

ADVANCED PLACEMENT CALCULUS (AB)

Description: This course is intended for students who have a thorough knowledge of algebra, geometry, trigonometry and elementary functions. The purpose of the course is to prepare the student for advanced placement into college calculus. The content of the course is driven by The College Board Advanced Placement Course Development Syllabus. Students are required to have completed an Honors Pre Calculus class. Exceptions, on a case by case, may be made for exceptionally motivated students who have completed a PreCalculus course with a grade of at least A-. Students are required to complete a summer reading and problem solution assignment prior to admission into the course. The assignment is due on the first day of class in September. The graphing calculator is used extensively throughout the course while a balanced approach of graphical, numerical and algebraic methods is stressed. Problem solving is introduced early and integrated throughout the course as connections to other topical areas are made through practical applications. Models and technology are used when appropriate. Throughout the course, students are encouraged to use the language and symbology of mathematics.

Student Summer Preparation: During the summer preceding enrollment in AP Calculus AB, students are required to read The Idiot's Guide to Calculus, listed below, and to complete the problems in the first eleven chapters. These chapters introduce limits; continuity, including one-sided limits; the ideas of a derivative and differentiability; and the limit definition

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Daily Schedule: Quizzes are held every day. The quizzes are modeled after the multiple choice questions on the AP Calculus AB Exam, although they are frequently presented in "calculation form" with no answer choices given. The quizzes are presented in a form which resembles the AP Calculus AB Exam, to ensure that the students are accustomed to the Exam format. Daily quizzes last 5-10 minutes and are completed in mechanical lead pencil. At the completion of each quiz, students use a pen or colored lead pencil to mark their papers. We review the quizzes immediately and questions are answered. The class discusses any errors they have made so we can all learn from each other. Students submit their work and I give the solutions to the day's quiz with the solutions worked out step-by-step, printed on blue paper, to each student. Students are advised to keep a file of their "blue sheets" to use in preparation for the AP Calculus AB Exam.

Technology:

- Each student is expected to have a TI-83, TI-83 Plus or TI-83 Silver calculator and is required to bring the calculator to class every day.

- b. I use the TI-Presenter to display my graphing calculator on the classroom television screen. I use wet-erase colored markers to write on the television screen to accentuate tangency, limits, cusps, functional behavior at infinity, etc.
- c. I use an overhead projector with transparencies to review old AP exam questions, solutions and rubrics.

Objectives: The student will be able to:

- Define and apply the properties of limits of functions
- Define continuity and determine where a function is continuous and discontinuous
- Find the derivative of an algebraic function by using the definition of a derivative
- Understand the relationship between continuity and differentiability
- Apply formulas to find the derivative of algebraic, trigonometric, exponential, and logarithmic functions and their inverses
- Apply formulas to find the derivative of the sum, product, quotient, inverse and composite (chain rule) of elementary functions
- Find the derivative of an implicitly defined function
- Use the first and second derivative to analyze graphs of functions and know the corresponding characteristics of f , f' and f''
- Apply the derivative to solve problems, including: tangent and normal lines to a curve; curve sketching; motion; related rates of change; linear approximations; differentials; and, optimization problems
- Find the indefinite integral of algebraic, exponential, logarithmic, and trigonometric functions
- Understand, interpret and generate slope fields for a family of functions.
- Understand the properties of the definite integral, including: the definite integral as an area and as a limit of Riemann sums and the Fundamental Theorem of Calculus
- Compute an approximate value for a definite integral by using Riemann sums, the Trapezoidal Rule, and, Simpson's Rule
- Apply the definite integral to solve problems including motion problems with initial conditions; the average value of a function; area between curves; volumes of solids of revolution about the axes or about lines parallel to the axes using disk method and washer method; volumes of solids with known cross-sectional areas; growth and decay problems; and, solutions of separable differential equations

Course Content:

A. Chapter 1: Limits and Their Properties. Students begin Calculus with an intense study of limits and continuity. On the first day, I introduce the concept of limits by discussing running a mile. Not long ago, it was thought that no human could break the 4 minute mark. Running the mile in 2 minutes is presumably impossible. 3:50 is possible. Somewhere in between is the true limit. Then I ask the students if they would pay \$1 for a raffle ticket for a \$1000 prize. Would they pay \$2? Would they pay \$5? Some students have different limits than others. Eventually, we get to an amount which no one is willing to pay. During the first week, I use the TI-Presenter to display a function with a removable discontinuity at $x = 1$. From the standard window $[-4 \leq x \leq 4], [-3 \leq y \leq 3]$, the discontinuity is not discernible and a novice may conclude incorrectly from the graph that the function is continuous at $x = 1$. However, when I set the window limits for x as $[0.99 \leq x \leq 1.01]$, the

“hole” in the function at $x = 1$ becomes clearly visible and I generally hear a collective “Ahhhhhh!” from the class. The students learn two important lessons: (1) the calculator has its limitations and calculator graphs cannot be accepted *carte blanche*; and, (2) it is important to combine the graphic, numerical and analytic approaches to finding limit and assess continuity. I follow up the first example with a discussion of the continuity of the function

$f(x) = \frac{(x-1)^2(x-2)}{(x-1)(x-2)(x-3)}$. First, the class uses an analytical approach to

determine function behavior at $x = 1$, $x = 2$, $x = 3$. Then, we investigate the graph of the function to confirm our conclusions. The student's learn to discern between removable and non-removable discontinuities, both graphically and analytically. Chapter 1 topics include:

1. A Preview of Calculus
 - a. Understand what Calculus is and how it compares to PreCalculus
 - b. Understand that the tangent line problem is basic to Calculus
 - c. Understand that the area problem is also basic to Calculus
2. Finding Limits Graphically and Numerically
 - a. Estimate a limit using a numerical approach
 - b. Estimate a limit using a graphical approach
 - c. Learn different ways in which a limit can fail to exist
3. Evaluating Limits Analytically
 - a. Evaluate a limit using properties of limits
 - b. Develop and use a strategy for finding limits
 - c. Evaluate a limit using dividing out and rationalizing techniques
 - d. Evaluate a limit using the Squeeze Theorem
4. Continuity and One-Sided Limits
 - a. Determine continuity at a point and continuity on an open interval
 - b. Determine one-sided limits and continuity on a closed interval
 - c. Use properties of limits
 - d. Understand and use the Intermediate Value Theorem
5. Infinite Limits
 - a. Determine infinite limits from the left and from the right
 - b. Find and sketch the vertical asymptotes of the graph of a function

The Chapter 1 Project is an analysis of trigonometric functions: $\sin x$, $\cos x$, $\tan x$ and $\frac{(\sin x)}{x}$. We graph the functions and analyze their behavior visually. We then experiment with drawing tangent lines at various points on the functions. And, we also make tables of values to determine limits, if they exist. The students see how the numerical, graphical and analytic approaches tie together and also gain an understanding of when one approach may be preferable to another.

- B. Chapter 2: Differentiation. The meaning and rules for calculation of derivatives are explored. As the first step in problem solution, I ask the students to translate verbal

descriptions, such as "the ladder is falling down the wall at a rate of 2 feet per sec" into calculus format, i.e., $dy/dx = -2$ ft/sec. Students are required to include units of measurement in all calculations. An "ah-ha" moment occurs when I graph $f(x) = \sin x$ on the interval $[-2\pi \leq x \leq 2\pi]$ on the television screen, using TI Presenter. At multiples of $\pi/6$ and $\pi/4$, I ask my students to approximate visually the slope of the tangent line to the graph of $f(x) = \sin x$. On the same screen, I then plot the values of the slopes they give me. When we have sufficient points plotted, I turn off the function and all which is remaining is the graph of the derivative, which they recognize as the graph of $f(x) = \cos x$. I then graph the function $f(x) = \cos x$, and go through the same process. Of course, this time the derivative is the graph of $f(x) = -\sin x$. I have found this to be a very helpful exercise in getting the students to truly comprehend this relationship between $f(x) = \sin x$ and $f(x) = \cos x$. The chapter project centers on optical illusions. Chapter 2 topics include:

1. The Derivative and the Tangent Line Problem
 - a. Find the slope of the tangent line to a curve at a point
 - b. Use the limit definition to find the derivative of a function
 - c. Understand the relationship between differentiability and continuity
2. Basic Differentiation Rules
 - a. Find the derivative of a function using the Constant Rule
 - b. Find the derivative of a function using the Power Rule
 - c. Find the derivative of a function using the Constant Multiple Rule
 - d. Find the derivative of a function using the Sum and Difference Rules
 - e. Find the derivative of the sine function and of the cosine function
 - f. Use derivatives to find rates of change
3. The Product Rule and the Quotient Rule
 - a. Find the derivative of a function using the Product Rule
 - b. Find the derivative of a function using the Quotient Rule
 - c. Find the derivative of a trigonometric function
 - d. Find a higher-order derivative of a function
4. The Chain Rule
 - a. Find the derivative of a composite function using the Chain Rule
 - b. Find the derivative of a function using the General Power Rule
 - c. Simplify the derivative of a function using algebra
 - d. Find the derivative of a trigonometric function using the Chain Rule
5. Implicit Differentiation
 - a. Distinguish between functions written in implicit form and explicit form
 - b. Use implicit differentiation to find the derivative of a function
6. Related Rates
 - a. Find a related rate
 - b. Use related rates to solve real-life problems

The Chapter 2 Project is a study of optical illusions created by families of curves. We find dy/dx at different values of x , and students discuss the relationship between dy/dx and the visual illusions we perceive.

C. Chapter 3: Applications of Differentiation. The applications of differentiation are studied. As a reinforcement of the relationship among $f(x)$, $f'(x)$, and $f''(x)$, on 3" x 3" pieces of paper, I print 2 dozen graphs, e.g., $\sin(x)$, $2\sin(x)$, $\cos(2x)$, x^2 , x^3 , e^x , $\ln(x)$, $1/x$, \sqrt{x} , etc. I also print graphs of the first and second derivatives of these functions. The pieces of paper are all mixed up and the class, as a team, must match all the three graph sets, without knowing which are the original functions and which are the derivatives. They use tape to affix the graphs to the wall; the graphs can easily be moved if the students change their minds. This is a great exercise in working collaboratively, in meshing graphic and algebraic approaches to function analysis, in presenting mathematical arguments to persuade classmates, and, in using curve characteristics to distinguish between similar looking functions such as $\ln(x)$ and \sqrt{x} . Chapter 3 topics include:

1. Extrema on an Interval
 - a. Understand the definition of extrema of a function on an interval
 - b. Understand the definition of relative extrema of a function on an open interval
 - c. Find extrema on a closed interval
2. Rolle's Theorem and the Mean Value Theorem
 - a. Understand and use Rolle's Theorem
 - b. Understand and use the Mean Value Theorem
3. Increasing and Decreasing Functions and the First Derivative Test
 - a. Determine the intervals on which a function is increasing or decreasing
 - b. Apply the First Derivative Test to find relative extrema of a function
4. Concavity and the Second Derivative Test
 - a. Determine the intervals on which a function is concave up or concave down
 - b. Find any points of inflection of a graph of a function
 - c. Apply the Second Derivative Test to find relative extrema of a function
5. Limits at Infinity
 - a. Determine finite limits at infinity
 - b. Determine the horizontal asymptotes, if any, of the graph of a function
 - c. Determine infinite limits at infinity
6. Curve Sketching

Analyze and sketch the graph of a function, using the following concepts: x -intercept(s), y -intercept, symmetry, domain, range, continuity, differentiability, vertical and horizontal asymptotes, relative extrema, concavity, points of inflection, and infinite limits at infinity
7. Optimization

Solve applied optimization problems, including maximum volume, endpoint maxima, minimum distance, minimum area and minimum length.

