is developed. My thoughts are that students learn to reason by practicing and by taking the time to think. I view the journey to logical thinking, through mistakes, struggles, differing approaches, and joining concepts, as the quest more than the ultimate numerical answer to a question. The calculator, I think, is a tool to augment mathematical reasoning, not replace it. However, I do have concerns with a student who, for example, announces that the probability of some event is 310%. My concern is not so much the impossible answer, but more so that the student does not recognize that his “answer” makes no sense. I wonder how other mathematics teachers feel about this subject; I wonder if they spend much time thinking about it.

So, I set out to discover the perceptions of Algebra teachers on how mathematical reasoning is developed and if calculators have any effect on the development. I do not think that there is a one-size-fits-all answer to this question, but I am hoping to find some threads of enlightenment through my interviews.

I do not personally have an "all or nothing" opinion on calculators in the classroom. I do not believe that calculators should be banned from classrooms; nor do I hold that calculator should be allowed for anything and everything. Somewhere in between these two extremes, I wonder is there is a reasonable balance. In my classroom, I see students who have used calculators during their entire high school (or even earlier) careers. I want to know what the teachers of the underclassmen think; what are their observations; what are their opinions; and, what has formed those opinions.

I have to recognize that my own experiences and beliefs will color my thinking no matter how much I try to approach each interview as a clean slate. However, I have never actually discussed these topics with my participants; so, I have no reason to believe that they will be influenced by my opinions, because they do not know my opinion. But, they might think that they do. I told each of my interviewees that I really want their opinions; I do not want them simply to agree with what they think I want to hear. All three of my interviewees are independent thinkers and I do not anticipate that this will be a problem. We have very congenial working relationships; we discuss issues and affably disagree all year long.

Assumptions

People who enter the teaching profession do not do so for the betterment of their bank accounts, especially teachers at independent schools, which pay less than public schools. Through my experiences and observations, I have come to assume that these teachers are teachers because they truly want to be teachers. They care deeply about their students and want to provide the best educational experience for them. I have no evidence to the contrary in the research for this paper.

I believe that Algebra teachers will know how mathematical reasoning is developed in their students. They will have observations, opinions, and experiences to support this knowledge.

I also assume that because these people have nothing to gain and nothing to fear from my interviews, that they were truthful with me. I have no reason to believe that did not happen. However, I was surprised by the one teacher's long pauses and seemingly measured responses. She said that she wanted to give me concise answers and give me what I wanted. Even though I assured her that what I wanted was her unfettered opinions, she would not or could not let go of her goal to be concise.

Even though I teach only juniors and seniors, I assumed that my colleagues' experiences would be similar to mine with the possible exception of less mature behaviors observed in their younger students. I thought the group of teachers may differ on some small matters but that they would be a generally homogenous group. When I asked Abby if she would consent to an interview, she eagerly agreed, as did Beth and Connor. She then asked me if I was interviewing any teachers from public schools. When I replied that I was not, she dismissed her own question and remarked that I would probably get different responses from public school teachers anyway because "they think differently than we do." I was not about to let that remark go without asking for an explanation. She replied that they have less leeway in their lesson plans; they have to stick to SOLs; they have more restrictions on their time; they are unable to give as much personal attention to students as we are able to do; and, their classes are huge compared to ours. She spoke with authority, as she had taught in public schools for ten years. But, from a selfish standpoint, I assumed that public school teachers would probably not have the in-depth observational experiences which I assumed the teachers at my school did have, if, for no other reason, because our class sizes are so small. Our approaches to after-school availability, communicating with parents, writing reports on each student several times a year, and weekly faculty meetings to discuss any child who is showing signs of slipping are simply not possible for a large student population. I could be way off track in assuming that public school teachers would have neither the time nor the opportunity for insightful observations, but currently I am not in a position to investigate.

Researcher Relationship

I chose my three participants for several reasons. I know them well and have a very good working relationship with them. They are all intelligent and insightful, each in her/his own way. I interviewed Abby first; she teaches the range Algebra classes in high school and in eighth grade.

Abby is in her mid-forties and has taught for about 16 years; she started in public schools and moved to private about 6 years ago. She has mostly taught middle school, but has moved up to the high school level in the 4 years ago. She worried that she did not know anything which would help me and felt that she might say something silly. With the range of Algebra classes she teaches, she has a breadth of experience which no other interviewee had. Several times, I assured her that I was looking for her opinions, observations and insights. She seemed somewhat relieved by that; but, I could tell that she was trying to phrase her sentences with proper grammar. Several times during the interview, she lamented, "oh, that sounded dumb." I have no idea why she felt that way because nothing she said was anywhere near to being dumb. I had expected her interview to be interesting but I never expected her to be so talkative and to have the depth of insight and thoughtful observations which she explained to me. I know she is quite intelligent; however, she usually is not so animated. She answered each of my questions multiple times, for the differing levels of her Algebra classes.

I interviewed Beth two days later. She is in her mid-forties and teaches Algebra, PreCalculus and Honors PreCalculus. She has about twenty years of high school teaching experience, all in private schools. She has taught at day schools and at boarding schools. She is a certified teacher but prefers the private school setting because of the smaller class sizes, the close-knit faculty, and the friendly relationships with students. My interview with her was surprising, not at all for the content, but for the brevity of her responses. Even though I continued to try to extract responses from her, her interview was well below one hour. She paused quite a bit and thought before she answered each question. She got talkative a few times; but, mostly, she seemed intent on giving deliberate answers, with clear flow of thought, despite my encouragement to ramble. Apparently, she thought she knew what I wanted. I thought I knew what I would get. We were both wrong.

My third interviewee, Connor, is a man who is in his late twenties or early thirties and who has been teaching for about 6 years. He is also the school's swimming coach and boy's lacrosse coach. He has a well-developed sense of humor and uses it daily, especially in his Algebra and Geometry classes. He is passionate about his students learning mathematics. He holds daily study sessions in his classroom during his preparation periods and invites any of his students to see him at those times. There is always a student in his classroom getting help. He also frequently asks the other math teachers about their experiences so that he can try different approaches in his classroom. Connor is a loquacious person and during the interview, several times, I had to get him back on track.

I had envisioned friendly conversations between colleagues. Abby was concerned that because I teach Calculus, her concerns with Algebra were not really very interesting to me. She asked if I as going to be taking my research to higher levels and apply it to math courses which are of interest to me. I assured her that Algebra was of great interest, otherwise, I would not have chosen this topic. I also told her that her input, her observations, opinions and experiences, would help guide where my research would go from here. She said that was "kinda cool." Abby accepted my assurances and calmed down and rambled just wonderfully. However, Beth was not swayed from her determination to give me "good data." And, she did give me much to ponder. However, it was more of an effort for her than I had ever expected or intended. Abby's interview was an easy, and somewhat rambling, but very insightful collection of thoughts. It was almost hard to tell when the interview was over, the tone of the conversation did not change, still two colleagues talking. It was about the same with Connor; he was talkative and energetic, as he usually is. Beth, however, seemed much more relaxed after the interview was over. She had many good insights; I still cannot understand why she seemed so on edge. I have the closest friendship with her; perhaps, she was worried she would not give "good data" to her friend.

In my conversations with these teachers, I have come to respect not only their teaching abilities but also their dedication to their students. The interviews have confirmed that they have insights and knowledge which I did not have.

Data Collection and Analysis

When I had finished the interviews and completed the transcriptions, I stared at a stack of 90+ pieces of paper and thought about what I should do next. I wrote what I thought were the salient points of each interview on Post-It notes, about 250 of them, using a different color for each teacher. After much arranging and rearranging, I had the notes assembled into different piles. Then, I stopped in my tracks, realizing that I was categorizing these ideas instead of listening to them. I read through them and the transcripts again and thought about what all of these teachers telling me. I discarded most of my categories (a few of mine matched my revised system) and resurfaced with the major ideas which the teachers had discussed: long hand work; understanding processes and concepts before using technology; checking for plausibility; and, responsibility to teach technology and teach it appropriately. There were also a few interesting teacher reflections which I believe warrant inclusion.

My conclusions are that high school Algebra teachers at this particular school believe:

- mathematical reasoning is developed by doing problems by hand;
- calculators can be an aid in developing mathematical reasoning but only if the teacher employs the calculator properly;
- checking for plausibility is essential for students to develop sound mathematical reasoning; and,

- technology is part of our students' everyday life. We have a responsibility to teach them how to use it correctly in order to prepare them for their college courses and for life.

Calculating By Hand Before Using Technology: The ideas that mathematical reasoning is developed through practice with long hand computations, and that calculators should not be introduced until the concepts have mastered, go hand in hand. Teachers believe that it is appropriate to allow students to use calculators for more complex or labor-intensive problems only after they have mastered the concepts using simple values and solving long hand.

Connor stated he "sees very little math reasoning" in his Algebra I Part 2 class. These are the slower track students and his remark is not terribly surprising to me. Even though Connor leads them through problem solutions in class, he says that he believes that their lack of success is due more to lack of interest than lack of ability. However, he believes that he is leading them to develop reasoning skills, and that is his goal. Abby describes these as "attacks skill," having several lines of approach to use. If one approach does not work then, the student has another approach to try. Beth summed up the views of all three teachers in saying that the ability to reason comes from doing problems in class without a calculator. While it may seem tedious, the teachers all say that they emphasize to their students that this is the way to learn. They remarked that they use "simple numbers" in the long hand problems. This builds both the students' ability and their confidence. When harder versions of the same types of problems are introduced, and calculators are allowed, the students have already cemented the requisite reasoning in their thoughts. In a recent conversation with Dylan, the middle school math teacher at this school, he told me that he wants his students to start their Algebra careers by "engaging their brains." When teaching quadratics, Connor does not advance to "weird numbers" until he is certain that the students can handle "regular" numbers. When they are ready, he will give them problems with more challenging solutions and show them that the process is the same. He says, "I want them to see that just because the numbers aren't nice and neat, that does not mean that the process is any different." Beth says that her students only get their "ah ha" moments while doing work by hand. Interestingly, all the teachers used a graphing example to illustrate their point. Abby believes that "sketching gives the students an expectation of what the graph should look like." Beth takes it a step further and has her students form shapes of common functions with their arms. She calls it polynomial dancing. Connor does a similar activity but he is limited to linear functions, so he focuses on positive or negative slope and steep or shallow slope.

Hand graphing quadratics, Abby says, leads the student to think about what direction the parabola will face and if it will be wide or narrow. In previous conversations with Abby, she has mentioned the importance of hand sketching graphs several times. This year, she had her advanced class make large exemplars of basic function shapes out of colored tape, which are affixed to her classroom wall. She clearly feels that visual recognition of functions is an important skill in the development of mathematical reasoning. I have walked past her classroom several times and seen polynomial dancing going on.

Furthermore, sketching quadratics helps students to develop an awareness of the number of solutions which a quadratic function will have; then, they learn to relate the number of solutions not only to the graph but also to the function itself. Connor purposely does not teach his students how to graph function on their calculators. He insists on hand graphing because he wants them to understand the process. He says, "I want them to hand graph versus going on there hitting keys and plugging in their functions and then hitting the graph key. That doesn't teach them anything."

All teachers agreed that graphing by hand could be a bit tedious and they had to stick with relatively easy numbers to use until the students had grasped the process, or at least the procedure, before they would allow graphing on the calculator. As Beth remarked, she wanted to ensure her students understood a concept, not if they could simply press a few buttons on calculator. Beth's experiences have convinced her that when classes are heavily calculator-based, the development of mathematical reasoning is weak at best. She has noticed that the basic mathematical skills have diminished and that students cannot multiply something as straightforward as 3×25 in their heads; they will reach for their calculator. On the other hand, Connor allows his students to use their calculators for basic arithmetic; he reasons that they will always have a calculator available to them and they have a comfort level knowing they can use it. His students are usually two years younger than most of Beth's students.

The teachers focus on process and understanding of concepts, above calculating the correct answer. This undoubtedly would surprise some students, but it certainly speaks to the importance which the teachers place on conceptual understanding. Abby referred to "number sense" several times, indicating that students need to develop it in order to be successful in their math careers. It all comes back to number sense, to understanding the balancing of an equation, to comparing positive and negative numbers, and relating roots and factors of polynomials.

Estimating and Plausibility. Estimation occurs before problem solving; it is thinking about what you expect the answer to be. Plausibility checking occurs after the problem has been solved; it is asking if your answer makes any sense. All three teachers emphasize checking for reasonableness but not so much on estimating. Abby feels that, starting in lower school, estimation can appear silly for those who can readily discern the exact answer. So, she does not often ask her students to estimate.

Connor says that his students' ability to estimate is still developing. Sometimes their estimations are "off the wall;" but, he says, "they at least actually estimate, they will at least actually look at it." Abby asks her students if their answers fit with reasonable thought for them. For example, after her class has solved a quadratic equation, she will ask the class to graph the function and compare the graph results with their calculations, asking, "do your answers look like that, what the graph shows?" Beth emphasized that checking for reasonableness is a skill which needs to be taught. Her experience has been that higher level students will regularly check; but, for some of her students, that idea never clicks. Connor's freshmen are learning to check but they are not yet doing so with regularity. However, they have difficulty discerning the plausibility of relatively large numbers such as deciding if it makes the most sense for the moon to be 2,500, 25,000 or 250,000 miles away from Earth. He attributes this not to a lack of ability, but to a lack of interest, especially with the slower track students. Abby also commented that students are less inclined to check for plausibility if they are working with a calculator, because she says, "they trust the machine." Slower track students frequently rely on rote procedures, so, they have not developed a context for discerning if their answer is reasonable. Beth believes that conceptual development can sometimes be out of sync with class placement; we place students in Algebra at such a young age, sometimes as early as 7^{th} grade; and, the students do not yet have the necessary capability to master some of these concepts. Student who are "lost," as Abby says, may get a feeling that their answer is not correct, but they could not justify it beyond saying that it is too big or too small. Some students may realize that their answer is unreasonable but they have nothing else. But, at least they connect that their answer is not plausible.

Teachers are that students place too much trust in their calculators and do not feel the need to check for plausibility as much as when they compute by hand. An input error, therefore, has less probability of being discovered. Teachers feel that they need to continue to emphasize the reasonableness check.answer is not plausible.

Responsibility to Teach Technology Appropriately. "We are Atlas holding up the world," says Abby. Because our students will use their calculators in science classes and in their college classes, we have a responsibility to teach our students well how to use graphing calculators. She in determined to teach calculator use effectively so that her students do not get "lost." She is equally determined that they will understand the mechanics behind it. She say, "They have to do both. We start with teaching it by hand where the calculator isn't even out at all, and make sure that they can visually sketch a line of best fit and visually pick off two points from that line and calculate the slope and complete the whole process."

Using a calculator, allows students to solve problems that are more "real world," where the interest rates are not easy integers and distances are not in exact feet. Abby says, "it allows them to do problems that they cannot mentally check as easily. If you are not using a calculator at all, the problems have to be too simple." Beth's classes use the calculator to complete more examples and for easy comparison of the shapes of graphs in graph families. This helps students to see the relationship between values in an equation and the visual appearance of the graph.

Teachers believe that the reasoning behind concepts must be grasped before calculators are introduced; otherwise, the technology can be a hindrance to the development of mathematical reasoning. That is why Beth insists that her students "really need the basics to be strong before they went on to the toy. So, in order to not set them up for failure, I really want to focus on having strong basics which they could build on."

Teachers agree that while calculators are wonderful, time-saving and labor-saving gadgets, they are no substitute for human thinking. We have to teach students to view a calculator as a tool, not as a magical solution machine. Connor feels that the students understand that the calculator does not have any ability to make leaps of judgment, that it has to obey rules. Beth and Abby are not so sure.

