

Analyzing Trade

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Analyzing Trade: Chapter 1, Preliminaries to Trade Theory

Antecedents

The antecedents of trade theory are found in the ideas of *mercantilism* and, particularly, in Adam Smith's rejection of these ideas, the latter involving the introduction of the notion of *absolute advantage*. Mercantilism was a prominent school of economic and political thought in the 17th and 18th centuries that developed against a background of European wars. It reflected the imperative of these wars, but it also reflected the concerns of the businessmen who served as its pamphleteers. So whereas Spiegel (1983) depicted mercantilism as "economic warfare for national gain," Dudley Dillard (personal communication) stressed the "confusion of private wealth with social wealth" and the entry of "the point of view of business into politics." Both of these insights are important.

Mercantilism can be reduced down to three central ideas.¹ First, there is a fixed amount of wealth in the world in the form of precious metals, particularly gold. Second, gold is necessary for the political purposes of supporting monarchies and fighting related wars. Third, the means to accumulate gold were to be found in mining and in the pursuit of a (non-official) balance of payments surplus. With regard to the last of these, Mun (1924, orig. 1664) famously stated: "The ordinary means... to encrease our wealth and treasure is by Forraign Trade, wherein we must ever observe this rule; to sell more to strangers than we consume of theirs in value" (p. 171). These elements of the mercantilist ideology fed naturally into colonial pursuits, with disastrous consequences.

Adam Smith's *Wealth of Nations* was a sustained, intellectual assault on the mercantilist ideology. As part of this assault, Smith introduced the notion of *absolute advantage* with the words: "If a foreign country can supply us with a commodity cheaper than we ourselves can make it, better buy it of them with some part of the produce of our own industry, employed in a way in which we have some advantage" (1937, p. 424). While there are a number of possible interpretations of this notion, technological efficiency comparisons across countries in a particular sector is a prominent one.²

¹ A more thorough review is given by Heckscher (1967).

² See van Marrewijk (2008), for example.

The analysis of trade was given an important subsequent boost by David Ricardo and his notion of *comparative advantage*. This appeared in his *Principles of Political Economy and Taxation* (1951, orig.), which was a well-argued attack on Britain's corn laws that proved to be fundamental to the subsequent evolution of economic thinking. In a footnote in Chapter 7 of that book, Ricardo stated: "It will appear... that a country possessing very considerable advantages in machinery and skill, and which may therefore be enabled to manufacture commodities with much less labour than her neighbors, may, in return for such commodities, import a portion of its corn required for its consumption, even if its land were more fertile, and corn could be grown with less labour than in the country from which it was imported." In contrast to absolute advantage, the comparative advantage notion is set out as a comparison across *both* countries *and* goods. Its dimensionality is therefore greater than that of absolute advantage and, as we shall see, this is the source of its power.

Ethics

The mercantilists, Smith and Ricardo all found international trade to be closely related to the primary issues of their era. All had prescriptions for policy as well. Policy prescriptions necessarily involve comparison across potential states of society and, therefore, inevitably require what ethicists call *outcomes assessment*. Trade theory and its applications constitute exercises in outcomes assessment. The question arises, however, as to what measures are used to assess alternative outcomes. In practice, trade economists have embraced what Bhagwati, Panagariya, and Srinivasan (1998) accurately term the "traditional" approach in which the only recognized object of value for outcomes assessment is the utility of consumption levels of commodities and services.³ From the standpoint of ethics, this is a narrow view, and other alternatives are possible and, in some instances, preferable.⁴

³ In practice (e.g. in applied trade policy modeling), there is a further restriction to *legally*-consumed commodities and services, a distinction which, in light of significant international flows in illegal drugs, weaponry, and sexual services, is not as insignificant as one might first believe.

⁴ See Reinert (2005).

This “traditional” approach to outcomes assessment, it must be admitted, evolved based on what Viner (1953) called “methodological preferences,” dictated by ease of analysis and data gathering. Commodities and services are traded, so defining utility over this space is convenient. Viner (1953) also recognized that “too few objectives being recognized as properly operative” is part of a “tendency toward over-simplification” and that he included the “single goal” *utility* in his list of such tendencies (p. 2). It is important to keep in mind that this is what the foundations of trade theory and applied trade policy analysis entail.

Calculus

When the value of a variable depends on the values of other variables, we can express this in functional notation:

$$y = f(x_1, \dots, x_n)$$

where: y is the dependent variable and x_1, \dots, x_n are independent variables.

Consider a function with *one independent variable*:

$$y = f(x)$$

The *derivative* of this function measures its slope at a particular point of value x :

$$\frac{dy}{dx} = f'(x) = \text{slope of } y$$

There are three relevant value ranges for this first derivative:

$$\frac{dy}{dx} = f'(x) > 0 \Rightarrow \quad y \text{ is increasing as } x \text{ increases}$$

$$\frac{dy}{dx} = f'(x) < 0 \Rightarrow \quad y \text{ is decreasing as } x \text{ increases}$$

$$\frac{dy}{dx} = f'(x) = 0 \Rightarrow \quad y \text{ is at a stationary point}$$

Next, consider a function with *more than one independent variable*:

$$y = f(x_1, x_2)$$

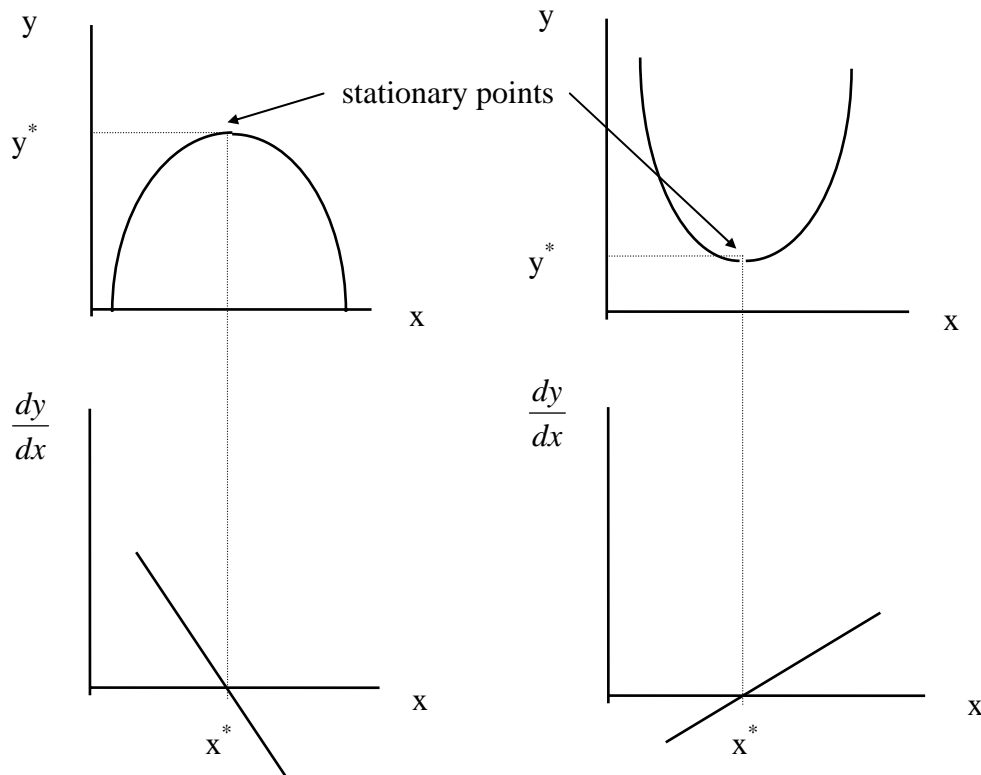
The *partial derivative* is the derivative of the function with respect to *one* independent variable, *holding the other independent variables constant*. For the case of two independent variables, there are two partial derivatives:

$$\frac{\partial y}{\partial x_1} = f_1 = \text{slope of } y \text{ along the } x_1 \text{ dimension}$$

$$\frac{\partial y}{\partial x_2} = f_2 = \text{slope of } y \text{ along the } x_2 \text{ dimension}$$

Derivatives are important when we engage in *optimization*. Let us first consider the case of *unconstrained optimization with functions of one independent variable*. To make things simple, we will confine ourselves to *smooth functions*, that is, to functions without any kinks.

Visually, we have:



Note that superscript “*” generally denotes the optimum value of a variable. For both a maximum and a minimum of a smooth function, the *necessary, first-order condition* is that the function be at a stationary point or that:

$$\frac{dy}{dx} = f'(x) = 0$$

Given that the first order condition holds true, we have the following *sufficient, second-order conditions*:

For a maximum:

$$\frac{d^2y}{dx^2} = f''(x) < 0$$

For a minimum:

$$\frac{d^2y}{dx^2} = f''(x) > 0$$

Next consider *unconstrained optimization with functions of more than one independent variable*. In particular, suppose we have the smooth function:

$$y = f(x_1, \dots, x_n)$$

The *necessary first-order conditions* are:

$$\frac{\partial y}{\partial x_1} = f_1 = 0$$

$$\frac{\partial y}{\partial x_2} = f_2 = 0$$

The intuition here is that these first-order conditions identify the top of a hill or bottom of a valley. That is, we need the surface to be flat no matter what direction we travel. The first-order conditions identify a stationary point. However, the second-order conditions are beyond the level of this book.

Finally, let us consider *constrained optimization*. A typical problem in microeconomics is the following:

$$\text{Maximize: } y = f(x_1, \dots, x_n)$$

$$\text{Subject to: } g(x_1, \dots, x_n) = k$$

Here the x_1, \dots, x_n cannot take on just any value. Intuitively, this is like trying to climb to the top of a hill and then encountering a fence. The way we deal with this problem is with the *Lagrangian multiplier method*. Specifically, we work with the *Lagrangian function*:

$$L = f(x_1, \dots, x_n) + \lambda [k - g(x_1, \dots, x_n)]$$

where λ is an additional variable known as the *Lagrangian multiplier*.

Note that when the constraint holds, $k - g(x_1, \dots, x_n) = 0$ and $L = f(x_1, \dots, x_n)$.

Maximization proceeds by finding the first-order conditions for the Lagrangian:

$$\frac{\partial L}{\partial x_1} = L_1 = f_1 - \lambda g_1 = 0$$

...

$$\frac{\partial L}{\partial x_n} = L_n = f_n - \lambda g_n = 0$$

$$\frac{\partial L}{\partial \lambda} = L_\lambda = k - g(x_1, \dots, x_n) = 0$$

Note that the last first-order condition is none other than our constraint.

These first-order conditions represent $n+1$ equations in $n+1$ unknown variables, namely x_1, \dots, x_n, λ . The solution to this system of equations has the following properties:

1. The x_1^*, \dots, x_n^* satisfy the constraint
2. The x_1^*, \dots, x_n^* maximize the L and (given 1 above) therefore $f(\)$.

Next, let's look at the interpretation of the Lagrangian multiplier. From the first n first-order conditions, we can write:

$$\frac{f_1}{g_1} = \dots = \frac{f_n}{g_n} = \lambda$$

Note that the numerators above represent the marginal benefits of increasing the x_1, \dots, x_n , so we can rewrite the first-order conditions as:

$$\frac{MB_1}{g_1} = \dots = \frac{MB_n}{g_n} = \lambda$$

Now we must deal with the denominators. Totally differentiate the constraint:

$$g_1 dx_1 + \dots + g_n dx_n = 0$$

If $dx_1 = 1$, we then have:

$$g_1 = -g_2 dx_2 \dots - g_n dx_n$$

This tells us that g_i = the *marginal cost* of increasing x_i in terms of the reductions in all the other x 's required by the constraint. Given this, we know that the first-order conditions can be interpreted as:

$$\frac{MB_1}{MC_1} = \dots = \frac{MB_n}{MC_n} = \lambda$$

That is, the Lagrange multiplier gives the ratio of marginal benefit to marginal cost for each independent variable. Constrained maximization requires that all these ratios are equal.

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Analyzing Trade: Chapter 2 Microeconomics

Someone once said that trade theory was “microeconomics with wheels,” and this is not a bad characterization. But to be more specific, trade theory is microeconomics in *general equilibrium*, where markets of different types interact with one another. It is useful, before plunging in, to take a step back and resurrect some basic microeconomics in order to see how its elements can be fit together into a relatively powerful analytical apparatus. This is our purpose in this chapter.

Production Functions

Many explanations of trade hinge on countries’ relative abilities to produce certain commodities and services. Consequently, the theory of the firm is a crucial component of trade theory. Indeed, we can begin our foray into trade theory with the neoclassical *production function*:

$$Q = f(L, K)$$

where L represents units of labor used in production, and K represents units of physical capital or a set of non-labor inputs.