8. Differentials
 - a. Understand the concept of a tangent line approximation
 - b. Compare the value of the differential, dy , with the actual change in Δy .
 - c. Estimate a propagated error using a differential
 - d. Find the differential of a function using differentiation formulas

In Chapter 3, we have two chapter projects. The first is introduced after we have studied increasing and decreasing functions and before we study concavity. The class analyzes the Law of Refraction in relationship to rainbow angles. We find the minimum angle of deflection for angle α .

The second project is introduced after optimization problems and before the study of differentials. The project concerns the water levels of the Connecticut River. We study a graph of the rate of change in feet per day versus time. From this, we ascertain when the river is rising most rapidly, when the river crested at the highest level, etc. The students are reminded that the graph is of the rate of change, not the water level.

- D. Chapter 4: Integration. Students progress to Integral Calculus. The chapter project strengthens the learning of abstract concepts by demonstrating the Fundamental Theorem. We spend several days investigating Riemann sums, determining how to write the function in

terms of i for $\sum_{i=?}^n f(?)$, and comparing left-endpoint sums, right-endpoint sums and mid-point sums. Students compare the sums for different values of n and determine how well the Riemann sums perform as predictors of area. Chapter 4 topics include:

1. Antiderivatives and Indefinite Integration
 - a. Write the general solution of a differential equation
 - b. Use indefinite integral notation for Antiderivatives
 - c. Use basic integration rules to find Antiderivatives
 - d. Find a particular solution of a differential equation
2. Area
 - a. Use sigma notation to write and evaluate a sum
 - b. Understand the concept of area under the curve.
 - c. Approximate the area of a plane region using upper sums, midpoint sums and lower sums
 - d. Find the area of a plane region using limits
3. Riemann Sums and Definitive Integrals
 - e. Understand the definition of a Riemann sum
 - f. Evaluate a definite integral using limits
 - g. Evaluate a definite integral using properties of definite integrals
4. The Fundamental Theorem of Calculus
 - a. Evaluate a definite integral using the Fundamental Theorem of Calculus
 - b. Understand and use the Mean Value Theorem for Integrals

- c. Find the average value of a function over a closed interval
 - d. Understand and use the Second Fundamental Theorem of Calculus
5. Integration by Substitution
- a. Use pattern recognition to find an indefinite integral of a composite function
 - b. Use a change of variables to find an indefinite integral
 - c. Use the general Power Rule for Integration to find an indefinite integral
 - d. Use a change of variables to evaluate a definite integral
 - e. Evaluate a definite integral involving an even or odd function
6. Numerical Integration
- a. Approximate a definite integral using the Trapezoid Rule
 - b. Approximate a definite integral using Simpson's Rule
 - c. Analyze and approximate errors in the Trapezoid Rule and Simpson's Rule

The Chapter 4 Project focuses on demonstrating the Fundamental Theorem of Calculus.

$$F(x) = \int_0^x \sin^2 t \, dt$$

Using $\int_0^x \sin^2 t \, dt$, we use a numerical approach to calculating the value at various values of x , from 0 to π . We use the integration capabilities of the graphing calculator to graph $F(x)$. We then use the differentiation capabilities to graph $F'(x)$. This is followed by a discussion of how the two are related and why.