All agree that calculators should supplement work done by longhand, however, teachers' practices on allowing calculator usage varies. Beth allows calculators only after a concept has been firmly grasped. She introduces more difficult problems, in which the answers may be complex numbers or radicals instead of a nice, neat integer, and allows her students to their calculators only after she is sure they know what they are doing. She also likes using the calculator for quick comparisons of graphs, to show where asymptotes are located. She said, "calculator use is ok in class once the process is learned; we can do more examples and quickly do a variety of comparisons."

Teacher Reflections. A pleasant surprise was that during and after each interview, every participant had a sort of reflection aloud. All thought that the interview had actually benefitted them because they were now thinking about their own practice.

Connor said that he was now thinking about how he could use calculators to help his students next year, saying, "I want to do a lot more being conscientious and deliberate in my use of my calculator. My idea would be saying that technology is scary and it's hard to deal with because we have changed, it changes the people who teach. The people who are teaching now usually learned without calculators and now have to deal with lots of people who understand calculators innately."

Beth remarked about how my questions had started her thinking about her own teaching in more depth than she has before. She apologized (unnecessarily, I assured her) for the long pauses she took before answering. She said that she never really put some of her thoughts into words and that doing so was making her think about her teaching practices and beliefs. "It's been really good for me to talk about this," she said.

Abby said that the interview had helped her to realize that she needs to repeatedly stress to her Algebra I students why she allows the calculator for some things and not for others. She added, "I wonder if I don't think about mathematical reasoning enough or maybe I just don't think in that phrase. … I think the whole math department should talk about this during faculty work week in August. I am really thinking about my classes and why I am doing the things I do there. I am going to think about this some more." I saw Abby several days after her interview and she told me that she had been thinking about mathematical reasoning for several days, and that she is thinking about that as she prepares for the next school year.

Validity

The interviews were held in the summer when the teachers did not have mountains of work to complete. Abby's daughter had just been married; both Beth and Connor were looking forward to relaxing vacations just days after the interviews. There were no major stressors in their lives of which I am aware. So, I believe they were able to concentrate on the interview questions and not rush through. I have no reason to doubt the truth of anything they told me.

My only real concern was that they may, in their desire to help me, give me information which they thought I wanted to hear. Other than Beth wanting to be concise, as she repeatedly said, I have no reason to believe that the data I was given was anything but the teachers' true opinions.

Lessons Learned

I learned several practical facts about qualitative research. A short list of questions on an interview guide is a good thing, given that the questions are open-ended, inviting to elaboration, and have good follow-up questions. Eating lunch before the interview is a good idea; both interviewer and interviewee can concentrate on the interview, not their growling stomachs. The audio recorder must be equipped with a slow-down mode to make transcription possible in a reasonable amount of time. And, assuring your interviewees that their recording will not be shared with anyone seems to alleviate some concerns, even when the interviewee has neither anything to fear nor anything controversial to say. These things, of course, can be written down and shared with others. But, what I learned about myself is far more important to me.

I never thought of myself as a know-it-all, but in my classroom, I am the authority. I had to learn to shed that authority role. When I was transcribing my interviews, I found myself wanting to correct the grammar. I did correct some minor (my opinion) grammatical errors, but did not disturb the essence of what the person was saying. I left some of the stumbling in the transcripts because it was a part of how the people talked. You can almost see the synapses firing as they start a second thought without finishing the first. I had to give myself a few virtual slaps on the hand and stop being the teacher and keep being the listener.

I found that allowing myself to wander through the thoughts and words of others without my red correction pen, without fixing, without teaching, or making it fit a procedure of a line of thought which I have explained, to be both a relief and an eye-opener. It was like picking up puzzle pieces and not knowing how or if they would fit together, and not worrying about how they might fit into any framework which I may have. And, I found that being able to accept ideas which were not in concert with mine was a little harder than I had anticipated. During the interviews, several times, I felt the urge to say, “Yes, that’s nice, but …” and insert my own opinion. Fortunately, I managed to keep my mouth shut and appreciate viewing the world through another person’s eyes.

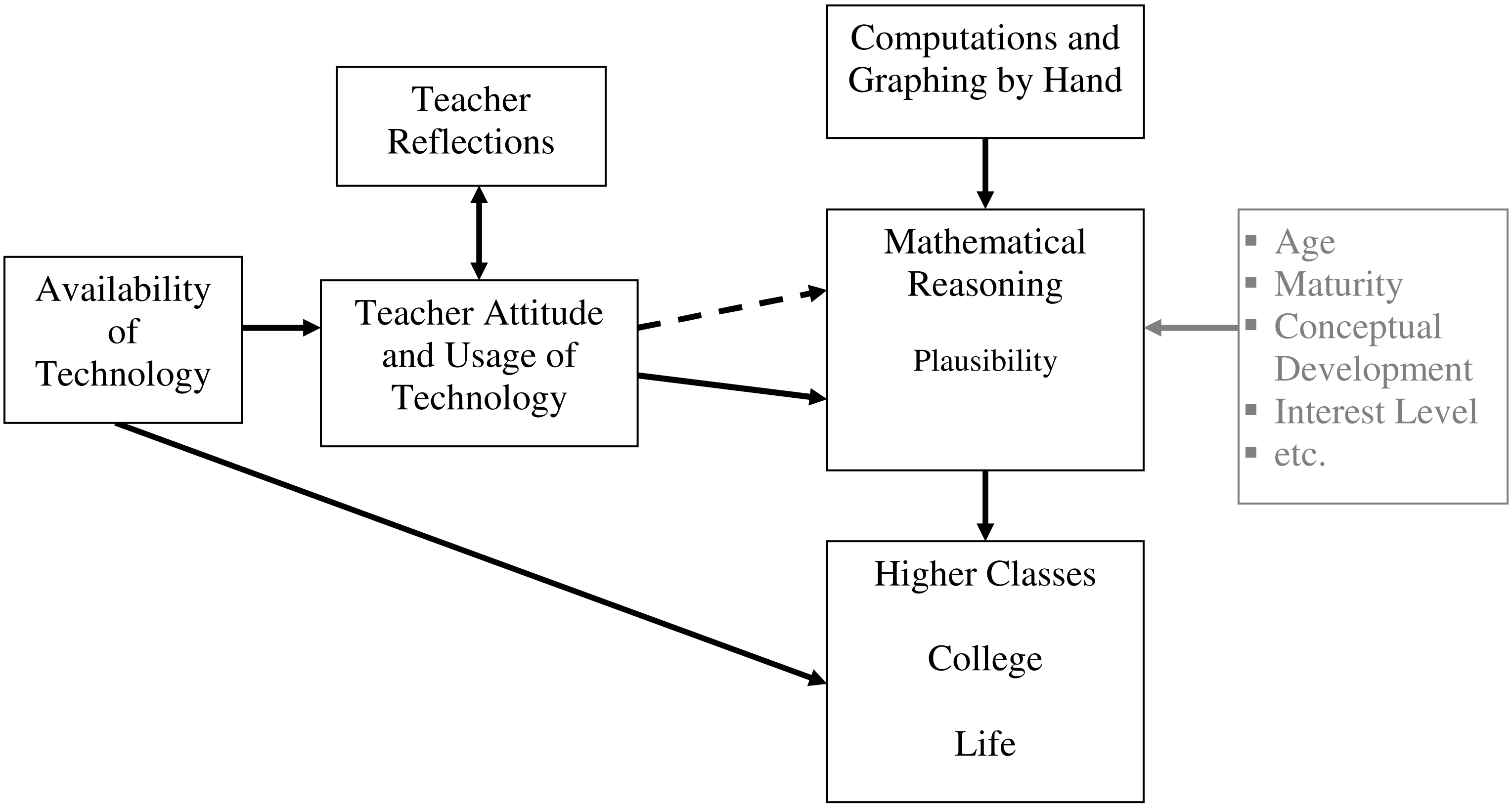
I learned not to pre-judge my participants, no matter how well I think I know them or how good our collegial relationship is. I am humbled by my colleagues and somewhat embarrassed that I unwittingly underestimated them by not realizing the breadth of their insights and the scope of their understanding. By no means did I think they were anything less than intelligent, capable and caring teachers. But, I failed to recognize how intuitive and reflective they are. I had no idea how deeply each of them has thought about how their students develop logical reasoning. Each was able to name a particular students and/or a particular lesson or class to illustrate several of their points. And, all these wonderful things are going on in classrooms right across the hall from me and I did not know.

Qualitative research is not a spectator sport. I cannot stand on the outside as if I am counting jelly beans in a jar. A qualitative researcher is part of the research. It took a while to wrap my brain around that. And, qualitative research changes the researcher. How could it not? I am learning from other people's experiences and observations. I encounter their biases. I listen to their reflections. This all impacts my own thinking. Maybe it bolsters my opinion or my bias in one way; maybe it makes me think twice about something I believe. It definitely causes me to reflect on how I think and what I think and why I think the ways I do.

While I had a smorgasbord of data, I soon understood that not everything can be included. There were several topics in this short project which I wanted to cover but there simply was not enough time or space. It is like being at a fruit stand and seeing piles of ripe tomatoes or deep burgundy cherries; you want to grab them all. But, you can't do that. So, in choosing what to focus on, I must leave a few of those prize morsels behind. But, they give me ideas for further research.

Appendix A

Concept Map



- Mathematical reasoning is developed by doing calculations by hand. Checking for plausibility is part of mathematical reasoning.
- A student's ability to mathematically reason affects their success in college and in life Plausibility checking is par to mathematical
- Use of technology in the classroom can be beneficial to mathematical reasoning or not, depending on the teachers' approach.
- Technology will affect our students for their entire lives; we have a responsibility to teach them well.
- Teacher reflections can change the teacher's classroom approach
- There are other factors which affect the development of mathematical reasoning, such as maturity, which are not discussed in this paper

Appendix B

Interview with Abby on Monday, 28 June 2010

Time: 1 hour, 22 minutes

Me: Let’s talk about the mathematical reasoning which you see your Algebra students employ. Tell me how your Algebra students mathematically reason; and, if it's different for Algebra I, Algebra II, Advanced Algebra and Algebra III, then make that distinction.

Abby: I think that they would think that the most logic they ever do is just with solving equations because so much of it is you’re either simplifying one side or the other. So, so many of the commands that they are asked to perform, the directions, are either simplify or solve. And, then they have to analyze what processes they're going to use. Is it working solving an equation with radicals? Is it solving an equation with quadratics and what are the methods that they would need to use? So, a lot of it is attack skills, I think, I look at it as. They understand word attack skills from when they were in elementary school. So, I think a lot of it is for them to figure out, “where do I get started”. What type of problem is this and where do I get started? And, from that point I think that they are mostly following a procedure that they're trying to trigger to remember in their mind and then, … it's interesting because, … mathematical reasoning, … I just think of it as their basic skills of where do I start, where we go with it?

Me: What do you mean by attack skills?

Abby: Well, you know, with word attack, it's looking for the beginning sounds and different ways to decode by syllable. So, I look at solving equations, they need to figure out, they need to recognize a framework that's going on. Is it a quadratic? Am I going to expect two answers? I'm sorry, up to two answers? Is it an equation that I have to isolate the radical first or is it an equation where I can combine like terms, if it's a simpler equation. Is it something I'll need to use the strategy of completing the square? Is it something that I'm going to go straight to quadratic formula. So, I think a lot of it is where do I get started, because you see a lot of low students look at a quadratic and immediately try to isolate the variable and they're putting, you know, they are subtracting the constant and putting it on the right hand side when they shouldn't be. They should be recognizing that it is quadratics. So, I look at a lot of those kinds of things as having a framework to know, to recognize, what type of problem they are dealing with. Because, as I say, often, the directions are only simplify or solve. And, then, if it is a word problem, being able to write an equation that appropriately matches, and, then solve.

Me: Do you think that your students have good mathematical reasoning? Do they attack appropriately?

Abby: I think that from the middle up they do, not the lowest third.

Me: When you say middle up, do you mean the middle classes?

Abby: Middle ability.

Me: Oh, middle ability in each class?

Abby: In each class, yes. I would say that if you divided each class in three groups of low, middle high, from middle up, they do. The low group, you know, they're trying anything that they might ever think to try. They might have one method, oh, I always, you know, combine, I always put the constant on the right hand side, or something. They might have some little strategy that, that's the only one they know.

Me: Do you think that they are then just looking for an answer but their answer doesn't actually have to make sense?

Abby: Oh, I think frequently the answer does not make much sense. For that confused group.

Me: For the other two groups, do you think it does make sense?

Abby: I think it does. And that's the exciting part for them and for the teacher, that they say, yes I see how this whole puzzle fits together, I see the point of how this puzzle fits together. And, then making that further connection that if it is a word problem, recognizing is it a work rate problem, is it a, just a system of equations or something like that, that they see the application.

Me: Do your kids ever estimate?

Abby: I ask them to, when they work something, I verbally will ask them to say, does that fit with any reasonable kind of thought to you? And, again, going back to quadratics, did you expect two answers, did you expect one answer, did you expect, you know, whatever, or is their answer reasonable. Well, that's your next question here, about plausibility, that I'm getting in to. If they got just one answer, that's a key thing in Algebra II, and Algebra I later, did you expect to get two answers? Because if they are factoring and solving with zero product property, often kids can get one answer and just have totally missed the point of what's going on there. So, that's part of the checking the answer for plausibility, not numerical plausibility, definitely, I'm sorry, not always numerical plausibility, but number of answers, is that plausible?

Me: I just want to recap. So, you are saying that they would check to see if they expected to get 0, 1 or 2 answers?

Abby: Right.

Me: Maybe based on the discriminant or something?

Abby: Or even just based on is it an *x*-squared equation. Getting down to that level of basics. Or, if it's an absolute value equation.

Me: But, you are saying that they are not actually checking if the values make sense?

Abby: At that point, No. The first thing I ask them to check is did you get the number of answers that you expected. And, then with the values, depending on, I don’t know if you want to start talking about the calculators or not, but if they are looking for an, for example, on a quadratic, if they are looking for an answer that comes out to an irrational number and they are actually going to, say, four with decimal places instead of an exact answer with the radical, if they have a calculator available for that section of a test, I’ll ask them to go and graph it and say, do your answers look like that, what the graph shows, even if I’ve asked them to do it with quadratic formula, just to say, check for reasonableness or plausibility or, you know, your estimate based on, does it match up with the what the graph might show.

Me: And how do they do with that?

Abby: They do pretty well, although I’ll say that I don't often give them the option to have a calculator available when I'm asking them for an answer that they had to calculate with, say, quadratic formula. I don't want them to be able to go to the calculator and go to 2nd CALC and find the zeros, when I've ask them to work it out as a process.

Me: Why don't you let them do that?

Abby: Because I want them to know both the 2nd CALC zeros and the process. Because why use quadratic formula if you just have to give your answer to four decimal places? And, you can force them not to be able to do that by asking for an exact answer with simplest radical form.

Me: Are you saying that you want them to know how to use the quadratic formula?

Abby: Yes. So, I allow calculator usage based on what I am asking for. Because I also do test and want to make sure that they know how to use the calculator to get the answer to four decimal places of precision just from 2nd CALC, because they need to know the shortcut ways, too, technology ways. But, if you want the estimating in an Algebra I kind of way, because they are totally different, you know, with Algebra I and Algebra II. With Algebra I, I do ask them to estimate or get an idea, at least check their answers for plausibility. And, I guess I don’t often ask them to estimate. I more often ask them to check their answer for plausibility.

Me: Do they know what that means?

Abby: I think so. I think they do. In fact, I think they know checking for plausibility more than they know estimating. And, I think part of that is because from lower grades up, I used to teach elementary, and in the textbooks when they ask you to estimate, if you were a good math student, it looks really silly to estimate because you already see the answer. I mean, if you’re a half decent math student, the answer, the exact answer, is just right there in front of you. So, when they're asking sometimes for estimation, in third or fourth grade, it's an exercise that seems silly to them because the answers already there. But, with word problems, and again I don’t know that I asked them to estimate. I asked them to look for plausibility instead, after they have done the work.