Recall that the important notion of the *marginal product of labor* is defined using a partial derivative as:

$$MP_L(L, K) = f_L(L, K) = \frac{\partial Q}{\partial L}$$

Also recall that this is the slope of the *total product of labor curve*. Ricardo’s notion of the *law of diminishing returns* is reflected in the fact that:

$$\frac{\partial MP_L}{\partial L} = f_{LL} = \frac{\partial^2 Q}{\partial L^2} < 0$$

The marginal product of capital is defined as:

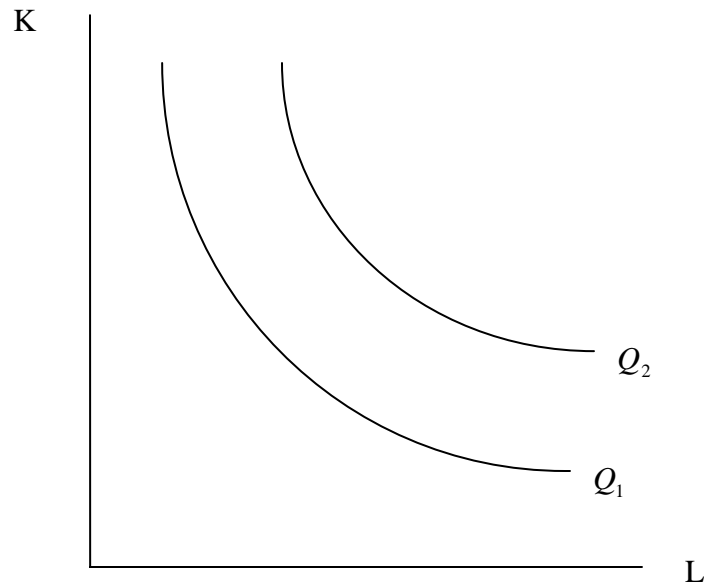
$$MP_K(L, K) = f_K(L, K) = \frac{\partial Q}{\partial K}$$

and the law of diminishing returns holds for it as well.

It is very helpful in understanding trade theory to keep in mind that both of these marginal products are themselves functions of the current inputs of labor and capital.

Consequently, if the inputs of labor or capital change, there will be impacts in the marginal product diagrams, appearing either as a movement along a curve or as a shift of a curve.

The production function is typically represented in two-dimensional space using the isoquant diagram:



In this diagram, the isoquant corresponds to a particular level of output such as Q_1 . This is a cross-section of the production function parallel to the L-K plane at a height of Q_1 above the plane. Movements along the isoquant represent substitution between the labor and capital inputs holding the output level constant. An increase in the level of output would correspond to a jump to a higher isoquant, such as Q_2 .

It can be shown the isoquants have three properties: they are downward sloping; they do not cross; and they are convex to the origin. By “convex to the origin,” we mean that a line segment connecting any two points on the isoquant lies above the isoquant itself.

To understand the isoquant a bit more, it is helpful to totally differentiate the production function:

$$dQ = MP_L dL + MP_K dK$$

We then restrict the firm to a particular isoquant so that:

$$dQ = 0 = MP_L dL + MP_K dK$$

$$MP_K dK = -MP_L dL$$

Cross multiplying gives us:

$$\frac{dK}{dL} = -\frac{MP_L}{MP_K}$$

This is an expression for the slope of isoquant, which is related to the ratio of marginal products. If we multiply this equation through by minus one, we get what is called the marginal rate of technical substitution or *MRTS*:

$$MRTS = -\frac{dK}{dL} = \frac{MP_L}{MP_K}$$

We will use the *MRTS* at a number of junctures in our consideration of trade theory.

An issue related to the spacing of isoquants is that of *returns to scale in production*. Returns to scale are defined in terms of the *homogeneity* of a production function. Let $\Theta > 0$. Then $Q = f(L, K)$ is homogeneous of degree n if

$f(\Theta L, \Theta K) = \Theta^n Q$. If $n = 1$, then the production function is homogeneous of degree 1 and exhibits *constant returns to scale*. For example, if Θ were 2 and $n = 1$, then we would see that doubling all the inputs results in the output level also doubling. In the case of constant returns to scale, a set of isoquants (known as an *isoquant map*) would be spaced such that output is proportional to the distance along a line from the origin of the isoquant diagram.

The cases of $n > 1$ and $n < 1$ correspond to *increasing returns to scale* and *decreasing returns to scale*, respectively. For example, if Θ were 2 and $n > 1$, then we would see that doubling all the inputs results in the output level more than doubling.

Cost Minimization

Microeconomics generally operates under the assumption that firms engage in profit maximization. Profit maximization, in turn, implies cost minimization. In our two input production function, costs come in the form of labor and capital, with the prevailing wage

(w) and rental rate on capital (r) coming into play. For any Q , w , and r , the firm chooses L and K to minimize costs total costs:

$$C = wL + rK$$

The constrained optimization problem is:

$$\text{Minimize: } C = wL + rK$$

$$\text{Subject to: } Q = f(L, K)$$

As discussed in Chapter 1, this requires the Lagrange multiplier method, and the Lagrangian function is:⁵

$$G(L, K, \lambda) = wL + rK + \lambda[Q - f(L, K)]$$

Note that if the constraint holds and $Q = f(L, K)$, then minimizing the Lagrangian function G also minimizes total costs $C = wL + rK$. The first-order, necessary conditions are:

$$\frac{\partial G}{\partial L} = w - \lambda MP_L = 0$$

$$\frac{\partial G}{\partial K} = r - \lambda MP_K = 0$$

$$\frac{\partial G}{\partial \lambda} = Q - f(L, K) = 0$$

Dividing the first equation by the second equation gives us:

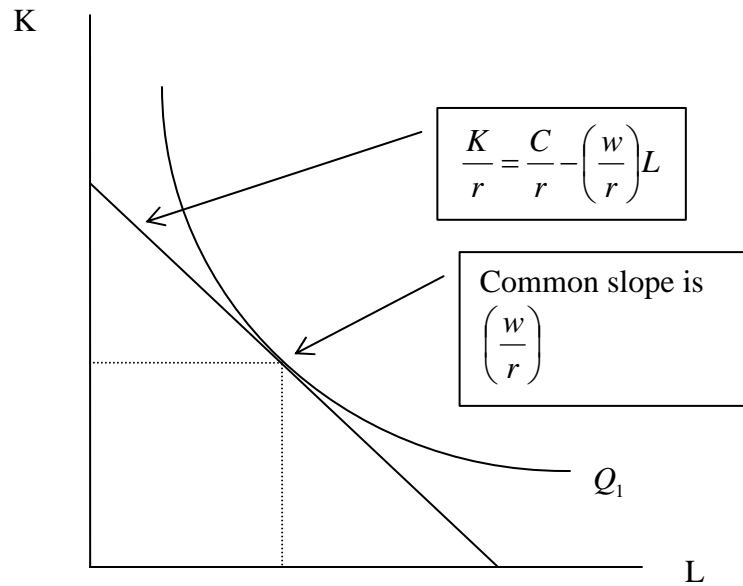
$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

This *cost-minimization condition* can be rewritten in terms of the marginal rate of technical substitution:

$$MRTS = \frac{w}{r}$$

Cost minimization can also be viewed as finding the lowest *isocost line* for any Q , w and r . This is depicted in the following diagram.