E. Chapter 5: Logarithmic, Exponential, and Other Transcendental Functions. Students build on their basic knowledge on integration and advance to the integration of more complex functions. At this point, I begin regularly giving the class a free-response question from a previous AP Exam. They work independently for 10-15 minutes. Then, we compare solutions and methodology. This exercise gets the students thinking about not only justifying their calculations, but also justifying their logic for the methodology they used. Students thrive on helping each other and learning from each others' errors. Sometimes, I will allow 10-15 minutes for independent work. Then, I have the students switch papers. I show the grading rubric on an overhead transparency. We go through each step of the solution step by step and the students grade each other's papers according to the rubric. Students learn an incredible amount and gain confidence from each other in these exercises. Chapter 5 topics include:

1. Differentiation of the Natural Logarithmic Function
 - a. Develop and use properties of the natural logarithmic function
 - b. Understand the definition of the number e
 - c. Find derivatives of functions involving the natural logarithmic function
2. Integration of the Natural Logarithmic Function
 - a. Use the Log Rule for Integration to integrate a rational function
 - b. Integrate trigonometric functions
3. Inverse Functions

- a. Verify that one function is the inverse function of another function
- b. Determine whether a function has an inverse function
- c. Find the derivative of an inverse function
4. Differentiation and Integration of Exponential Functions
 - a. Develop properties of the natural exponential function
 - b. Differentiate natural exponential functions
 - c. Integrate natural exponential functions
5. Bases Other than e and Applications
 - a. Define exponential functions which have bases other than e
 - b. Differentiate and integrate exponential functions which have bases other than e
 - c. Use exponential functions to model compound interest and exponential growth
6. Differentiation and Integration of the Inverse Trigonometric Functions
 - a. Develop properties of the six inverse trigonometric functions
 - b. Differentiate inverse trigonometric functions
 - c. Understand and apply the basic differentiation rules for elementary functions
 - d. Integrate functions whose Antiderivatives involve inverse trigonometric functions
 - e. Use the method of completing the square to integrate a function
 - f. Understand and apply the basic integration rules involving elementary functions
7. Hyperbolic Functions
 - a. Develop properties of hyperbolic functions
 - b. Differentiate and integrate hyperbolic functions
 - c. Develop properties of inverse hyperbolic functions
 - d. Differentiate and integrate functions involving inverse hyperbolic functions

In Chapter 5, we have two chapter projects. The first is introduced after we study differentiation and integration of exponential and logarithmic functions with bases other than e . The students use graphing utilities to estimate slope. Using the piecewise

function $f(x) = \begin{cases} |x|, & x \neq 0 \\ 1, & x = 0 \end{cases}$, we investigate why the formula $\frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}$ can

be used to approximate the derivative. Students again see that the calculator does have limitations and they realize that relying on the calculator without analytical thinking can easily lead to approximating the slope of a graph incorrectly. The second project is introduced after differentiation and integration of inverse trigonometric functions. We use a hyperbolic cosine function as the model for the construction of the St. Louis. We are able to determine the height and width of the arch.

- F. Chapter 6: Differential Equations. Students proceed to Differential Equations. As an activity, the class completes the Slope Field packet from the AP Calculus course web site. For a second activity, on the television screen, using TI Presenter, I graph a family of functions in the form $f(x) + C$. Each student is given a value of x for which she/he calculates

the slope of the tangent line. Each student then draws a small slope line segment for the tangent line on each of the functions. We then erase the functions and are able to view our slope field. The slope field handout from the AP Calculus course website is used. Chapter 6 topics include:

1. Slope Fields and Euler's Method
 - a. use initial conditions to find particular solutions of differential equation
 - b. use slope fields to approximate solutions of differential equations
 - c. use Euler's Method to approximate solutions of differential equations
2. Differential Equations: Growth and Decay
 - a. Use separation of variables to solve simple differential equations
 - b. Use exponential functions to model growth and decay in applied problems
3. Separation of Variables and the Logistic Equation
 - a. Recognize and solve differential equations which can be solved by separation of variables
 - b. Recognize and solve homogenous differential equations
 - c. Use differential equations to model and solve applied problems
 - d. Solve and analyze logistic differential equations
4. First-Order Linear Differential Equations
 - a. Solve first-order linear differential equations
 - b. Solve a Bernoulli differential equation
 - c. Use linear differential equations to solve applied problems

In the Chapter 6 Project, students are given a differential equation which models weight loss based on weight, calorie consumption and time. We find the general solution for the differential equation and then work on particular solutions given original conditions of a person's weight and caloric intake. We then graph the solutions and try to ascertain the "limiting" weight of a person. We also determine time required for a desired amount of weight loss. We complete this process for several fictitious people.