Me: Do they do a pretty good job with that or do you see any problems with it?

Abby: Well, I think that they do a pretty good job with the plausibility. But students who are struggling might say, that answer is not plausible, but that's all I've got, and stop right there. But, at least they connect that my answer is not plausible.

Me: Do they give good reasons why their answer isn’t plausible or do they just say that?

Abby: The ones who are totally lost, I think they have a feeling that it's not right. I mean, when they look at it and they can say it's too big, too small, maybe, but I don't know that they could justify all that well beyond it's too big or too small, or I have a gut feeling, I have no clue. But I think kids are pretty good about saying whether the answer is plausible, the kids I have taught.

Me: This is throughout all levels of Algebra?

Abby: Yes, I think they're pretty good about saying whether their answer is plausible.

Me: Now calculator usage in your classroom, how does that work and tell me some of your observations.

Abby: Ok. With Algebra II and Advanced Algebra, I think it's really important to teach calculator usage with the graphing calculator because my view of the math, especially high school, is that we might exist for kids to study it for the joy of studying and the beauty of math, but we also exist to be able to support the sciences. We're Atlas holding up the world there. And, they will use their calculators so much in their science classes and in their college classes, that I think we have a responsibility to teach them well how to use the features of the TI-83 or 84, whichever. So, I want to make sure I teach them how to effectively use the calculator for when they're doing linear regressions, things like that. Yes, you could plot by hand and sketch your line of best fit, pick two points on there, but I don't think that's very real-world. I think that's an exercise leading to the application of it. So, if you were doing some experiment, you would get all this data and you would put it in List 1 and List 2, when you would do the linear regression model on the calculator. So, I want to make sure that we teach them that effectively, how to use that, so that when they get to their college science class, they are not lost and asking a friend how to do that on the calculator. But, they need to understand the mechanics behind it. So, they have to do both. We start with teaching it by hand where the calculator isn't even out at all, and make sure that they can visually sketch a line of best fit and visually pick off two points from that line and, you know, calculate the slope and the whole process there.

Me: Tell me about how you think your Algebra students understand the solutions which their calculators deliver.

Abby: I think often they don't, I think on calculator work there, they don't check their answers for plausibility as much. I think that if they've got word problems that they had to analyze that, they will check their answer for plausibility a little better. I think using the calculator for some of their things, … I don't know that they check their answer as well for plausibility, … interesting, … I hadn't thought of that, really.

Me: I wonder why that is. Do you have any idea?

Abby: Well, they trust the machine.

Me: Do you think they would recognize if they got an absolutely ridiculous answer?

Abby: I think depending on the problem and how well versed they were in that type of problem, they would. And, also depending on a time constraint. If it were a test and they were running out of time, I think they stopped checking for plausibility. They're happy to have something. I think they are pretty good at checking for plausibility. But then there are times, you know, when you are doing some word problem that comes up with a Biology example, they might not be as good as checking for plausibility there. I mean, certainly they should. If they say, I've got some sample and it's decreasing that's easy to check. … Interesting.

Me: What do you think the effects of calculators in your classroom are?

Abby: Hmm. … I think it allows them to do problems that they cannot mentally check as easily. You know, if you're not using a calculator at all, the problems have to be too simple, when you get to Algebra II and Advanced Algebra.

Me: For all the students, you think?

Abby: The lower Algebra II, maybe not. But, I think the calculator brings them a little closer to real world and a little closer to what they might see in college science classes.

Me: So, how do you think the calculators are affecting their logical reasoning, their ability to problem solve, their ability to set up a problem?

Abby: I think in the Algebra II and III, it does not hurt their ability to reason, because they've got a larger concept that they're dealing with on a problem, or a more difficult one. It's not like in Algebra I, if they were using a calculator for a one step or a two step equation, I often don't have them use a calculator for solving one and two step equations because they need to internalize these are the steps for basically undoing that order of operations for isolating the variable that I'm doing. So, I'm totally a different place for Algebra I with solving equations. I would rather they not use a calculator at all at that point, because it highlights to them what they are doing in the equation.

Me: Not using a calculator highlights it?

Abby: Not using a calculator highlights that. And, if you use simple numbers there, I think that is a good thing because it clarifies for them, these are the steps, undo the addition or subtraction, undo the multiplication or division, if you are looking towards the beginning of Algebra I, So, I would rather use simple numbers there and no calculator when they are learning internalizing those steps. Because if they have a calculator, they just start punching buttons, even a four function calculator.

Me: Are you saying you don't think calculators at all in Algebra I?

Abby: In the early stages of working with solving equations, including distributive property, variable on both sides, they tripped over the integers, which is too bad, over signed integers, but I think that's where they need to build that number sense. And, they've got to get it right.

Me: Do I understand you correctly then that if they use the calculator you think they would not develop that requisite number sense?

Abby: Yes, from the beginning, I think, at the very beginning, they need to not use the calculator. Once they've mastered, I think that’s the next step is to put on harder problems, where it would be appropriate to use just a four function calculator, so that they can make sure that they have the steps. But also with a harder problem, they can't intuitively, visually see the answer as easily, and they would have to check for reasonableness, or they would have to confirm their answer, check their answer. So, I like to have it be a progression, where they start out with Algebra I not using a calculator for solving equations, and then, go to the calculator for harder problems. The other use of the calculator in Algebra I with solving equations is very often if they have an equation that has compatible numbers, and for example, let me see if I have a good one. (Abby draws Figure 1). If they have X over 5 equals 45, visually a lot of students say, oh, yeah, fact family, X is 9, because they say, I've got division by 5, I've got 45, must be 9. And, even if they have a calculator for that, they will jump to, yeah, 9 is the answer, calculator or no calculator. So, sometimes, I like letting them have a calculator on some ones like that because it even encourages them to go faster and say, oh, yeah, 45 divided by 5, it's 9. And, that's not what they need to be doing there and I'll intentionally give them problems that look, that are part of compatible numbers like that, so that they are forced to slow themselves down and not jump to an answer. And, there, I don't think it matters whether they have a calculator or not. Sometimes a calculator helps them makes an error that I want to highlight with them.

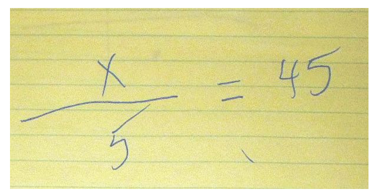


Figure 1

Me: So, even if they use a calculator they come up with nine?

Abby: A lot of times. Because their mind jumps to, oh, I know that. I've got division, got 45, got a 5, must be 9.

Me: Now, when you say compatible numbers, do you mean easily divisibility or numbers which are factors of the largest number?

Abby: Yes. When they are looking at fact families, the multiplication fact families, they have, say, (Abby draws Figure 2) 5-9-45, and they have a family, a little house, and they could have 5 times 9 is 45, or 9 times 5 is 45, this comes from elementary, or 45 divided by 5 is 9, or 45 divided by 9 is 5. When they have those fact families, and it's good for them to have that number sense saying, these numbers kind of go together. But it also can lead them astray that they knew. They need to look at their ways for solving equations.

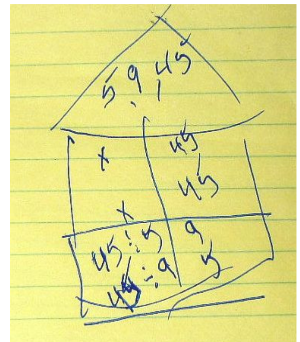


Figure 2

Me: So, order of operations, is that a difficulty area?

Abby: Absolutely. Because they visually can jump to a connection that they should not be jumping to sometimes. And, textbooks test them on this sometimes also by putting numbers that, again compatible numbers, if you had 5 times 8 plus 2, I don't know if I'm going to get one, ….

Me: You could make it 12 divided by 4.

Abby: Yeah, the 8-12, that's good. (Abby writes Figure 3). And, depending how it's laid out, sometimes visually it's laid out that if you, you know, if you use math a lot, your plus becomes such a marker for you as separating your terms, but before they've internalized that very well, if they see in the center here with this 8 and 12, 8 plus 12, if they are such easily combined numbers, you know, they jump on 20. So, if you had a 7 plus 3 in the center, they will, can jump to that, to 10, because visually they are drawn into it.

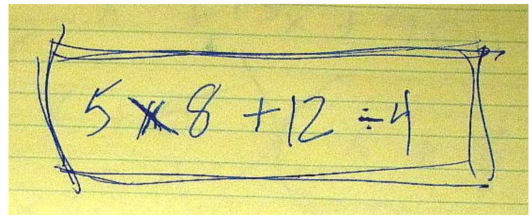


Figure 3

Me: How does the calculator affect their solving of that problem? Or, does it?

Abby: I don't think on this one, on this example, I don't think the calculator has an effect there.

Me: They would use, they would have the same reasoning?

Abby: If they made an error, they would have the same error either way on this one. So, this one I don't think the calculator affects.

Me: But if they typed that into the calculator like it is written, they would get the right answer, right?

Abby: At this level, they are only using the four function calculator that wouldn’t do it.

Me: So, you use four function calculators in Algebra I?

Abby: They use that early on, four function. You’re right, that's true and that's part of intentionally what we do. This example here with a four function, that would probably be a pre-Algebra example not an Algebra

I. I mean they would do this in Algebra I also.

Me: But, still, in Algebra I, you use four function calculators?

Abby: They use both. The Algebra I starts with the graphing calculator really when they get to the linear equations. So, for solving equations, if they haven't bought a graphing calculator yet, they're still using four function. So, they are about 80% using graphing calculator and 20% four function and they are not well trained in Algebra I to have that graphing calculator with them everyday, which means they are more likely to have that four function calculator, early on for order of operations and solving equations.

Me: I assumed that all the students used TI-83s.

Abby: Well, the other part of that is kids who have accommodations are allowed to use the four function calculator pretty much any time unless you are specifically testing whether they can add and subtract integers, which goes to a different thing about calculator use. Still with Algebra I, they make so many errors on adding and subtracting positive and negative integers and to get around kids who have the accommodation use of four function calculator, I'll often set up a section with no calculator where they have to add, say 5 plus negative 8, and not tell the answer but just tell whether it's positive or negative. So, there is a way to get around kids who have those accommodations but still test "do they know the positive or negative." So, they would just have to give whether it's positive or negative as an answer.

Me: But, the child can still use the calculator?

Abby: On this one, I say no calculators for anybody, including those with accommodations, because we are specifically testing, do you know the concept.

Me: Well, I guess using a calculator would defeat the purpose…

Abby: I know. (Laughter) Exactly. And, they need that, I'm sorry, I didn't remember if that fits (pointing to the interview guide).

Me: We do not need to follow that order, that is just a guide.

Abby: Ok. Well, that's from the lower-level grades, and just, again, you'd think, I know, you'd think that they'd know this stuff, which we teach them at the younger grades. But even using the words like "positive times and negative," and having them write that the answer would be a negative, just different ways help them internalize what their number sense ought to be.

Me: Ok, you have talked about your Algebra I in detail. What about Algebra II? Please tell me about calculator use in your Algebra II classes.

Abby: In Algebra II, I allow calculators for almost everything, in part because I want them to learn how to use it and not trip over themselves with it, and this is all with a TI-83 graphing calculator. The only time I don't allow a calculator specifically is if they are working with different functions and developing those expectations of what a function should look like, whether it's linear, if they see a function written in function notation, to be able to match it to a graph. So, if you've got examples of a linear function, of quadratic, x cubed, x to the fourth, and be able to match it to some sample graphs. I do not allow a calculator there, or, if they are describing quadratics and they need to describe whether the graph of the function opens up or down or, what else, an example of, do you expect a function to be a wider parabola or more narrow parabola, which, you know, just general matching of a function equation to a graph. So, those are the kinds of sections that I don't let them use a calculator because they need to develop expectations of what these functions ought to look like.

Me: And, your reason is? Allow me to elucidate on the obvious, your reason is because they could easily graph it and see it, right?

Abby: Oh, yes. They do need to develop those expectations so that when they see any function, they immediately have expectations of what it is. It’s like number sense for a younger group. It's function sense.

Me: Thinking again about Algebra II, we’ll stick with them for a while and then move on to your Advanced Algebra and then Algebra III, ok? For your Algebra II, what effects do you think the calculator has on their mathematical reasoning, their ability to problem solve?

Abby: Hmm. … I think it's such an integrated tool by then that I don't think it has as much effect on having them internalizing what they're looking for. That's a weak sentence. They will see. I think that …

Me: Just think out loud.

Abby: … Right. I think that a lot of students come into Algebra II not having learned a lot about the TI graphing calculator. Some schools with their middle school Algebra I use calculator a lot and some schools use it almost none. And, so, in the Algebra II, I think a lot of that is training them how to use the calculator, just mechanics, you know, beyond add, subtract, multiply, divide and taking square roots and things. So, some of it's training. So, I don't know, … on solving equations, … on that topic, … I don't know that it matters for them developing number sense. On graphing, I think that if kids are weak, they will try graphing anything to see if they get any hope of what they're doing, if they are weak. And, I think they will often look at a graph and try to give roots just by visually estimating the roots off the graph, and those, again, are the weakest students who won't even think at that point to go to 2nd CALC and look for zeros. That's part of it. These are kind of misuses of the calculator. Sometimes with factoring, kids will go fiddle with the calculator, I guess, and they'll catch that there is some connection between the roots and the factors, if you're looking at the low kids. But, of course, then they will get it backwards. They'll say, the factors are, but give the roots, right, or, if they are truly confused, they will look at the *y*-intercept and say, that must be one of the answers. So, the calculator can lead them if they are grasping at straws, on an assessment. They might grasp at straws and the calculator gives them a straw to grasp at. So, I don't know that the calculator has any effect on their learning, I think that they were missing it all along. I don't think the calculator did that. It might help them think that they are saving face on a test to be able to put something down. But I don't think it has a direct effect on their learning there. Where I do see, and this part I do feel strongly about, kids have the weakest, I'm sorry, half the kids in Algebra II have this very weak concept of what a base is when you're working with powers in a negative base. And, when they key something in the calculator and if it's a negative base, they'll forget the parentheses, and that is the first lesson I teach when I am working with eighth-graders in Algebra I, about how to put in that base. It's an early lesson in just order of operations and it's one of the first lessons that I make sure, when we pull the calculator out for Algebra II, to highlight that your negative base has to be in parentheses, and it just seems one fourth of the students have a hard time remembering that throughout everyday math that they use. And, I don't, it's interesting because I don't know that the calculator, the calculator hurts them a little bit on that, using the calculator. Because sometimes they will write it down incorrectly if they have no calculator for a problem that has a negative base. They will write it down incorrectly but do it correctly. So they'll write negative 5 squared (-5^2) and write 25 as the answer. So, they will do it correctly even though they have written incorrectly. So, the calculator kind of guarantees that they do it wrong because they don't key it in with the parentheses, but if their hand writing it wrong, they are still wrong on having written it with, right, as the opposite of 5 squared.

Me: Now when they write negative 5 squared (-5^2) and they get that answer of negative 25, do they realize that it's wrong?

Abby: No, especially if they are, for example, doing several operations at once But, even if they write negative 5 squared (-5^2) and get negative 25, and again, that's where they're not checking their answer or thinking through. They're just accepting, I keyed it in, that's what it is. And, it's a lesson that we come back to repeatedly. But, it's one, it's a hole they fall into frequently.

Me: All of them?

Abby: A fourth of the students. I would say a fourth of the students consistently fall into that hole.

Me: And the other 75%?

Abby: They get it. They occasionally, you know, will forget the parentheses around the base.

Me: And, those same 75%, if they write negative 5 squared (-5^2) in their calculator and got negative 25, would they recognize that it is wrong?