⁵ Note that we here use G for the Lagrangian function rather than L as in Chapter 1 to prevent confusion with the labor input.



The “2x2” Model

The above description of production focused on a particular sector, but in trade analysis, we are often interested in multiple sectors at a time. Trade theory usually approaches multiple sectors by considering the case of two sectors. An initial consideration of this gives rise to a two-sector, two-factor model known as the “2x2” model. By combining the two goods markets together and linking them to the two factor markets (labor and capital), we make a leap into general equilibrium analysis. We denote the sectors with subscripts 1 and 2. Production in this model is described in the following four equations:

$$Q_1 = f_1(L_1, K_1)$$

$$Q_2 = f_2(L_2, K_2)$$

$$L_1 + L_2 = L$$

$$K_1 + K_2 = K$$

A few things must be noted here. First, the two production functions are assumed to exhibit constant returns to scale. Second, we have a notational difference with regard

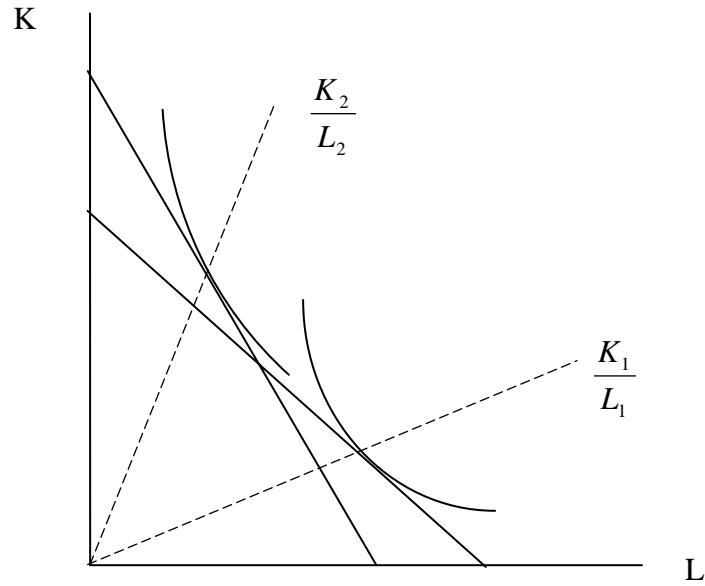
to factors of production. L and K are not the total amounts of factors of production used in the economy. L_i and K_i are the amounts of the factors of production used in sector i .⁶

To make things interesting, we are going to assume that sector 1 is *labor intensive* relative to sector 2 or, to put it the other way, that sector 2 is *capital intensive* relative to sector 1. Note that these *factor intensity* terms (labor intensive, capital intensive) are associated with *sectors* of the economy. This will prove to be important. Think: factor intensity \Leftrightarrow sectors. What do we mean by sector 1 being labor intensive and sector 2 capital intensive? This can be represented as:

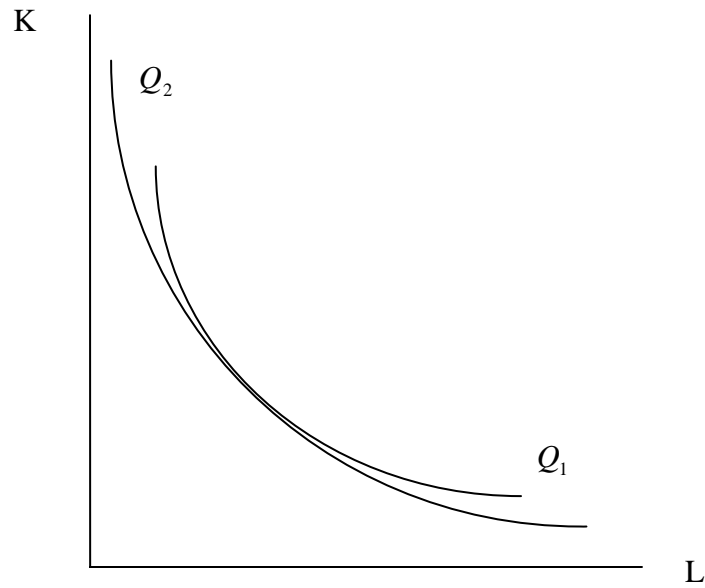
$$\frac{L_1}{K_1} > \frac{L_2}{K_2} \quad \text{or} \quad \frac{K_1}{L_1} < \frac{K_2}{L_2}$$

If these inequalities hold for all possible w/r , then we have what is known as the *strong factor intensity assumption*. When the w/r ratio changes, firms tend to substitute one input for the other. The strong factor intensity assumption states that these substitutions will be such that the inequalities always hold. It is important to be able to visualize labor or capital intensity in the L-K space of the isoquant diagram. This is depicted below for two cost-minimizing points, one for each of our two sectors. As we can see here, the dashed line for sector 2 is steeper than that for sector 1, indicating a greater degree of capital intensity. Why, steeper means being closer to the K axis of the diagram. Similarly, the dashed line for sector 1 is flatter than for sector 2 and is nearer the L axis, indicating a greater degree of labor intensity.