G. Chapter 7: Applications of Integration. Students proceed to applications of integration. Before calculating volumes of solids with known cross sections, we work in teams and use Play-Dough, small plastic rulers and "invisible" thread. My students are given the equation of a triangle or semicircle, and are instructed to form a solid such that every cross section which is perpendicular to the base is a semicircle or an equilateral triangle. We use the invisible thread to cut the cross sections to show that the desired shape of the cross sections has been achieved. Teams complete several constructions. This 3-dimensional, "hands-on" exercise gives the students a thorough understanding of the concept which they cannot get from the textbook, even though the textbook images are quite good. Also, the competitive spirit of these students usually emerges in full force during this exercise. Chapter 7 topics include:

1. Area of a Region between Two Curves

- a. Find the area of a region between two curves using integration
 - b. Find the area of a region between intersecting curves using integration
 - c. Describe integration as an accumulation process
2. Volume: Disk Method
- a. Find the volume of a solid of revolution using disk method
 - b. Find the volume of a solid of revolution using the washer method
 - c. Find the volume of a solid with known cross sections

For the Chapter 7 project, students are put into groups of three and each group is given a large plain bagel. We assume that the shape of the bagel and the center hole are concentric circles. The shape of the bagels are assumed to be disks with height equal to one inch. The students may place the disk (bagel) anywhere on the coordinate axes and choose an axis of rotation. Their goal is to decide the equations, which when rotated around an axis, will produce the 3-dimensional shape of the disk (bagel). Once they have their equations and axis of rotation, they use integration to determine the volume of the bagel. Groups also receive one sheet of construction paper. They use the paper to construct a rectangular box without a lid. The volume must be the same as the volume found for the bagel. The bagel is then torn into small pieces and placed in the box. No stretching or squishing of the bagel pieces is permitted. If their integral is correct, the pieces of the bagel should fit into the box perfectly. This is a fun project which the students always enjoy. At this point in the course, they need some fun. During the project, edible bagels are also provided. ☺

H. Post AP Exam. In this after AP Exam topic, students find areas of surfaces of revolution; calculate work done by constant and variable forces; and, study centers of mass. Topics are:

- Arc Length and Surfaces of Revolution
 - Find the Arc Length of a Smooth Curve
 - Find the Area of a Surface of Revolution
 - The F Statistic and F Distributions
- Work
 - Find the Work done by a Constant Force
 - Find the Work done by a Variable Force
- Moments, Centers of Mass, and Centroids
 - Understand the Definition of Mass
 - Find the Center of Mass in a One-Dimensional System
 - Find the Center of Mass in a Two-Dimensional System
 - Find the Center of Mass of a Planar Lamina
 - Use the Theorem of Pappus to find the Volume of a Solid of Revolution

Evaluation: All students are required to participate in the nationwide AP Calculus AB exam. No other Final Exam in the course will be administered. Evaluation is based on homework,

quizzes, chapter tests, projects and the semester examination. Longer chapters will have more than one chapter test.

Text and Supplemental Materials:

1. Larson, Ron, and Hostetler, Robert P. (2006). Calculus of a Single Variable. Eighth Edition. Boston: Houghton-Mifflin
2. Kelley, Michael W. (2002). The Idiot's Guide to Calculus. Indianapolis: Alpha Books
3. TI-83 or TI-83 Plus or TI-83 Silver Graphing Calculator
4. Previously released AP Calculus exam questions
5. Supplementary books and materials
6. Graph paper
7. Mechanical Pencil
8. Colored pencil (not red) or pen with colored ink (not red).
9. Small ruler
10. A pencil pouch is recommended but not required.