Abby: They recognize it most of the time. There are occasions when they are doing something harder, more intricate, and trip anyway. But not many.

Me: Not to harp on this, but if it was, let's say, a distance problem and they get an answer that the distance from here to the barn is negative 25 yards, how would they think about that?

Abby: They would get distance needs to be positive and they would switch it to positive 25 yards. However, if it were was a problem that had multiple steps in it, they would still have the wrong answer because one of the steps would have been positive 25 instead of negative.

Me: Now, let me get this straight. Algebra I are generally eighth graders and freshmen?

Abby: Yes.

Me: Mostly freshmen?

Abby: Uh, half and half. No, two-thirds eighth graders

Me: And, Algebra II, which we just talked about?

Abby: They are mostly sophomores and juniors. And, the juniors, technically would have been the weaker math students. Because they would've taken Algebra I as freshmen, Geometry as sophomores, and Algebra II as juniors. However, because they are another year older, sometimes they are not a weaker math student because things have, over time, sunk in.

Me: So, it’s 50-50 sophomores and juniors?

Abby: Probably a few more sophomores than juniors.

Me: Advanced Algebra?

Abby: Those are almost all sophomores. Usually there's one or two freshmen out of 35 kids, who took a Geometry class elsewhere, and Honors Geometry elsewhere as eighth graders, Algebra I as a seventh grader. And, then there usually are three or four juniors who for whatever reason took Algebra I as a freshman, Geometry as a sophomore and were ready for Advanced Algebra as a junior.

Me: So, your Advanced Algebra students, same questions I’m going to ask you. How are calculators used in your Advanced Algebra classes and then include in that how you think the calculator usage is affecting their ability to problem solve and think rationally.

Abby: The Advanced Algebra, is all, those students are already well focused on how do I apply this, they are really good at wanting to know how to apply it. They are usually concurrently enrolled in a Chemistry class, or sometimes a Physics class. So, they are thinking applications almost all the time. Some of them will still moan when we get to a good strong section of word problems, but that's okay, they still like them, … or they hate them, but they know. They know, that’s what they’re doing. So, they've already got a good eye towards how am I applying this. So, they are more versed, more practiced, and more inclined to check an answer for whether it's reasonable. They are also less likely to reach for the calculator and start pushing buttons to say, yeah, I can find the answer here somewhere. That happens more with Algebra I in solving some kind of either word problem or solving an equation. They just keep keying in operations until they say, I'm sure that's the answer, that looks good. So, they are less inclined to misuse the calculator to start with and when they're working with word problems they are potentially more likely to go down a path where they have tried a couple of things on the calculator and they've mentally committed to this is my process and end up in the wrong place. But usually it's because they did not understand, say how to set up a work rate problem to begin with, something like that, or a mixture problem.

Me: Do they then recognize that their answer is not reasonable?

Abby: Well, if it's some specific type of problem set up from a word problem, like work rate or mixture problem, if they didn't know how to start, even if they recognize it's not reasonable, usually they've lost where to go with it. So, I think the calculator there is neither a help nor a hindrance if they didn't start with, I mean it's such a, those two types of problems, there is just such a such a method to starting them, and if they haven't learned that method, then they're gone. I mean it's not such a major application problem, it's do you know how to do it or not. So, I don't think it hurts them on that part. I think that with that advanced group, I think that the calculator just becomes another set of skills that they need, and it is a key set of skills though. They've got to know, when am I going to enter data in List One, List Two, and go for quadratic regression. Is this the type of question that's asking me for this equation in that kind of manner? Is this the type of question that's asking me, giving me three or four points Abby sketches Figure 4) and saying specifically, find a quadratic function to fit this, and knowing where to go on the calculator. So, now you're teaching the calculator as a method, as the technological method, to find their path. And, I think they do check to see if it's reasonable and a lot of this, that gets through to them, is saying when you're doing SATs, you should make a little sketch and plot those three points that you're given, so that you can have an expectation. If they are telling you it's quadratic, you can have an expectation, it's going to have to open up, so you know your lead coefficient is a positive number and it's going to have to have some sort of, you know, get your expectations. I think they still develop, I try teach them to still develop an expectation even when they're going to the calculator, that they need to know the process on the calculator, the steps, but also do a little quick hand sketch so you can build some expectations there before

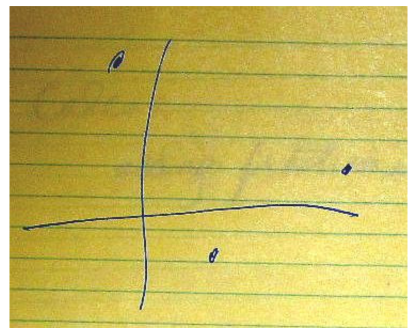


Figure 4

you even start. The one time that I think that kids need all along, and this is really from fifth grade on, that they need to not use a calculator so that they can develop some number sense, is with multiplying and recognizing factors of a number, because when they get to eighth grade with factoring quadratics to a binomial times a binomial, and we rarely give them ones that are prime, I mean, if they are, then they are simple ones. But, to do that factoring, even if they have a calculator there, they have to have some pretty good number sense, if they have, let me make sure I give a sample that works but not be too simple, whatever, (Abby draws Figure 5), if they have $(x+5)(x+8)$, if that's how it ends up factoring, if they've got the $x^2+13x+40$, even with the

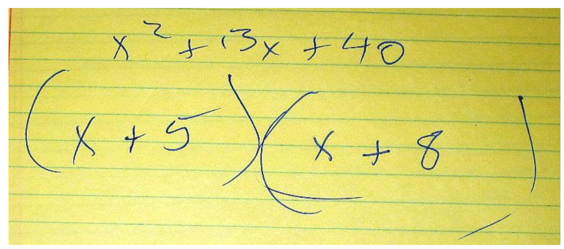


Figure 5

calculator in front of them, they've got to have some number sense, how can I multiply to get 40 in a way that adds to get 13. If they don't have that number sense, even if they have a calculator, they won't have a good thought of where to get started. So, that's something I don't know that third, fourth fifth, sixth grade teachers build enough. I don't know if they realize the importance of how, yes, anytime you can reach out for a calculator and multiply 5 times 8 and get to 40. But, they don't necessarily unpack that well enough to know, the teachers, to know that the kids have got to have that number sense of what are the multiple different factors of 40. They do it as an exercise to teach and then move on. But, the kids who have calculator accommodations can be missing out by not building that number sense well enough and that affects them for so many years with Algebra I ,Algebra II, and I don't know how much trouble that gives them when they get to the Pre-Calculus. Anyway, I don't know much trouble that gives them later. I think that the kids who do not develop that, … that feel for those numbers, for those factors, yes, we can go through, and we definitely teach them how to do, we call it T-diagrams, (Abby draws Figure 6) where they list all the factors of, say 40, so, 1 times 40 is 40, 2 times 20, 4 times 10, 5 times 8, and they go through an organized list and, yes, you could process that as if you were computer, saying this is how you do it, but if they don't build that intuition then they are always struggling with the process, rather than having built any intuition for it.

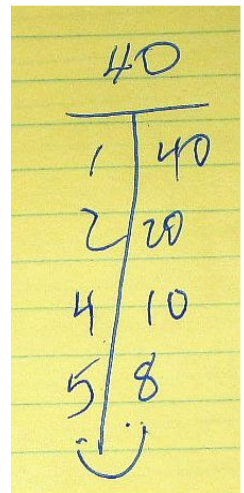


Figure 6

Me: Are you drawing a little smiley face at the bottom?

Abby: You do. Because if you make an organized list, you know, by the time you're done you can go around the corner, and do 8 times 5, but it’s going to be the same thing. I don't know that they internalize that smiley face. (Laughter)

Me: Algebra III, tell me about calculator usage in your Algebra III classes.

Abby: With Algebra III, so much of that is the rational functions and the conic sections and then working with the sequences and series, and if we have time, we include working some with logarithms, if that gets in there. So, for the rational functions, again without a calculator, they need to look at a function and have some expectations of what it's going to look like. They need to have an expectation of whether it's going to be linear, quadratic or whether it's going to be a rational function with some breaks, with some asymptotes, they have to have some expectations without a calculator to start. The second part of that is when they are factoring rational expressions, again that goes back to the Algebra I, they still have to be able to factor quadratics, to be able to simplify and then graph. So, again without a calculator, they've got to be able to factor.

Me: Can they factor?

Abby: Um, every, well, … they need to be re-taught factoring year after year, unless they're in the top group of students. And, I think that Algebra II teachers historically, for the, I don't know, for the decade that I've taught either Algebra I or Algebra II, you know, some teachers along the line say, if they would only come to me knowing how to do fractions. Well, Algebra II teachers say, I can't believe they can't factor. But, when look developmentally how kids learn, they are reintroduced year after year to order of operations from third grade on, at increasing levels of complexity. So, it's not a surprise that Algebra II should have to re-teach or teach again, how to factor. I don't think that's wrong, I don't think that's a flaw from the students' standpoint or from a curriculum standpoint. Some Algebra II teachers are so often stunned it seems, I can't believe they don't know how to factor. Well, it isn't unbelievable, they need to be re-taught that, and they're taught it, they might not realize it if they don't teach Algebra I, they might not realize that you are teaching it with harder problems. You know, you reintroduce it, remind them what they already know with some simpler problems, you know, with all positives or a little bit of negatives, whatever, and then they get the harder problems. So, I don't think that's a surprise to have to re-teach factoring in Algebra II or again in Algebra III, a reminder. And, I don't think it's anything, you certainly don't want to tell the kids, I don't believe you don't know this. And that it happens, it does, it's sad, it happens. But, we don't say that to fifth-grade students when you have to remind them about order of operations that they first learned in fourth grade. You expect that you have to work with fractions over a couple of different years. You expect that you have to work with order of operations, or even place value, from second grade on. So, we should expect to have to teach again, and I won't say re-teach, teach again and build factoring skills.

Me: How do the kids react, just as an aside, if they are told something like?

Abby: They develop a concept early on, I'm in the low math group. And, either they get over it by some point, usually by the time they are juniors if they are in Algebra II, they are working hard at that point. And, they are realizing, I'm not taking any math in college unless I have to, I'm almost, I can see the light and at the end of the tunnel. I have this much to do to get out of here. And, they work at it generally. I find the juniors who are not slackers even in a lower math class, and, in fact, a lot of times that's when it kicks in, that their math work ethic or their math basic skills kick in better. It is horrible, it is horrible, when they hear that and I'm telling you they hear it. The teacher's hand needs to be slapped when that happens. And, it slips out. And, it is such a subtle difference to some teachers to catch on that you can say, oh, you know this, let's look at again, instead of saying, you guys ought to know this, how come you didn't learn it yet. And just that little bit, that little, it's almost a slip of the tongue, but it's not, it's important and it hurts some of the kids. It does.

Me: For some, that will roll right off their backs and some will remember it for the rest of their lives.

Abby: Yes, I know.

Me: So, back to calculator usage in your Algebra III classes.

Abby: Ok, with that group, that's when it becomes important to be able to graph a rational function and interpret the graph that they got, even if they're not taking a rational function and being able to identify asymptotes, horizontal and vertical, even if they're not at that stage, but being able to take a rational function and appropriately key it into the calculator with appropriate parentheses, and then being able to interpret the graph for identifying where the vertical asymptotes and horizontal asymptotes are. So, the calculator is integrated into the skills that they need to be able to use. They also could do this without a calculator if they were given a graph already on paper to be able to identify the horizontal and vertical asymptotes. And, an additional layer is if they are just given the function, they might have to simplify it themselves to be able to graph it. Without a calculator you're going to have to graph a certain point and realize, … and have some of concept about where a vertical asymptote might be, so to select some points that they can use to be able to graph by hand or get a sketch and have some expectation of what the graph might end up looking like. So, there is a start where they have to do some pieces of it without a calculator and some pieces with a calculator and being able to interpret the graph that they see on the calculator.

Me: And the effect of the calculator on their thinking?

Abby: I think it gives an additional level of richness to their study. I sure didn't have in high school or college. (Laughter) I got to go use the computer some, but anyway, to graph, you know, but it was basically a computer program to simulate what the TI-83 does now. But, I think it adds an additional level of richness to their study, an additional level of reality to their study, because if they do go on in college, they will be using either programs for, you know, more complex things, if they become meteorologists or something, using more complex computer programs that will give them some graph to analyze or if they're just in their Biology, Chemistry, Physics classes, they need to be able to use that calculator. It's a math skill to be able to use the TI calculator and evaluate what they get.

Me: You've already covered this somewhat, but I'm going to ask it directly anyway. Do you see any connections between your Algebra students' at different levels, and your answers for your four different levels may well be different, do you see any connections between your Algebra students' ability to reason and solve problems and their use of a calculator? Do you think there is any possible dependence?

Abby: I think at the Algebra I level, there is a dependence on either a four function or a graphing calculator. At the Algebra I level, there is a little over dependence on the calculator because their number sense is not as strong as maybe it should be. For example, with factoring, with understanding factors of a number, to be able to factor a trinomial into a binomial times a binomial. So, in Algebra I, I think we could be doing a better job from, say, fourth grade on pulling the calculators out of their hands a little more often. Not always, but a little more often. I don't think it needs to be "no calculators in fifth or sixth grade," I don't think that's appropriate either, because there are some kids who have disabilities that they're just not going to get very far if they don't have a calculator. They are going to be stymied at very simple problems, so for Algebra I, I think that we could build some, … build some skills more that they would use a calculator less. But, for Algebra II, I think the calculator opens up their capabilities to be training them better for what they need for college, to be training them to get better for what they need it for, if they go on any professional career that would need more technology in it.

Me: Now, when you had mentioned, I want to go back to when you were talking about the elementary school, and you said that you do not advocate taking calculators out carte blanche, but they have to be integrated sometimes because some of the kids who have accommodations need them. What about the other kids? Do the other kids need them sometimes, too?

Abby: I think that the calculator can let you work with larger problems so it doesn't always look like little baby problems with integers from negative 20 to positive 20. And, it helps them work with problems that they do have to check for reasonableness a little better, so, by using a calculator and giving larger problems, that's where your estimation comes in to say, when I check for reasonableness, if I estimate, you know, say your answer is 7392, if you plug in 7000 to the original equation with some estimating, you would say, oh yeah that's a reasonable answer. So, I think the calculator allows them to get to harder problems that build number sense in a different way, whether, to test whether and answer is reasonable.

Me: Ok, let’s talk about Advanced Algebra.

Abby: In Algebra II and Advanced Algebra, the same thing and the Algebra III as a follow-on to Algebra II, that it lets them get to more real-world applications and more real world types of word problems so that you don't have some interest problem that always uses simple nice, round numbers, or some exponential or half-life kind of problem because they're going to be using it in their science classes. We need to have it 360, all around, that they're using some good numbers.

Me: In Algebra III, I should have asked you this before, but who is in your Algebra III classes? Juniors? Rising seniors?

Abby: They are rising seniors, in the summer class, just that one. Rising seniors, some rising juniors and, as a category, it’s kids who either bloomed a little later and were not ready for Advanced Algebra but there is also, the reason I started the summer class is, I feel like that as a school, we under-place new students and they, … there are new students who probably should have been placed Advanced Algebra who were in Algebra II instead. And, I hate the thought that they would miss Calculus or miss even Pre-Calculus based on “we weren't quite sure (where to place them).”

Me: Now, from Algebra III they go to Pre-Calculus?

Abby: Yes.

Me: Nobody goes from Algebra III to Honors Pre-Calculus?

Abby: We've had a couple. We had, we had three out of the five students last summer went on to Honors Pre-Calculus and two of them did well and one was a, gosh, is it judgmental, a slacker.

Me: Did that person go down to PreCalculus? Or, did s/he stay in Honors Pre-Calculus?