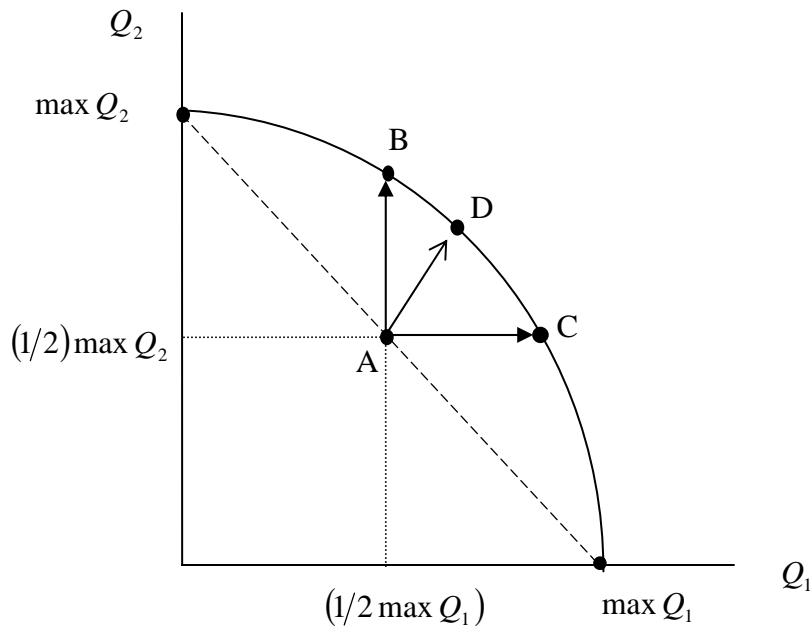
⁶ There is an interpretive issue here that proves to be important. L and K might be the “full” employment levels of labor and capital, or they might just be some constant, initial levels.



There are some instances where the strong factor intensity does not hold. The diagram on the next page, for instance, is one where a factor intensity reversal takes place due differential rates of substitution between the two sectors (higher in sector 2). At high values of w/r , sector 2 is more capital intensive than sector 1, but at low values of w/r , sector two is more labor intensive that sector 1.



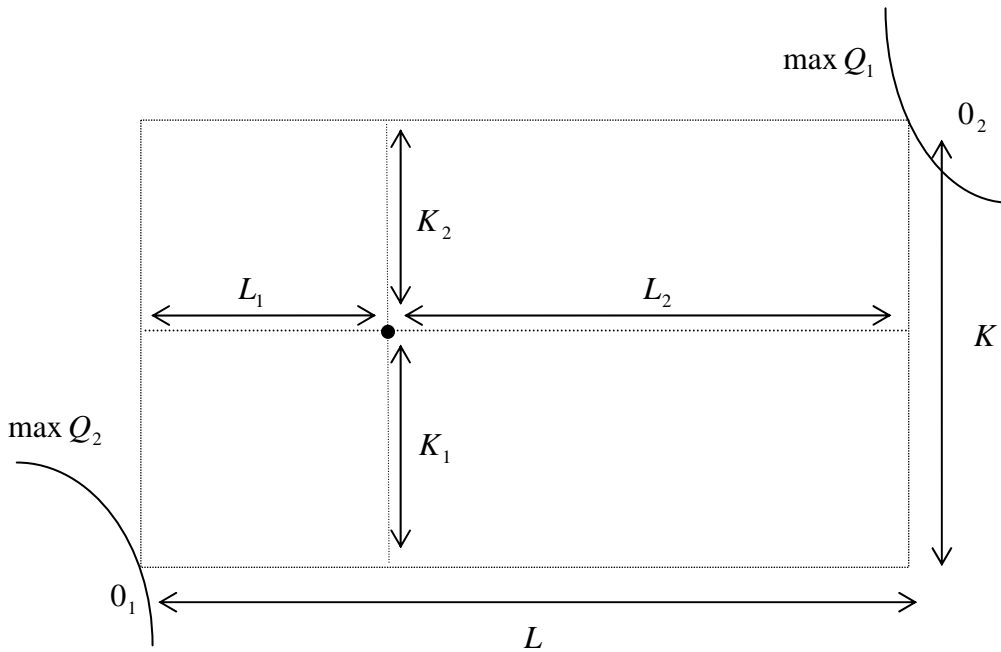
The first order of business in the 2x2 model is to get a sense of the shape of its production possibilities frontier or PPF. Recall the a PPF is a diagram with quantities of output on the axes, in our case, Q_1 and Q_2 . The PPF reflects the endowments of the two resources, L and K , and the technology present in the two production functions. Imagine that we took all the resources available and dedicated them to the production of good 1. This would give us a level of output $Q_1 = \max Q_1$. Similarly, if we took all the resources available and dedicated them to the production of good 2, this would give us a level of output $Q_2 = \max Q_2$. These represent the two endpoints in the PPF diagram, connected with a dashed line.



Markusen et al. (1995) suggest a useful way to assess the shape of the PPF, namely to begin by splitting up the resources evenly between the two sectors. Given that we are working under constant returns to scale or homogeneity of degree 1, the associated levels of output will be $(\frac{1}{2})\max Q_1$ and $(\frac{1}{2})\max Q_2$. This puts us as point A in our PPF diagram. From point A, however, we consider making some changes. We take some labor away from the capital-intensive sector (2) and give it to the labor-intensive sector (1), and we take some capital away from the labor-intensive sector (1) and give it to the capital-intensive sector (2). Each sector is then receiving additional amounts of the resource in which it is intensive. This is a better use of resources and therefore moves us to the northeast of point A. That this movement is possible tells us that the PPF is not linear through points $(\frac{1}{2})\max Q_1$, A, and $(\frac{1}{2})\max Q_2$. Rather, the PPF is *concave* to the origin. That is, a line connecting any two points on the PPF will lie below the PPF.

Due to the differences in factor intensities of sectors 1 and 2, the PPF has the concave shape depicted in the figure above, and the concave shape of the PPF reflects the increasing opportunity costs of increasing the output of one good, opportunity costs being measured in terms of the amount of the other good given up. If the factor intensities of the two sectors were the same, the PPF would be linear.

To better understand the efficiency characteristics lying behind the PPF, we are going to develop the *Edgeworth-Bowley box diagram*.⁷ This diagram is illustrated below. The dimensions of the box are determined by the total endowments of labor and capital in the economy. The lower left-hand corner is an origin for sector 1, and the upper right-hand corner is an origin for sector 2. Any point in this box diagram divides the total labor endowment L into L_1 and L_2 . It also divided the total physical capital endowment K into K_1 and K_2 .