Abby: I don't know. I don’t remember where they ended up. But, I think they went down to regular Pre-Calculus. But, it was a kind of thing where they had the focus in the summer when it was one class, and when it got to be the school year with every student in the building and so much action going on.

Me: So, it wasn't a question of a child's ability?

Abby: Right, it was not. I hate to use that word but … a slacker.

Me: Let me ask you what you mean by slacker.

Abby: (Laughter)

Me: I assume you mean by slacker somebody who has the ability but doesn't use it.

Abby: Right. And, it really was a case of being wrapped up in so many other things and prioritized inappropriately.

Me: As a high school student will frequently do.

Abby: Sure.

Me: Ok, so, anything else on the connections between your Algebra students' ability to problem solve, set up a problem, and reason and the use of calculators?

Abby: I think with Algebra I, the kids who have good math ability sometimes will see a word problem and the problem will be, you have a huge range of ability, and the problems will be a little simple for them, and they won't want to show their steps or their work and so they will jump to an answer and if the calculator is there, it makes it easier to jump to an answer. But that's good for them to go get a wrong answer and then maybe they'll learn, don't jump to an answer by reaching for the calculator, it is worthwhile to show your steps. But, then that's on us to make sure we follow up with harder problems where they have to show some intermediate steps.

Me: Have you ever had any experiences with your students, where you said, either aloud or to yourself, that you couldn't believe they were using a calculator for that?

Abby: Oh, definitely. And, often I will say to them, try that without a calculator. I bet you can do it.

Me: What types of problems are those?

Abby: Usually they are types of problems which will need a calculator later or we will be doing a lot of problems that have a calculator basically required. And, they'll be in the habit of going to the calculator first. Does that makes sense?

Me: Yes, it makes sense. Let me ask my question a different way. Are they using the calculator to find 10% of something, are they adding five and two with the calculator? What types of things are they doing that make you say, either to yourself or to them, you should be able to do that without a calculator.

Abby: Usually in Algebra II or Advanced Algebra, it's squaring a number, something that's from 1 to 15 squared, and that troubles me a bit and that's when I will comment. Because they ought to have that at their fingertips better.

Me: My last question: for what Algebra II topics is the calculator most helpful to your students? Why? Well, actually, that's several questions.

Abby: I think really making that link between what they've been doing with functions and making that strong link to what that particular functions looks like and what their expectations are. I think that's the biggest topic where having a calculator is important. I think also what Algebra I is working with systems of linear equations, they need to know the multiple ways for finding a solution, graphing by hand with the simple ones when they first are working with that, that's important that I wouldn’t want them to use a calculator. But then building on that concept later when they go to substitution and elimination, not needing a calculator necessarily, but then coming back to graphing more complex problems that they couldn't graph by hand, you know, with larger numbers or something and being able to go to 2nd CALC and finding the intersection. So, I think that just opens doors for them to think bigger. So, in Algebra I, I think that’s one example. Also, the calculator is a tool for Algebra I to quickly build some concepts of what a linear equation is going to look like. Is it going to a positive or negative slope? Where is the *y*-intercept going to be, you know, have some mental concept of where the *y*-intercept would be. So, I think, let them use it as a tool to build some sense of what their linear equations are going to look like, and, again when they get the quadratics, if they graph a whole bunch in a row after they worked by hand, it helps reinforce and build the concept of, yeah, I still see that *y*-intercept but I really care about the roots now. Did I expect one root, two roots, imaginary roots? So, I think it helps them build some number sense in that way with functions.

Me: What impact or effects does the calculator have on your students’ mathematical reasoning and do you have any particular stories you want to relate?

Abby: I think with the Advanced Algebra, the vertical and horizontal asymptotes are an hard honest-to-goodness brand-new topic to them and I think the calculator really helps there for them to build what is a totally new skill for them. They still have to be able to do all the work algebraically, but I think that helps reaffirm for themselves what they've done as they're doing homework because, you know, on a test if they would have a section where they don't get to use a calculator and they would have to identify asymptotes there. So, I think that it helps, you know, build a bigger picture for them of their mathematical understanding when it comes to that. Also, we hadn't mentioned anything about working with matrices and that’s something again they have to be able to do by hand, but I think that being able to do it on the calculator is important also. And it just lets them then go to word problems where they do so much of the concept building in the algebraic part of it and then set up their matrices and then go to a calculator maybe to solve.

Me: Ok, so are you saying that some students can develop understanding by using smaller matrices themselves, and then once they understand the concept, then they could use a calculator to reaffirm?

Abby: Well, with the matrices it's not so much reaffirming because I don't know that they have a good, intuitive feel for the answer. It lets them spend more time on setting up the system of equations from a word problem without, and setting up of more complex system of equations, that I would not want them to solve very often, … that I would not want them to have solve by hand algebraically. So, it lets us focus on the word problem part of it and not quit there, but then also go solve it. And, again, I think it lets them get to more real-world applications or one step closer to real-world applications. Let's them get one step closer there.

Me: Any detriments? Do you ever see anybody using it as a crutch?

Abby: I see that mostly with the eighth-graders or Algebra I students. And, that's, it's kind of sad, because they say, I will always have one available, but they just, the main problem there comes to factoring a trinomial to a binomial times a binomial. That's the hardest part I see. And then also just earlier on in Algebra I, combining like terms or solving equations where they mess up their positive and negative integers, and, yes they can always go to a calculator if you are looking for mad minutes where it just say, okay add these 30 problems, you know, 5 plus negative 8, if they have a calculator, of course they can do it. But, then when you get to solving equations where they are combining like terms, they mess it up, because they may not have built that concept as well as they should have.

Me: What is a mad minute?

Abby: Little second graders have to solve, and there is no reason to say they should solve 30 addition problems in one minute, you know, they have easier ones and harder ones, but it is, it sets a pace for them to say, how many can you get done in a minute, to see if you have internalized them well. And, then for third-graders, it's multiplication facts or division, simplifying fractions, and mad minutes can be really cruel because kids' speed is not always the best indicator, but you do have to develop some facility with doing this. So, it can be a real cruel thing for elementary students. But if you manage them well, it can help them develop their skills better. I think more, … more elementary grade levels are going to computer games instead. There's less a lot less negative feelings about not getting all 30 problems done in a minute.

Me: That's a lot in a minute.

Abby: It is.

Me: That's two seconds each.

Abby: Well, you know, if its 2 plus 3, 7 plus 0, …

Me: Yes, but it takes at least a second to write down each one.

Abby: Yes, I know.

Me: Anything else you want to tell me about?

Abby: I think we should have the kids have a roundtable discussion on this.

Me: Is there anything else which is important to you, that's on your mind?

Abby: This helps me realize that I need to repeatedly hit those Algebra I's with why we're doing things with the calculator and without. And, I need to build that for them a little bit, because I tell them, you know, why you need to show your steps when you're solving problems, but I think that would be a worthwhile thing. I wonder if I don't think about mathematical reasoning enough or maybe I just don't think in that phrase. That’s the only question that got me, because I don’t use that phrase very often. I think the whole math department should talk about this during faculty work week in August. I mean, … I am really thinking about my classes and why I am doing the things I do there. Interesting … going to think about this some more.

Appendix C

Interview with Beth on Wednesday, 30 June 2010

Time: 42 minutes

Me: Let's talk about the mathematical reasoning which you see your Algebra students employ. Tell me how your Algebra students mathematically reason; and, if it's different for the different levels of Algebra, then make that distinction.

Beth: (Looks uncertain)

Me: And take your time there is no rush.

Beth: I'm not sure specifically what you're referring to by mathematical reasoning if you're talking about an equation being imbalanced and so when you're manipulating an equation, you know?

Me: What I mean is, how do you see them reasoning? Do they know why they are doing things or do they just throw numbers around, not knowing what they're doing, or are they thinking logically, is what I'm getting at.

Beth: Okay, my observations have been that some of the students think logically and some of the students want to employ a rote procedure, which I do not promote because that's easy to forget and that's easy to confuse. Whereas if they understand the structure and the logic behind what they're doing and that will stick with them and there will be fewer mistakes.

Me: Do they estimate?

Beth: Well, a lot of that depends upon the intelligence of the student. Your lower-level students can get negative answers for measurement and it doesn't send off bells, whereas the students that don't struggle as much with mathematics, that are more logical thinkers and are generally placed in higher level mathematics classes, that does send off a bell that something is wrong with their answer because they've gotten a negative number instead of a positive number.

Me: Now when you say lower-level students, I want to be clear that I understand what you mean, even though we know each other well, I don't want to assume, I want Beth according to Beth, not Beth according to Mimi.

Beth: Right (Nods head)

Me: So, when you say lower-level students do you mean the different math classes or within a class? Do you mean strata within every class? Which one are you referring to? Or are you referring to both?

Beth: Well, it could refer to both but I'm talking about the students who are not taking the regular, non-advanced path. I'm talking about the students that are lower than that. I mean the kids on the slow track. But, the students who are in the regular classes, some of them will make that mistake; they are not thinking logically and they are not evaluating, "is this a reasonable answer" and, generally, the students that are in the Honors classes and the students that are at the top of the regular track will look at their answer to see, … to see if it's reasonable. And, that's something also that you have to train them to do. But, for some of the students it just never clicks.

Me: So, are you saying that even if you train them, some of them still won't use it, won't check for reasonableness or plausibility?

Beth: Because if they're getting through the math by doing rote procedures then whether or not it's a reasonable answer, they do not have a context for that.

Me: The next part of this question is: do they try multiple approaches? And, when I say "they," I think that you are going to answer me in a couple of different ways depending on the kids.

Beth: Yes, your Honors students build, try multiple approaches, they'll try something and if it doesn't work, they'll go another way and then another. But, that level of thinking diminishes with the intellectual ability of the student.

Me: What are your views on using calculators in Algebra classes?

Beth: I think that they're good. I think that the class needs to be a combination of doing things by hand; and, I think that that learning needs to be initially by hand without the calculator. And, I see it as the difference between learning the structure or the discipline of the mathematics versus learning the applications of mathematics, and there is crossover between the two. The calculator is great when you're doing applications; but, if you don't understand the structure and the discipline, again what comes in is “is this a reasonable answer?” So, I think they both have a place in the classroom and that they are both necessary.

Me: Are there any times when you will not allow calculators?

Beth: Yes

Me: What would those be?

Beth: I want to know that the students understand how to do something and the reasoning and the logic behind the concept that they're working on, as opposed to, well, an example would be changing degrees to radians or vice versa. Well, you can push a button on the calculator and have it do that for you. But, if you do it by hand you need to understand the relationship between them. So, very often my tests will be no-calculator or it will be divided into sections. When I had longer class periods, generally every test had a calculator and a non-calculator section. Here at our school, because of the shorter class periods, I found that trading the sections back and took up too much time, so a lot of my tests were non-calculator only.

Me: I just want to be really clear. Why do you make them non-calculator?

Beth: It's because I am not testing “can you do this procedure on the calculator,” I'm testing the knowledge of the concept or topic.

Me: Do you think that by not using calculators, I don't want to use the word forcing, that you are engendering the development of mathematical reasoning? I don’t want to put words in your mouth.

Beth: Oh, that's always my goal in teaching, because if it was all calculator based, I’ve seen when classes are heavily based on the calculator, the development of mathematical reasoning is weak at best. And, where do our mathematician's come from if they don't understand the logic? And, where do the people come from who are going to take this wonderful tool and improve it if they don't understand the mathematics behind it? So, while I think that the calculator has a place in the classroom, I don't think that you could do without it. And, even with having a calculator in the classroom, what I think, what I am saying is that the basic mathematical skills have diminished and that students cannot multiply 3×25 in their heads, and they don't even do it on paper, simple mathematics, you know 45 divided by 5 and they will reach for their calculator. (Frowns)

Me: Is this is at all levels?

Beth: This is even at the Honors level, which is why when they have a section on a test that is non-calculator, I do allow them to use four function calculators, because they can't do simple arithmetic anymore because of calculator usage.

Me: Are you saying think this is directly related to calculator usage in the earlier grades?

Beth: Sure, definitely.

Me: Please tell me how you think your students understand the solutions which their calculators give them? And, please give me examples.

Beth: Okay … (looking pensive)

Me: And you can give different examples for different classes or different students... it's not that there's only one answer here. Feel free to ramble.

Beth: I'm not much of a rambler (laughing), … when we are doing application problems, we've done them in class, we've correlated by hand and by calculator. So, the calculator, say it is an example of projectile motion with a quadratic, and so they know what shape they should be looking for on the graph, and if it's not going to be that shape then they know there's something wrong. And, when they are looking for the answer, they know that they could do it by TRACE on the calculator, but of course, that's not very accurate because of the limitations of the calculator itself. And, so they know the different ways to get an answer on the calculator and we talk about the domain and range and things like that, so, when they are solving problems on the calculator, they've got understanding because we've talked about that in class … um … Well again, that's a range of kids. But pretty much, because my classes are heavily on the non-graphing calculator side and then the calculator is a supplement. And, so when we are doing data analysis, we are going through and we are talking about the meaning behind what is showing up on their calculators and what we are looking at. For example, the correlation coefficient, we talk about what this represents and why we are looking for something which is close to negative 1 or positive 1, and if it's negative what that would represent. So, we're talking about it in class, what this represents, and now this year with data analysis, I did not have them do the line of best fit by hand, …, but, um, ….

Me: Tell me why you didn't do that.

Beth: Classes are very short here.

Me: So, there was no pedagogical reason, it was just time constraints, is that right?

Beth: Yes, that's it. Purely based on time available.

Me: So, you were talking about how the students understand the solutions at their calculator calculators give them.

Beth: Yeah, I think because there is a heavy emphasis on doing it by hand and on what the calculator is doing ….

Me: You had mentioned that your students would graph a parabola and if it is not facing the right way they'll know something's wrong.

Beth: Right, if the leading coefficient is negative then expect the parabola to be facing down.

Me: Do all kids get that, it's not just your kids this year, it's your experience with all your Algebra classes?

Beth: Right, yes, they pretty much do. The courses that I've been teaching with the Pre-Calculus have generally had a section on graphing and graphing your cubics and fourth powers and beyond, and we look at the families of graphs and what their shapes would be like and we do polynomial dancing, where we just have formulas on the board and they make their shapes with their arms, ... and ....

Me: You mean like this? (I make a U-shape with my arms) for a quadratic?

Beth: (Laughing) Yes, or that could be quartic or any even power. (Beth makes a Saturday-Night-Fever formation with her arms) This would be a linear. (Beth makes a Walk-Like-An-Egyptian formation with her arms) and this would be a cubic, or any odd power.

Me: (I try to form a V with my arms) Would this be absolute value?

Beth: I didn't throw in absolute value because that's too confusing with your parabolas, looks too similar. So, they are looking at whether or not the leading coefficient is positive or negative and are looking at what the highest degree is and they're looking at families of graphs and what the general shapes would they have, …and so I think they pretty well know what to expect the graph to look like when the calculator puts it up on the screen. And, so if they are getting a cubic shape, and they thought they were putting in an even power polynomial then they know, "oh something is wrong."

Me: Think about your Algebra I classes, when you didn't have the more advanced students. Let's say something similar in the same vein but simpler, like $y = -3x + 2$, something like that and the student graphs it and gets a positive slope, do those kids clue in that that could not possibly be right?

Beth: The Algebra I kids sometimes know. Usually that understanding doesn't come until the Algebra II level, so they've been introduced in Algebra I to linear and quadratic and much more, … more focused on the linear. Conceptual understanding of all the details of that seems to really take place at the Algebra II level.

Me: Does Algebra I graph by hand?

Beth: Yes. They were when I was teaching Algebra I last year. There was not a whole lot of use of the graphing calculator.

Me: And were you for that or against it? Do you think it's a good idea?

Beth: That was my choice.

Me: Oh, okay.

Beth: That was my choice.

Me: Okay, but I'd like to ask you to elaborate on why.

Beth: Because I think they really needed the basics to be strong before they went on to the "toy." (Laughter) These were kids who were struggling to get the basics and I felt like, if you if you give them the graphing calculator that makes it very easy, they were not going to go through the struggle. And, if and they are required to go on in mathematics, and so in order to not set them up for failure, I really wanted to focus on having strong basics which they could build on.

Me: Do you think that worked?

Beth: I think so. The teachers who were teaching them afterwards did not come back to me and say, "you need to teach them this or you need to teach them that," and recommend changes in what I was doing or how I was doing it.

Me: So, your reason for that was development of mathematical thinking?

Beth: Yes, most definitely.

Me: Again, I don't mean to put words in your mouth.

Beth: No, that's a good synopsis of my motivation … We used to use the calculator so that they were familiar with it, so that they do not get to Algebra II and they had no clue how to use it. But, we really focus a lot on doing things by hand.

Me: Do you see any connections between your Algebra students' ability to reason and solve problems and the use of the calculator?

Beth: I think the ability to reason and solve problems comes not from the calculator but from doing them in class. You can't just teach the structure of mathematics and expect the students with that structure, whether or not they have a calculator, to be able to apply that to a problem that is in context. You have to teach how to solve application problems, how to read the problem, how to pull out the information that they need from the problems, and so whether or not they have a calculator to us, I think it's not the key, I think it's <u>not</u> the key whether or not they can do the application process. Because there is a whole process in teaching them how to read the mathematics, how to teach them how to find the information they need. Teaching them how to approach setting up an equation, and until they get to that point, the calculator is useless. After they have the equation set up, then using a calculator is great, it lets you get to a lot more, it shortens the amount of time they spend on solving. So, you can practice application problems and be able to set them up in class, but using the calculator allows us to get through more practice problems. But again, knowing how to reach a reasonable answer and having cranked these things out by hand is an important prerequisite to know whether their calculator is giving them a reasonable answer.

Me: Do you think that calculators have had an effect on your Algebra students' mathematical reasoning?

Beth: They have had effects. The multiplication tables, the addition of subtraction skills have just gone down the tubes as far as learning the math and the thinking. I think that the influence of the calculator may be part of it. But, I think there is a large role societal influence plays in what is going on, in that everything has speeded up, students are on the Internet getting things instantly. They watch TV as opposed to reading books, they are not used to putting the time into something to come to a conclusion because in their world they get solutions very quickly, and their focus is shorter than it used to be. In the Honors Pre-calculus this year, students were complaining at an involved problem which was multi-step, and might take a whole page to solve the problem.

(We both laugh a one page solution being regarded as long)

Beth: I know, I know. The students were talking about how long each problem took to do and, there weren't all that many, so I thought it should have been interesting to them to do that long problem. The students did the problem, they did the length, but …., oh, ….

Me: They were not interested or they were?

Beth: I would say the majority of them did not get the satisfaction and mathematical pleasure out of doing a long problem that that students of 10 years ago got.

Me: Would you call that intrinsic gratification?

Beth: Yes, and it's because of the length, I mean because and I think that that's more of a reflection of what is going on in society.

Me: I want to go back to before when you were talking about when you don't allow calculators and I wanted to ask you how the students react to that.

Beth: A few will, more than a few, would say they wish they were allowed to use their graphing calculators more on tests. They like using their graphing calculators, and although their viewpoint is not entirely accurate, if you go back and add them up, their viewpoint was based on their emotional feeling that they didn't get to use their graphing calculator enough and so their comments would be, “we never get to use our graphing calculators.” Extreme, as teenagers will often be. (Laughter)

Me: Do you know, did they want to use the calculator because it's easy and fast or because they want to be confident in their answer or is it some combination of those or other factors?

Beth: Well, I think a lot of it is that they just liked using it. They’re using their cell phones, they’re using their computers, it's cool, it’s a gadget they like to use. And, interestingly, like you said about being confident about their answers, yes, they <u>trust</u> their calculators.

Me: To a fault?

Beth: You know, … I'm not willing to say, “to a fault,” but I think because I grew up without calculators, I still have the distrust of, not so much the calculator coming up with the right answer, but user input errors. And, so when I get an answer then I really examine it to make sure that I have input it correctly. Whereas the kids, I think, don't have that hesitation about the idea that maybe they made a mistake when they input things.

Me: How about, and this is a little bit of repetition, but how about situations in which the answer is not plausible?

Beth: Your Honors classes definitely pick up on that and the top of the regular classes will pick up on that, too. The other kids just aren't as intellectually strong.

Me: And, what portion of the classes would you say the top part is? Generally speaking, in your Algebra classes, the ones who don't get it that the distance from here to there cannot possibly be negative 25 yards.

Beth: The Algebra I classes is probably three quarters.

Me: Would not get it?

Beth: Right, would not get it. With Algebra II, probably a half for less because also I think part of that is conceptual and its conceptual development. And, so I think that by the time they get to the PreCalculus level, that development is there. Part of it also because we are pushing them to take Algebra as such it would young age, that the conceptual development that's necessary for some of these concepts has not occurred yet, even though we are giving them information So, that is part of the reason that at the PreCalculus level, I'll still see kids who, with equation they move *3x* from one side to the other and they truly just move it. And the concept of "this is an equation and it's in balance and you have to subtract *3x* from both sides to keep it in balance" is very hard to imprint at that level.

(Recorder shuts off and gives an audible signal. I see that the memory is full; so, I delete a few unneeded files and start the recorder again.)

Me: Hey were back. Sorry about that. You were saying that it was difficult to imprint the idea of balance in a equation in seventh and eight graders, is that right?

Beth: What I was saying is that I think that sometimes the students have learned that it's, it's hard, it's very hard, … no, … wait a minute, ok I know what I was saying, I was saying that it's very hard to remediate their understanding of how to handle an equation when they have learned how to handle equations before their conceptual development is there to understand the mathematical structure.

Me: This is the subtracting *3x* from both sides?

Beth: Right …

Me: For what Algebra topics do you think the calculator is the most helpful?

Beth: I think it is helpful for graphing, … I think it's helpful for data analysis …

Me: And when you say data analysis, do you mean List 1, List 2, let's do some regression?

Beth: Right, let's find the line of best fit.

Me: Okay thanks. I just want to make sure that I know what you mean. Because, we don't at this school, but some people say data analysis and they mean what we call PreCalculus.

Beth: Sure. I know what you mean. … Yeah, I have noticed that, too, and some people mean statistics, don't they? … And, yeah not statistics, I don't mean statistics but finding the line of best fit, whether or not it's truly a linear or if it's quadratic. And, I have taught data analysis in Algebra I and Algebra II.

Me: Why do you think is good, because it relieves you of the tedium of graphing by hand?

Beth: No, because, … yes, it relieves you of the tedium of graphing. But, physically and we've already learned that process and how to do that. But, now we can do more examples and more applications because the calculator can do the actual graphing very quickly and it's easier also to go back and forth to compare graphs. So, it's a timesaver, that's the big reason.

Me: How do you think that affects their mathematical reasoning?

Beth: Well … I think that … it can help their mathematical reasoning if the teacher is using it to develop that. If the teacher is using it as just a process, here's what to do, it's not going to help at all. But, if it's incorporated into an understanding of the topic, then, yes, because you can do more of them. And, they have a basic understanding of the graphing, which by hand is very important to do so that they get the idea of ordered pairs being a solution, multiple ordered pairs being solutions, and *x* is deciding what *y* is going to be according to the equation, the value of *x.* So, if you have gone through that, and they have that "Ah ha" moment that, "Oh, the parabola is curving because *x* is being squared." So, when I put in a negative for *x* , it's making it positive. And that "Ah ha" moment, I haven't found the kids to get that except by hand. I haven't found them to get that from the calculator because the calculator is a little magical; so, … but I think, … I think it can help the reasoning if the teacher uses it to do that.

Me: What impact, do you think, does the calculator have on the students learning and thinking and reasoning? And, give me some examples, please?

Beth: .....(thinking) I think it's had a positive and a negative impact ... The negative impact again is on basic skills … adding, subtracting, multiplying and dividing, … square roots ... and ... a positive impact that it's had is that you can use it to help develop understanding if you're looking at a problem and you use the calculator. You try to get the kids to come up with a path to take, and you say, "Well, let's try that on the calculator and see what we get and see if that makes sense, … uh … no, ok, let's try something else." So, it can help understanding that way because with their short ... short focus times ... short tolerance for things taking a page to work out, that trying different methods by hand they might give up, whereas having a calculator to try the different things is generating their problem-solving thinking skills.

Me: Is there anything else you think I should have asked you or anything else pertinent to our topics that you wanted to add?

Beth: … No … (Laughter) I can't think of anything right now.

Me: Did you find having the interview questions in front of you helpful, sort of a stress reliever or was it a distraction?

Beth: I didn't really read it. So, I didn't really pay any attention to it.

Me: If you could tell me one thing, "Next time you do this interview, you should … do this differently." How can I improve my interviewing?

Beth: Maybe if I had read the questions in advance, I would've given better answers, you would have gotten more concise answers. That's all, I just didn't give you a very concise interview.

When the recording was over, Beth remarked about how my questions had started her thinking about her own teaching in more depth than she has before. She apologized (unnecessarily, I assured her) for the long pauses she took before answering. She said that she never really put some of her thoughts into words and that doing so was making her think about her teaching practices and beliefs. "It's been really good for me to talk about this," she said. She still bemoaned not giving me what she viewed as "concise" answers.

Appendix D

Interview with Connor on Friday, 09 July 2010

Time: 1 hour, 17 minutes

Me: Let’s talk about the mathematical reasoning which you see your Algebra students employ. Tell me how your Algebra students mathematically reason; and, if it's different for the Algebra I and for Algebra I Part 2, then make that distinction.

Connor: There is a definite difference between the two groups when it comes to the reasoning. You know, what I see my Algebra I students are much braver, in ways, and they are much more willing to look at a problem and assess what they've learned before and hope and apply it. And, then that versus my Algebra I Part 2 students who rarely have the ability or the desire to look backwards in their study, my, you know, which may be a function of them being much less comfortable with the material, which is why they are in Algebra I Part 2 in the first place. And, my regular Algebra I students are by definition more advanced students and so have a much more of an understanding of where they came from and that's really an important part of how I design my classes. I want to show how math is evolutionary and how we evolve ideas, on how we develop ideas. And, so I really push that from day one. Okay, so we know this idea, what can we then derive; or, if we did this yesterday and I put this up on the board and I ask you tell me the range, they should hopefully look and say, "Oh, God, we talked about range yesterday, and there were *y*'s, and there is a "*y =*" up there, and maybe we can figure this out. So, I see this much more with my regular students, much more than I see it with my Algebra I Part 2 students.

Me: Could you please tell me the distinction between the two classes, Algebra I and Algebra I Part 2

Connor: Okay, Algebra I Part 2 is a is the second year of a two-year class for Algebra I. Again, with the understanding of the 12 chapters, Algebra I Part 2 will spend their eighth grade year covering chapters 1 through 6, and their freshman year covering chapters 7 through 12. And, the idea is, we give them more repetition; we can go at half the speed that we normally do, we give them a better understanding, with the expectation that they will finish at the same point as the Algebra I class. Under ideal circumstances, they will finish at the same point that my Algebra I classes, which do all 12 chapters in one year. That's the big difference between the two levels; and, so you'll see … but again, we were talking also talking about a spectrum, my Geometry students are the top 65% of the students, then you've got the next 25% are Algebra I and then bottom 10% are Algebra I Part 2s in the spectrum of freshman math classes.

Me: Is that generally always the case?

Connor: Yes that's always the case. That's how we differentiate the levels. So, coming in here freshman year, we really have four choices of classes, we differentiate the freshman classes that way in four levels, Honors Geometry, regular Geometry Algebra I in Algebra I Part 2.

Me: Now to take the Honors Geometry of Geometry, they have to already had Algebra I, correct?

Connor: Yes, the full Algebra I, all 12 chapters.

Me: So, let's get back to the mathematical reasoning which you see at those two levels of Algebra I. Please describe the mathematical reasoning that you see in your Algebra I Part 2 class. What mathematical reasoning do you see going on?

Connor: Very little … I'm serious, I see very little reasoning going on in that class, especially at the beginning of the year.

Me: And you are evidence of that is what?

Connor: It breaks down to they will multiply when they are looking at a division symbol; and, they are not good with working with negatives and fractions. They are very, very weak students but especially from what I understand, I have not been able to compare year on year yet (he just completed his first year of teaching at this particular school), but from what I have been told by our colleagues, this is the weakest group she (referring to Abby) has ever seen in her years of teaching Algebra I, because she had them as eighth-graders one year ago and two years ago. But, that group was very weak, better students in that group did at the end of the year, I think, show some real understanding of the material and were able to reason their way through things and were able to figure out what I was asking, look at the keywords in the question or in the directions and be able to identify the procedure to use. But, the weaker students in that group needed example problems in front of them to be able to do the work. So, for example, I mean, if I ask them, given the function $f(x) = 3x + 2$, give me the range over a domain of 1 to 4, for my best students, that's all they need to know to be able to tell me the range; my weakest students need to see an example of that for them to be able to do the problem.

Me: Now let me ask you two questions from where you are leading me. Your weaker students, when they have an example in front of them, do they understand the example or are they just plugging numbers and in the same fashion as the example?

Connor: They are plugging, … plugging.

Me: So, if they got a wrong answer, an unreasonable answer, would they know?

Connor: No, they would not pick up on that.

Me: You say that the stronger pull students in Part 2 will finally get it, and will know a procedure, do they have an understanding of what they are doing or do they have the procedure memorized?

Connor: It much more procedure memorization; so, they are a step up from the weaker students, … but, it's not a big step.

Me: Is the difference that they don't need to have an example of the procedure in front of them, they know the procedure?

Connor: Yes, that's right.

Me: Do they comprehend what they are doing?

Connor: No, I wouldn't say across-the-board. There are times they will look up there (gestures toward the whiteboard) and they will write a negative 3 on their papers instead of a positive 3, you know, if you were doing distance on a number line, because they just don't understand symbolism.   I can explain to them that we don't have negative distances but they don't really think of that little sign as being a negative; they think of it as being a minus sign.

Me: So, if they were working a problem and they got an answer that the distance between here to the barn is negative 25 yards, they would or they would not recognize that that is incorrect?

Connor: They would not recognize it, not at that level.

Me: Do your Part 2 student ever estimate an answer?

Connor: No, that is not in their vocabulary, that is not in their ability.

Me: How about checking for plausibility, do they do that?

Connor: Very rarely, very rarely to never.

Me: The stronger Part 2 students, too?

Connor: No, they don't do it either. When we are factoring trinomials, they just kind of try to put things up on the board, and the better ones know the procedure to do, but they don't understand the reason behind a procedure. They don’t understand that reasoning; they don't understand why we checked a back number why would check *c* and then we check *a* (in a trinomial of form $ax^2 + bx + c$). So that's, you know, they understand using a procedure.

Me: So, for the Algebra I Part 2 students, are you saying that they will perform the procedures correctly but are they doing it rote or do they understand why they are doing what they are doing?

Connor: They are doing it rote. They are not understanding the underlying principles behind the procedure, they memorize the procedure but they don't understand why the procedure exists.

Me: For your Part 2 students, if something doesn’t work, do they try another approach?

Connor: Occasionally.

Me: When would they do that?

Connor: When they are most confident about themselves. To explain, the Algebra I Part 2 students have been told, have been tracked, have seen that they are the lowest students in our school when it comes to mathematics. My own frustration with them tells them that, to be honest with you, at times. But I'd say they do not have the tool bag to identify that this idea is related to that idea. I'm trying to, … this of a good example, okay, the ideas of domain and range ties in with graphing, that does not really exist to them. Those are two different fields in their minds, whereas there's a huge correlation between them in our minds.