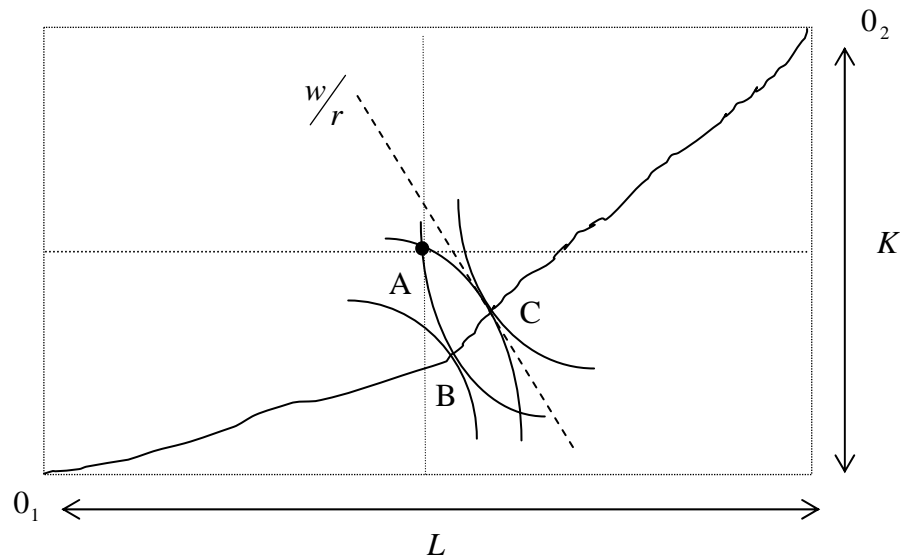


It should be clear that this box diagram, from the point of view of each origin is in the same space as the isoquants we considered earlier. That is, sector 1 isoquants can be plotted in the box diagram with reference to 0_1 , and sector 2 isoquants can be plotted in the box diagram with reference to 0_2 . For example, the diagram includes two isoquants, one for $\max Q_1$ and another for $\max Q_2$.

In order to proceed with our analysis of production efficiency, we will reproduce the box diagram below, drawn initially with point A in mind, namely $(\frac{1}{2})\max Q_1$ and $(\frac{1}{2})\max Q_2$, corresponding to the equal division of resources between two sectors as

⁷ See Humphrey (1996).

suggested by Markusen et al. (1995) as a starting point. From point A, it is possible to move down the sector-2 isoquant, and this initially increases sector-1 output up to a maximum point at C, after which the output of sector 1 begins to fall again. From point A, it is also possible to move down the sector-1 isoquant, and this initially increases sector-2 output up to a maximum point at B, after which the output of sector 2 begins to fall again. Points A, B and C all correspond to the similarly-labeled points in the PPF diagram above. Also, there is a point D that lies between B and C in the box diagram. Points B, C, and D are all *allocatively efficient* because at these points, it is not possible to increase the output of one good (given the resources and technology) without decreasing the output of the other good. Each of these points is characterized by a tangency between the isoquants. Point A, however, is allocatively inefficient, because one can increase the output of good 1 or good 2 without decreasing the output of the other good.



The collection of points along the curve connecting O_1 , B, C and O_2 is a *locus* (locus just means collection of points). Because it is a locus of allocatively efficient points, it is known as a production efficiency locus or PEL. The fact that the PEL is bowed downward in the box diagram reflects the fact that sector 1 is labor intensive and

sector 2 is capital intensive. If the factor intensities were reversed, the PEL would be bowed upward. If the factor intensities were identical in the two sectors, the PEL would be linear. Importantly, *there is a one-to-one correspondence between points along the PPF and points along the PEL*. Each of these diagrams is considering the same thing, but from the perspective of two alternative spaces, quantity space in the case of the PPF and input space in the case of the PEL. The case of the linear PPF corresponds to the case of the linear PEL.

As we have stated, the isoquants of sectors 1 and 2 are tangent along the PEL. Also, we had the condition from cost minimization that:

$$MRTS = \frac{w}{r}$$

In the case of the 2x2 model, this holds for both sectors, so we have:

$$MRTS_1 = MRTS_2 = \frac{w}{r}$$

This is illustrated in the diagram above for the case of point C.

It is worth exploring in a bit more detail the link between the PPF and the factor markets using a bit of differential calculus. To do so, we start with the production functions:

$$Q_1 = f_1(L_1, K_1)$$

$$Q_2 = f_2(L_2, K_2)$$

Totally differentiating these, we have:

$$dQ_1 = MP_{L_1}dL_1 + MP_{K_1}dK_1$$

$$dQ_2 = MP_{L_2}dL_2 + MP_{K_2}dK_2$$

These two equations introduce the first element into the relationship between the PPF and the factor markets, namely *technology*. The second element is *profit maximization under perfect competition*. When we view profit maximization under perfect competition from the point of view of the factor markets, we find that the marginal revenue product of labor must equal the factor price. In the 2x2 model, this implies that:

$$P_i MP_{L_i} = w \quad i = 1, 2$$

$$P_i MP_{Ki} = r \quad i = 1,2$$

Dividing these through by price gives us the “real” version of these profit-maximizing conditions:

$$MP_{Li} = w/P_i \quad i = 1,2$$

$$MP_{Ki} = r/P_i \quad i = 1,2$$

We now substitute these profit maximization conditions into the equations representing technology to obtain:

$$dQ_1 = \frac{w}{P_1} dL_1 + \frac{r}{P_1} dK_1$$

$$dQ_2 = \frac{w}{P_2} dL_2 + \frac{r}{P_2} dK_2$$

We factor out the prices to obtain:

$$dQ_1 = \frac{1}{P_1} (w dL_1 + r dK_1)$$

$$dQ_2 = \frac{1}{P_2} (w dL_2 + r dK_2)$$

Dividing, we see that the slope of the PPF is given by:

$$\frac{dQ_2}{dQ_1} = \frac{P_1 (w dL_2 + r dK_2)}{P_2 (w dL_1 + r dK_1)}$$

Now to the elements of technology and profit maximization under perfect competition, we add the final element of full or constant employment of our two factors of production, labor and capital. For constant total endowments, this implies that $dL_2 = -dL_1$ and that $dK_2 = -dK_1$. Therefore, we can rewrite further as:

$$\frac{dQ_2}{dQ_1} = -\frac{P_1 (w dL_1 + r dK_1)}{P_2 (w dL_1 + r dK_1)} \quad \text{or}$$

$$\frac{dQ_2}{dQ_1} = -\frac{P_1}{P_2}$$

This equation gives us the slope of PPF. Multiplying through by minus 1 gives us:

$$MRT = -\frac{dQ_2}{dQ_1} = \frac{P_2}{P_1}$$

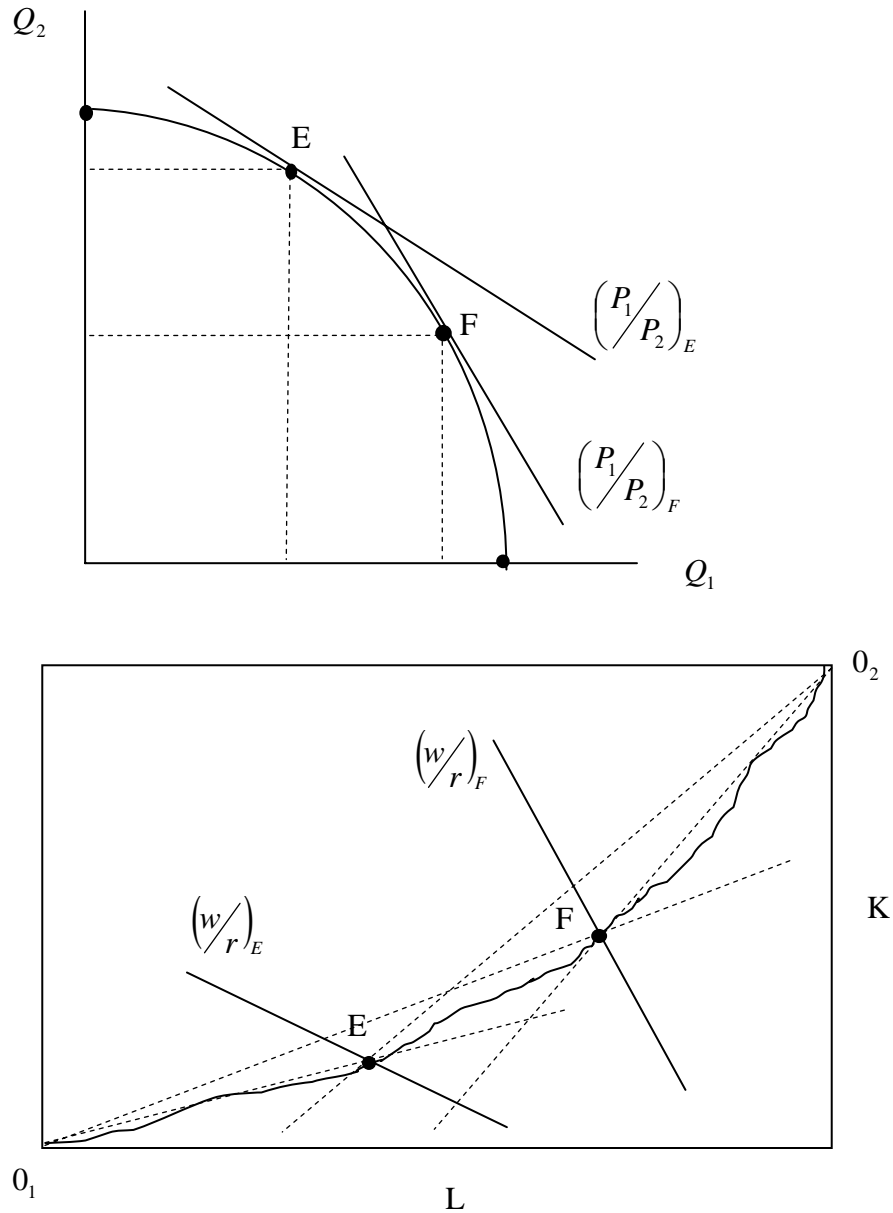
where *MRT* is the *marginal rate of transformation* along the PPF.

In other words, technology, profit maximization under perfect competition and full or constant employment together imply that there is a tangency between the PPF and the goods price line as illustrated below.⁸ This tangency is utilized in a great deal of trade theory and trade policy analysis.

We now have a picture of the supply side of the 2x2 model, allowing for differences in factor intensities in the two sectors. From the goods market perspective, the 2x2 model appears in the PPF diagram with a concave PPF and a tangency with the price line. From the factor market perspective, the model appears in the box diagram. These two characterizations are linked to one another.

It is useful to consider this supply side of the 2x2 model in terms of what occurs when we obtain an increased amount of one of the two goods, say good 1. This is depicted in the following diagrams as a move from points E to points F. Moving along the PPF from E to F involves an increase in Q_1 and, due to the presence of opportunity costs, a decrease in Q_2 . It is important to note what happens to the tangency with the goods price line. Increasing the output of Q_1 requires an increase in the relative price of this good. This is the general equilibrium equivalent of an upward-sloping supply curve that applies when there is only one good. Other than in the peculiar case where there are no opportunity costs of increasing output, an increased incentive is required to effect this increase.

⁸ There is an alternative approach to this result that proves to be important to more advanced treatments of trade theory. Here, the tangency condition is derived using a GDP function, $GDP(L, K, P_1, P_2)$ that is maximized subject to the PPF. See Chapter 1 of Feenstra (2004).



There are also some interesting relative price changes that occur in the box diagram. As the shape of the PEL indicates, the movement from E to F involves an increase in the output of the labor-intensive good, good 1. As we move to higher good-1 isoquants, labor is demanded in greater proportions that it is given up as we move to lower good-2 isoquants. This difference in proportions causes an incipient disequilibrium

in the factor markets that is resolved by an increase in $\left(\frac{w}{r}\right)$. Why does this change in the relative price of labor have this effect? As $\left(\frac{w}{r}\right)$ increases, both sector 1 and sector 2 substitute away from labor and toward physical capital along their isoquants, the combined effects of which is to bring the factor markets back into equilibrium. The dashed lines in the diagram from the two origins give the capital-labor ratios in the two sectors, and these have shifted towards the capital axes as the economy moves from E to F.

Adding the Demand Side to the 2x2 Model

The aspects of the 2x2 model that we have considered up to this point, deal exclusively with the supply side of the economy. For a complete general equilibrium system, we need to also consider the demand side. In standard microeconomic analysis, the demand side is modeled using a utility function. This serves a dual purpose of modeling demand and assessing welfare changes.⁹ The utility function is represented as:

$$U = U(Q_1, Q_2)$$

Recall that the important notion of the *marginal utility* is defined using a partial derivative as:

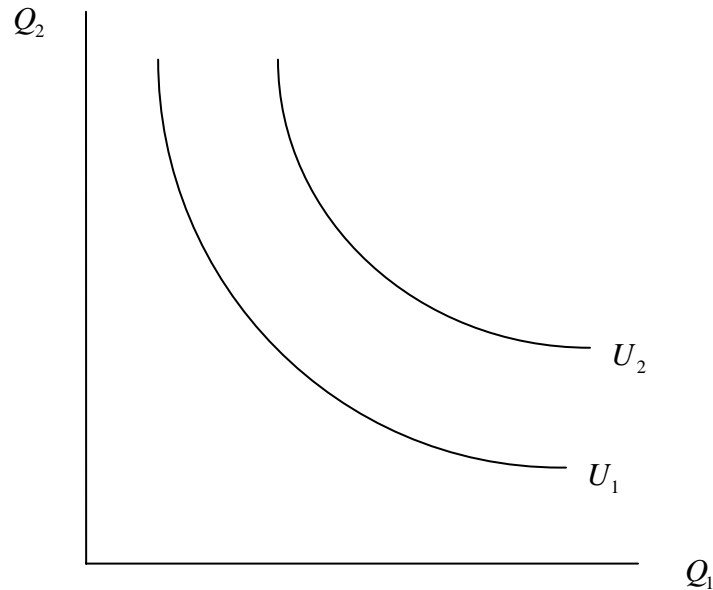
$$MU_i(Q_1, Q_2) = \frac{\partial U}{\partial Q_i} \quad i = 1, 2$$