Me: Do you know if they ever get that in their Algebra careers?

Connor: I don't know. I haven't seen that develop.

Me: Let's go back to your Algebra I class now and the mathematical reasoning that they employ. First, let me ask you if you tier your Algebra I classes, are they low, medium, high, or are they all about the same level?

Connor: There are few that are high, but they're generally all about the same level, not differentiated like I see my Algebra I part 2 students. The Algebra I classes are much more homogenous. The difference in their understanding, in that group, is the speed or how quickly they understand the reasoning, they understand the process, they understand the reasons behind the processes. My best students will get it on my first example; my average students will get it maybe at the end of homework; my weakest students will have to come in and ask for help. So, that' the difference, but when I teach it, I still teach the same to everybody with the expectation that some will understand it immediately, some will understand it … in a week.

Me: What percentage of your Algebra I class is in each one of those three tiers?

Connor: Roughly, I would say 30% are in the top tier, 60% are in the middle tier, and 10%, I would say, are in the lowest tier. And, it stays consistent throughout the year and like most classes you can tell during the first two weeks in September, who are going to be your stars and who are to be your anchors, who is going to talk too much and who is never going to talk ever. And, occasionally they change because they are freshman and they haven't figured out who they are in high school yet; their personalities have not been set, by the end of their freshman year they usually have been set.

Me: So, I was interested in what you said when you started talking about the Algebra I, you said when they are understanding the process and that's a phrase you did not use with your Part 2 students. So, let me ask you this, do your Algebra I students actually understand the processes?

Connor: On my best lessons, yes. I think goes back to me as a teacher, me showing that example, me showing that relationship. They do understand that process. And, I like to do lots of linking and mapping. I will put *x* and *y* up on the board and then we will put every word or symbol that relates to them underneath. And, I teach them that they are interchangeable. That *y* can be *f(x)*, that *y* can be range, *y* can be my dependent axis, and *y* is my vertical axis, that all of these words, in my mind, are interchangeable and because of the way I think, I will be writing y equals something, but then ask the students, "what is your *f(x)*? And, the Algebra I classes understand that those are related concepts, and understand that we can interchange those without changing the meaning. There are different ways, just as Eskimos supposedly have 27 ways to say *snow*, we have the same in mathematics, we have many ways of describing your dependent, your *f(x)*. So, that's your input-output, you could say.

Me: And, do you ever see that with your Part 2 classes?

Connor: No,….never.

Me: Do I understand then that your Algebra I students do not rotely apply procedures and that they actually understand what they are doing?

Connor: At times. I wouldn't say 100% of the time, but my best students will start deriving understanding. The best example is factoring. They have to see factoring, but I will eventually start putting up all sorts of weird factors, things where you have to factor out a negative 4, out of a trinomial, and it's pretty hard to factor out a negative 4. And, when they can do that, that to me, is when I know they understand the material. And, then I can start pulling together this concept and that concept and put those into one question and the students can do it. And, I see that usually by the end of most chapters. And, that is usually my A buster, on tests, the difference between getting an A or getting an A- or a B+ on my tests, is the kids who get A's on my tests can answer at least part of those problems and my, … whereas my A-/B+ kids can answer almost everything but then cannot be creative. And, I talk about being creative as a synonym for understanding the process, for understanding the reasoning behind them, because you have to be creative and take what you know and apply it in different ways.

Me: Algebra I students, do they estimate?

Connor: Yes, I see them most estimating what we are looking at word problems. And, sometimes their estimations are off the wall; but they at least actually estimate, they will at least actually look at it. Let's say there was a question about pizzas, they will always look at it and say, "500 pizzas." The answer may have been 300 pizzas, but at least they are giving me a number, at least they are thinking, "okay it might be something close to this." I see more of an ability and much more of a comfort with their mathematics, when I see the confidence between them and the Algebra I Part 2 students. And, I think I might be jumping ahead here a little bit, my Algebra I students, my regular students I should say, they are much more willing to try to do calculations in their head. The Algebra I Part 2 kids can do a little bit, … but really can't. And, even the regular students are very tied into their calculators, and need them, feel like they need them all the time. But, they are much more able to understand some tricks like multiplying by 11. I can show them that and they go, "oh, that's why that works." And, in my Part 2 to class, I don't even try teaching that material, I don't even try teaching that kind of trick.

Me: When your Algebra I kids estimate, do they just grab something out of the air or are they thinking when they estimate?

Connor: They are grabbing more out of the air. I can tell that they are making some attempt, thinking through the problem. But, they grab more, but they do reasonable grabbing. They are not sitting there and looking at a problem and going, "okay the airplane is traveling at 200 mph, and he's gone for one hour 45 minutes, and so we can say he's gone for about two hours so 400 miles total about." They are not doing that. They will muse over the 200 miles per hour part and then say, "oh 600 miles." It's off, of course, but they are at least thinking about it.

Me: Your Algebra I again, do they check for plausibility when they get their final answers?

Connor: They are learning to, they don't always. But they were learning to check for plausibility.

Me: So, if they got an answer that the distance between here to the barn is negative 25 yards, they would know that that was incorrect?

Connor: They would recognize that that was wrong they, especially the positive more than the negative, those kinds of things like scale or large number of versus small number, you know. Relatively large answer or relatively small answer, they are a little rough on. It goes back to the moon, is the moon 250,000 miles away or is it 25,000 miles away or is it 2500 miles away? That's the kind of things they sometimes mess up when doing estimations or checking their answers. They don't look at that and say 2,500 miles, that doesn't make any sense.

Me: They don't or they do?

Connor: They don't. They won't do that sometimes. Again, they are much more comfortable with a positive and negative and looking at dividing something, you know, and thinking the answer should be a smaller number, why is it a larger number? They are much more comfortable with that. They will look in a word problem, for example, and identifying that this number really does not make any sense in the scheme of the world.

Me: If they did that problem where the plane is flying at 200 mph and the plane is out there for an hour and 45 minutes and their answer was 1000 miles traveled, would they recognize that that was implausible or would they not?

Connor: My top students would, the top half of that group would and the other ones wouldn't, I think. I think, again, I don't think it's a lack of ability as much I think as a lack of interest at times. That's the other challenge I've been running into when dealing with the bottom third of the class academically, the interest isn't there now. Now, if we rolled over to my Geometry students, you'd see much more of an awareness of what we're asking. It's the Honors kids especially; they are the kids who are asking, "what is he looking for, what is Mr. Connor looking for as an answer to this question?" not necessarily to find the process or see the beauty of the question, but to answer the question so that we can get an A. So, they are much more aware of looking at an answer and saying, "does this really look like something my teacher would ask?"

Me: And those Geometry students are generally always sophomores?

Connor: No, freshman. My classes this year, I had, I would say 80% were freshman and the other 20% were sophomores.

Me: Oh, I was thinking maybe it was related to maturity, if the Geometry students were a year older.

Connor: Well, I do see that in my more advanced students in Algebra I. I do expect them to be asking those types of questions when they are Geometry students as sophomores. My Geometry or Honors Geometry classes are already aware of that.

Me: Your Algebra I students, do they try multiple approaches? If some procedure that they want to employ doesn't get them where they need to go, will they look for multiple approaches?

Connor: Yes, often what I see, a little more often, is that they well get stumped, try the problem again, get stumped, then ask me for help, and then if I give them a little shove in the right direction, they will come up with a procedure which they are supposed to use. They need a bit more promptings versus a more advanced student who does that on their own. That's very natural and, again, I, as a rule of thumb, I was a kid who never really asked the teachers for help. I ignored all that; and now as teacher I see, I realize, that I should have, and so I always make it as easy as I can't for anybody to come and ask for help. "Hey, Mr. Connor, I don't get this." I put music on, I bring food and give answers away and I want that. I encourage that, I am okay with the idea, especially on quizzes, to come in for help. I will put up the quiz question, without telling them that it was quiz question, and I am very happy if somebody, if word gets around, that if you go see Mr. Connor, he will let you practice quiz questions. They won't know that they're getting the actual quiz question, but I'd rather it be that way; that to me is the carrot for growing and seeking help. I want them to develop that idea that, okay, if I ask for help, the teacher gives me the benefit of the doubt, he helps me out. And, that's what I want to happen

Me: What are your views on calculator usage in your Algebra classes?

Connor: I am wide open, but I would rather them use it less than more. What I try to do is that I really try to model the ability to work without a calculator?

Me: How do you do that?

Connor: I do that, I will sit there and I will race them. They will do it on their calculators and I will do it on the board and I will beat them nine times out of 10 because I, … I just remember all the tricks, I've learned it, and to be honest with you, I probably practiced the problem out the night before (laughter) so I kind of know the answer already. So I want to model that take, and they'll be ... how did you do that? And, I'll say, here's my reasoning, here's the reason, here's how I thought about it, and whether it's doing the distributive property or you using the trick of multiplying by 11 or any of these estimation techniques, I always want them to see that it is easier without me telling them that it's easier. That would be, that's the nice idea, leading them to discover that it's easier and the other thing is they're freshman, half of them don't remember to bring their calculators anyway; so, there's no use for me to even have to have a blanket policy or ban calculators, just because they don't bring them enough for me to really ban them.

Me: Do any of your students use 4-function calculators or do they all have graphing calculators?

Connor: Occasionally, but today in this day and age, any cell phone has more than four function calculator ability, I mean, my Blackberry can graph. You know, you can, the android, I'm buying next week does everything like TI-83 can do. But, I don't see kids carrying 4-functions anymore, anywhere. The only 4-functions I've seen all year have been in my desk, the loaner calculators I have, because every cell phone these days has beyond the computing power of a 4-function.

Me: Do you let your students use the calculators on their cell phones?

Connor: In class and occasionally on quizzes but never on tests. I really don't let them use it on tests especially, but on quizzes which are much smaller, either you know it or you don't, in the timeframe that I give them, so, I generally, depending on the mood, to whether I let them use phone calculators.

Me: I am sorry depending on what, mood?

Connor: Yes, mood, my mood, the teacher's mood (self-deprecating laughter)

Me: Do you let them use regular calculators on a test?

Connor: Yes, but they cannot use one on a phone.

Me: What is your reason for that?

Connor: Preparation, and the fact that these days, every phone, every cell phone is Internet connected and if they were really ingenious they could Google, I don't think, actually to be honest with you, I don't think any of them know Wolfram|Alpha well enough to understand how it works (laughter). But, in theory, they could Wolfram, they could plug in the question into Wolfram|Alpha and they would get the answer and it is an awesome program. I have it on my phone and I have it on my computer at home and it is an awesome little function to use, but so that's why I don't, and actually all the new i-Touches and phone's have a Wolfram|Alpha app. And, you can, it plugs in and you can type it in, it's slow but you can, in theory type in an equation and then get an answer. I see kids do longhand though at times, I still see, Carl, there is an example of a kid has the mental ability to do the math, but he makes math errors in his mental calculations; so, he will sit down and do it longhand and he's in Algebra I and again not necessarily because of his ability but because of his organizational skills.

Me: Doesn't everybody have to take Algebra I?

Connor: Yes, but he had to repeat it this year. He took Algebra I is an eighth-grader and he had to repeat it again this year as a freshman. In my Algebra I class this year, he only did fairly well, but not because of a lack of ability it's because he's so disorganized.

Me: Now let's tie the two together. So, we been talking about mathematical thinking and reasoning and estimating and checking for plausibility and then we talked a little bit about use of the calculators. What I'd like to do is tie those two together and ask you how do you think your students interpret or understand the answers that their calculators give them?

Connor: I see most of our students using the calculators, I think it's a little purposeful on my part that I usually, I don't teach them about using the graphing function on their calculators as Algebra I students, I want them to hand graph. I wanted them to understand that process, so most of the work they're doing on their calculators is just addition and subtraction or multiplication and division, basic arithmetic is what they're using it for. Whereas, since the information they are plugging into their graphing calculators is relatively simple, so, they have do all the reasoning outside of the calculator before they can plug it into the calculator. So, for example, I will teach them how create a column chart on paper, you know, the *x*-values, the function, and then the *y*. And, where I will see them use the calculator is figuring out what the *y* is going to be by plugging the *x*-value into an equation, say $y = 3x - 2$, and so they are not necessarily doing, they are not doing the actual reasoning on the calculator, they are not plugging this into a function and then go into the TABLE function on their calculator to get the *y*. They are using the calculator to do just the physical calculations not reasoning behind the whole problem. They do the reasoning themselves.

Me: If the calculator gave them a ridiculous answer, would they recognize it?

Connor: Yes, I've, especially with the graphing segments and the functions that we work with in the second half of that year. They really get that idea. We do linear equations and we do quadratics. We spent about three weeks on quadratics this year. I would like to spend more; and, so, they understand quadratics and again I really don't show them how to use their calculators to graph quadratics. I would rather they learn how to graph quadratics by hand; I don't give them any really crazy quadratics, it's just basic simple ones so they can recognize the U-shape. And, I want them to hand graph that, I will let them use a calculator to find an *x*-value, but I don't necessarily show them. I don't say, "graph the function on your calculator." A few of them might be able to do it but most of them have not explored enough to be able to do that.

Me: If they were graphing $y = -3x + 2$ and they got a positive slope, would they know was wrong?

Connor: In the calculator or by hand?

Me: Either way.

Connor: In my regular class, the top 75-80% definitely do. My weaker students don't. And again, I think it's less of a lack of understanding, but less of a, more of a lack of paying attention, a lack of attention to details, I think is much more common. In their work, every once in a while, you'll see a kid get dyslexic and do every positive slope negative and every negative slope positive. But, it is not consistent and it is not all the time, I would say the vast majority of the time, they really look at a graph and say, that is either a reasonable graph, or say, that isn't a reasonable graph. And, sometimes we will sit there and we will use our arms to show the lines, you know, the positive or negative (Connor positions his arms to look like a linear equation), is the slope steep or shallow. We don't do that a heck of a lot but, you know, once in a while … breaks up the routine (Laughs).

Me: Beth calls that polynomial dancing.

Connor: Yes, I walked by her classroom a couple of times and saw them all in there waving their hands and it all looked pretty strange, .. like old style, … you know, disco dancing or something, but then I asked her and went , "Oh, ok."

Me: She likes doing that.

Connor: Yeah, but she has a lot more function types than I do (Laughs). When its just linear, it can get boring, you know.

Me: Does this apply to the *y*-intercept as well?

Connor: Yes, they look at the graph and decide whether the *y*-intercept is reasonable or not. And, that is drilled into their heads. I mean I drill that into their heads, in terms of this is how to look at it.

Me: Do you cover exponents of numbers?

Connor: Yes.

Me: Let me ask you about this in the calculator. Negative 5 squared (-5^2) is negative 25. But, $(-5)^2$ squared is 25. So, your Algebra I Part 2 students, do they understand the distinction between these two?

Connor: No, not really.

Me: If they get -25 for an answer to (-5^2), they will march on?

Connor: Yeah.

Me: And, they don't realize that it's wrong?

Connor: Yeah, they don't see that it is wrong.

Me: What about the Algebra I students? Do they understand the distinction between these two?

Connor: They do get that. Well, they don't have so much about how they have to type it into the calculator, but they do understand that there is a definite difference between partitioning off -5 with parentheses and leaving it off on its own.

Me: Do they understand the difference of how you enter it into the calculator?

Connor: They are learning that. And, usually, this is again, I think, this is probably a weakness on my part this year because I wasn't very specific, and teaching that and in using my graphing calculator this year. I didn't do a whole lot of that style of teaching. We do a lot of things our own. Sometimes graphing calculators as crutches. I'm thinking about how I can use them to help the students next year; I want to do a lot more being conscientious and deliberate in my use of my calculator. I want to show them how we use our calculator and what do these answers mean because these kids understand programming to a significant step much more than I did at that age. For me, at that age, it was a huge step to program a quadratic equation on a calculator and for these kids that's a very, very basic step. They are computer savvy and they understand that process. And, the kids, across the board, need to understand that the calculator doesn't have this ability to make leaps of judgment, it has to obey rules. The kids understand that idea. And, they are actually much capable of working with calculators than my generation was, and certainly earlier generations were. But, I think this is a group of students who, I mean I took C++ programming in college. These kids are taking C++ as high school freshman, if not learned earlier. But, they understand. And I think that's where the kids actually thrive on the process, Algebra I students thrive on the process, because they've learned these procedures, to run this program, have to open this, and then hit this button or to play games they double click, and they know that to get certain results, they have to do all these steps.

Me: When you say thrive on the process please tell me what you mean.

Connor: What I mean is that they, they do well to well once they see the procedure.

Me: Do they understand why?

Connor: With time. I think they are, I teach them by showing them how to do it and then showing them why it works and showing them, you know, can we relate to how we'd get to the answer. And, so that's why we do all these things. So, they would probably be a little bit more challenged to derive their own ideas and that's what I want them to do. Maybe I'm getting these backward, but I think, … I think if they know, they like that step by step, they could see how that works and our trick is to teach them how to write that stuff. Does that make sense?

Me: I think what your telling me is that they have found a comfort zone?

Connor: Yes, that's good, that's a good description, they have a comfort zone in the procedure.

Me: If there are aberrations from the procedure, let's say we are solving a quadratic by completing the square. It's pretty straightforward if the *a* equals 1; but, when *a* does not equal 1, there is an extra step. In this sort of case, when there is some sort of an aberration in their *comfortable* process, does that make them come to a screeching halt, scratching their heads, or do they look for and implement plan B?

Connor: My regular Algebra I can do that. My Algebra Is one Part 2 cannot. For them, it's steps 1-2-3 and that's it. My Algebra I's can adapt.

Improvise, adapt, overcome. That how I really want them to look at problems say, okay, can I take what I've learned and now adapt what I have learned already to answer this question and apply it. That would be my dream.

Me: Okay, now let's tie those three together. Thinking about your Algebra students' mathematical reasoning, ability to problem solve, and ability to apply knowledge to problem solutions, and then thinking about the use of calculators, do you see any connections?

Connor: I think the calculator used exclusively gets in the way in terms of truly understanding the material because now you're just thinking about I have to push this button and then this button and then I've gotten it. You learn, you think, your mind is trained to think the answer is only three button clicks away; whereas, if you use a calculator as a tool it can be incredibly powerful and I think that is the challenge, you have to teach students to view a calculator as a tool not the solution machine; it is not magical. It is pretty awesome but it goes back to the idea of what I was talking about, I want them to hand graph versus going on there hitting keys and plugging in their functions and then hitting the graph key. That doesn't teach them anything, whereas if I teach them how to hand graph I'm very comfortable with them going in there to the calculator to use it to help them do the calculations. Because they should be looking at these problems and doing the calculations in their heads, they are not hard, which is why haven't made the problems really hard, you know, with difficult numbers, I do not want that to get in their way of getting understanding.

Me: You don't want their not being able to do it in their heads to affect their understanding?

Connor: Yes because as much as you and I, we'd would love to think that everybody can do this math in their heads, it's no big deal, but the reality is that most of the world can't and, and won't and they don't need to. I mean you can type 9 * 12 into Google and it'll give you the answer. Literally Google will, will put 9 time 12 equals, what is that, uh, 108. I'm slow (laughter), I'm on summer vacation. I was estimating first; ten times 12 is 120; so, I was like, the answer's going to be a little under 120.

Me: That's estimation.

Connor: That's what I was thinking. And, that's no different than, I mean, you know, than Mrs. Rodriguez (one of our Spanish teachers) would, in her ideal world, everyone would speak Spanish and English. Most of us go on the Google translator and translate that Spanish document into English and we get the understanding we need. Most of us remember just enough of what we need to know and there are Web tools to teach us the rest. So, I'm okay with using calculators as a way to do calculations; I don't like, again it goes back for what level of kids I teach, I don't want them to do their quadratic graphs on the calculator. I want them to do that by hand because at least then they learn why the quadratic is shaped like this (traces a quadratic shape in the air).

Me: When you say that you let them use their calculators for calculations, are you saying that they set up the problem themselves, and they set up the order of operations and then you let them put that into the calculator; but if they don't set it up correctly then they won't get the correct answer?

Connor: Yes, that's exactly what I mean, exactly what I'm saying. I want them to be able to set the problem up and then use their calculator as a tool to help them do the calculations. And, then, yeah, they really haven't practicing their multiplication, their addition, whatever but they have demonstrated a knowledge of the process, they have an understanding of when I say, "graph this," they can do it, they can do it by hand.

Me: For what's Algebra topics do you think the calculator is the most useful?

Connor: I think it's most useful, …, let me think for a second here, …, I see it most useful when they are working with exponents, really. I think it's the way for them to look at exponents and algebraic sequences, see it as a tool for them to take the knowledge that they have and plug it. When we first do it, they take quizzes they will when they are not allowed to use calculators and there will be questions like 4^3. So, I want them to learn those things in their heads, but once they do, very quickly we go with large numbers with exponents. And, it can get tedious and so I am very comfortable with them using a calculator for those things and simplifying them; but, it can also be a double-edged sword when you start doing fractions with exponents, when there is an exponent in the numerator and the denominator, you know exponential numbers in both. That's when sometimes they'll get a little lazy and plug all that into the calculator and they know to put in the parentheses, and so they are showing an understanding of, … of why we have to put those parentheses in there, in order of operations and things like that. But, I think on the whole exponent thing, it helps them; it is a huge help for them when it comes to that because many of them understand that to the 2^5 is $2 \times 2 \times 2 \times 2 \times 2$, but not all of them can sit there and do it quickly. Some will sit there are and say 2×2 is 4, 2×4 is 8, 2×8 is 16, and 2×16 is 32. And, either you can do that or you can plug it into a calculator, as long as you can tell me that 2 raised to the fifth power is 2 multiplied by itself, with itself as a factor, five times.

Me: And do the students who type 2^5 into their calculators understand that that means $2 \times 2 \times 2 \times 2 \times 2$?

Connor: Yes, because we've taught that before, they've learned how to use the caret on the calculator.

Me: Have you ever, either aloud or to yourself, said, I can't believe you're using a calculator for that?

Connor: To myself.

Me: Oh, I didn't mean for you to differentiate between aloud and silently. Sorry, I wasn't clear. What types of things are they doing that you would think that?

Connor: Additions, divisions, the simple stuff they should be doing in their heads.

Me: Something like $2+7$?

Connor: No, not usually something like $2+7$ but larger numbers, and I'm surprised. But, I'm also a math teacher this is what I do for a living. Adding three digits numbers is not a struggle for me. But, I do say this to myself every day, every day.

Me: So, it's arithmetic they're using it for?

Connor: They're using, they use it for arithmetic. I don't see them using it much for, I mean, I don't see them using it for graphing, partially it's because I don't teach them a lot about how to use it for graphing. I want them to learn how to do that by hand. I'm very understanding and let them use it because, again, there are situations in life when I couldn't do some work without a research manual in front of me. So, I couldn't go back and program a computer right now without getting out my old textbooks and reading and relearning things. In most of life, we do have those tools. It's awesome to memorize the periodic table, for example, but most people the world, if they need to look something up on the periodic table, they have a cell phone with Internet access, and they can pull up that information very easily without having to remember H is for hydrogen, and so forth. But, I'm incredibly understanding about allowing them to use those tools to help them because they will be using those tools for their entire life. And, they will, I mean, this phone (pointing to my cell phone) probably has as much, if not more graphing, er …, computing power than the computers on the first space shuttle. I learned that, read that the other day, the i-Phone's have more computing power than the computers on the very first space shuttle. It has a GPS, a camera, and this is what we all carry in our pockets. So, there are a myriad of things that we as people who are of an older age who came up, you know, I came up as computers were coming into vogue, but they were still things that sat on desks not things that were in our pockets. We've come so far and that's the thing that we have to remember, as teachers, that we had to learn by hand because there was no other way to do it. Nowadays, they don't have to learn that, so I have to be willing to say, it's okay that they use it for arithmetic. They will learn the things that they really need to know by heart, they are, any kid, because 5 comes up so often, they can all sit there and do the multiplication table for 5. That's very easy to do and because they use 5 all the time, they remember. So, I'm okay, they have to go back and say 13x13 is 169; they all know the prime numbers now because we teach them because they're very common. But, you know there's a lot of other things out there that you and I could sit here and try to do on our own and use all the tricks in our head but we don't need to. From my perspective, there are so many things that we used to have to memorize just to get through daily life that we don't have to anymore. For example, telephone numbers, I call my girlfriend at least five times a day and I do not know her phone number. Who knows anybody's telephone number anymore? And, I remember, I was, I was one of the very last undergraduate classes where people were still using typewriters. I remember typing and having five copies of each school's application. College applications are now all done online. How quickly things change is really what I'm saying. So, I'm saying I'm okay with calculators and, again, I am much more okay with calculators doing the simple things because they will then, the kids, because I teach them the process. And, I fully expect that when they get to your level (Calculus) you're okay with them doing the graphing on the graphing calculator, because they've already learned that and I'm assuming that they've learned enough arithmetic that it's okay for them to use it. (Laughter)

Me: In tying everything together, we started off with me asking you to talk about mathematical reasoning and then we talked about calculators and we talked about connections. Now, I'd like to go back to the idea of mathematical reasoning. What impact does the calculator have on a student's learning, mathematical reasoning and thinking?

Connor: (long pause) This one is, yeah, got to think … Well, I see a couple ways. I see there is an expectation of immediacy now in knowledge and both positively and negatively. I think I don't necessarily mean that kids are lazy, they expect the answer to be given to right away, I think it's more of a process. I was listening to on NPR the other day they were talking about the Stanford University library and how it will be totally paperless in the next five years. They said that all their journals, everything is already digital, and one of the assistant librarians said that the entire legal department, that everything is changed now. So, no kid has to peel through have half a dozen tables of contents to look for that one article. We all have LexisNexis, we all have all these great search tools in front of us; so, the computer and the calculator have changed our ability to look for knowledge, and in taking a long tedious process of looking at lots of things which are really irrelevant and allowed us to focus on what's really, really important. I think that's what you are trying to get out of me, me being okay with letting the students use their calculators for arithmetic because I want them to focus on this, the process. And so, you can relate that to track and field, you can relate that to many, many parts of life, in that these tools have allowed us to be very immediate and taking bits of information and turning it into new bits, we're taking an idea and asking a question and being able to find something out. Wikipedia, for example is unbelievable, I love it. I absolutely believe it's one of the coolest things I've ever seen. And, I use it daily. I used it this morning for five different things. If you look at trigonometry, there is just mountains, the history, all kinds of information there. I mean, it also becomes a repository for the most random information, both useful and not useful. But, I think there is an immediacy of seeking an answer or idea, I think that's changed. I also think … their thinking has changed in that I think kids have learned shortcuts. I think this is less of a calculator than a computer thing. I think kids identify, because to search Google you have to use keywords. And, you have to look at the really important words. And, so I think kids, you know, understand a little bit more of those keywords more than they used to.

Me: And how does that apply to mathematical thinking?

Connor: Well, I think that it, again, it's the process of stripping away extraneous material and identifying what's important in any question, especially a word question in mathematics. You have, we all obfuscate our questions to some extent, otherwise it wouldn't be fun for us. (Laughter) And, but, I think, I think kids are learning to cut to the chase. I imagine they are learning to cut to the chase more often or more quickly or at least this is the way I find myself, because I am a closer their generation than not. And, you know, learning to look for keywords, key dates, being able to search within a document, and so when you see these math questions, you would have the same idea, you know, in looking at a proof, looking at the idea, they are very able to zero in on information, to narrow things.

Me: So, if you told them $y = x^2 + 2x - 3$, how do they look that up on Google? Let's say you asked them for a picture of $y = x^2 + 2x - 3$.

Connor: If that is all I said to them, they would be clueless. But, what they've learned, their vocabulary and learning words to them, maybe they are not going to become great writers, but they are learning vocabulary at an exponential rate because they have to be able to translate this mathematical equation into words. They have a look at this and say, "hmm, quadratic." And, I think they are getting there, to look up not just quadratic but parabola and graph. Certainly, not on day one, but they were getting there. And, by the end of the course they are there. We talk about, well, for example, over here (points to the wall where there are about a dozen photographs with geometric figures highlighted), what you're seeing up here is just a little bit, all of those things are. I told them to bring in examples of triangles, and over on the left are examples of parabolas. So, they know, they were looking, they are Googling "arch" and "parabola." And, so I think that there's a vocabulary that they are learning, and that applies to computers more, so that really doesn't answer your calculator question.

Me: So I'm going to try to pull you back in that direction, mathematical reasoning and the calculator, how does that ready availability of the calculator impact their mathematical reasoning?

Connor: (long pause) Hmm. I'm having a hard time thinking about how that works. (Pause) Let me think about that one and we can come back to it, ... if I can think of something..

Me: Ok, then I was going to ask you if you an had examples to illustrate that.

Connor and I then discussed my interviewing skills and whether he found having the interview guide to be a help or not. I also asked him if he had any other comments which he wanted to make about the interview topics which I did not ask him. He had none. So, I asked the final question again.

Me: What impact does the calculator have on your Algebra students' mathematical reasoning?

Connor: The question of the day. I think the hard thing for me is that I haven't seen a time without the calculator. I was taught, you know, I'm a young guy and I've only taught like six years now or been around schools (as an adult) for six years now. So, I'm of the generation that was not calculator-free. I am now teaching, using basically the same models I used in school as a student. So, I can't really say before or after so much, but I am less willing to say that there is a negative impact. I think there is a generational difference in the question when you say, "Well, I didn't have it and I grew up to be a math teacher, so this could do nothing but hurt." And, I think there is a selection bias in the sense that we are looking at people who ask that question are people who are always interested in math, you know, we are people who love math, who want to teach it, who want to learn about it and the kids who are still interested in math, who love math, are still interested, and they now have calculators which allows them to do more with their thoughts than ever before. Whereas, kids who weren't interested are never going to be interested in math, and, they probably become a little more reliant on the technology and probably do a little bit less reasoning and have less understanding. But the kids who are better at it, who really all are motivated by it, they probably do more, I think they probably do more and better work. I don't have any exact anecdotal or experimental examples of that, but I would imagine that to be the case. That would be, … well we are probably spreading the range, but that top level is getting a lot farther a lot faster than that bottom range is probably falling. And, so we've seen the general ramping upward with that calculator technology. I would imagine that is what is happening.

My idea would be saying that technology is scary and it's hard to deal with because it, we have changed, it changes the people who teach. The people who are teaching now usually learned without calculators and now have to deal with lots of people who understand calculators innately. Just as I'm going, now, and at some point kids will be using, … you know, … internal computers or who knows. And, I'm going to be sitting there saying, you know, that's not learning. And, because they have all the knowledge in their brains, but, idea of learning changes. It may not be memorizing it, but it's going to have to be sorting the information. And, it's stripping away, cutting away what is extraneous and what is useful. And, whereas, we had less things that were distracting us. But, I think it's just the process of learning changes. We all wondered what we did before e-mail; we must've done nothing with our lives, but we didn't. Our days were filled and now we have so much more e-mail, it's making us do more things, but it's making us deal with things more quickly; and, it's allowing us to do more in other things. So, I think it is making us more efficient and changing the way we think. But, I think it's probably for the better. I can't imagine that we would develop technology that would evolve to where it will hurt us in the future.

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