We represent utility in goods space, the same space as the PPF, using *indifference curves*. In this diagram, the indifference curve corresponds to a particular level of utility. This is a cross-section of the utility function parallel to the $Q_1 - Q_2$ plane at a height of U above the plane. Movements along the indifference curve represent substitution between the goods 1 and 2 in consumption holding the utility level constant. An increase in the level of output would correspond to a jump to a higher indifference curve, such as the movement from U_1 to U_2 .

It can be shown the indifference curves have three properties: they are downward sloping if the two goods are actually “goods” and not “bads”; they do not cross; and they

⁹ The latter function of using the utility concept has been called into question on general terms by Hicks (1969) and in the case of international trade by Reinert (2004).

are convex to the origin. By “convex to the origin,” we mean that a line segment connecting any two points on the indifference curve lies above the indifference curve itself.



To understand the indifference curve concept a bit more, it is helpful to totally differentiate the utility function:

$$dU = MU_1 dQ_1 + MU_2 dQ_2$$

Restrict the household to a particular indifference curve:

$$0 = MU_1 dQ_1 + MU_2 dQ_2$$

Or:

$$MU_2 dQ_2 = -MU_1 dQ_1$$

Cross-multiplying gives us:

$$\frac{dQ_2}{dQ_1} = -\frac{MU_1}{MU_2}$$

This is the slope of the indifference curve in the above diagram. Multiplying through by minus one gives us what is referred to in microeconomics as the *marginal rate of substitution* or MRS:

$$MRS = -\frac{dQ_2}{dQ_1} = \frac{MU_1}{MU_2}$$

The microeconomics of the household proceeds using the formulation of utility maximization subject to a budget constraint. This is shown as follows:

$$\text{Maximize: } U = U(Q_1, Q_2)$$

$$\text{Subject to: } P_1Q_1 + P_2Q_2 = I$$

The Lagrangian function for this constrained optimization problem is:

$$G(Q_1, Q_2, \lambda) = U(Q_1, Q_2) + \lambda(I - P_1Q_1 - P_2Q_2)$$

The first-order, necessary conditions of this Lagrangian function are

$$\frac{\partial G}{\partial Q_1} = MU_1 - \lambda p_1 = 0$$

$$\frac{\partial G}{\partial Q_2} = MU_2 - \lambda p_2 = 0$$

$$\frac{\partial G}{\partial \lambda} = I - P_1Q_1 - P_2Q_2 = 0$$

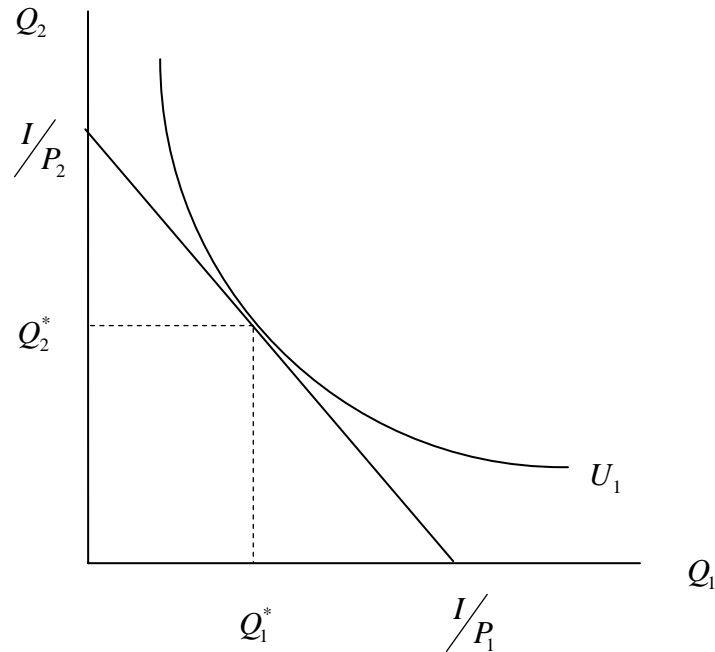
If we divide the first equation by the second equation, we get:

$$\frac{MU_1}{MU_2} = \frac{P_1}{P_2}$$

Or:

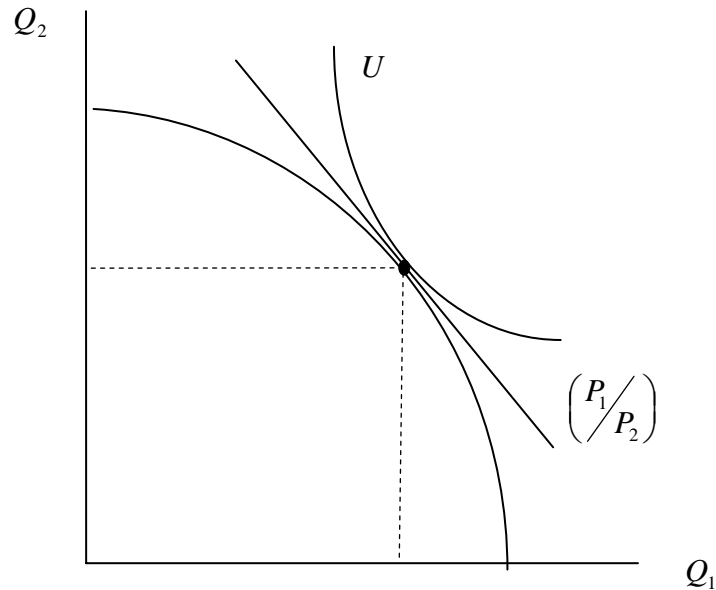
$$MRS = \frac{P_1}{P_2}$$

This is the utility maximization condition for the household, and it can also be interpreted graphically as a tangency condition between an indifference curve and a budget constraint. The household is envisioned here as attempting to reach the highest indifference curve given its budget constraint and thereby determining its optimum levels of consumption for goods 1 and 2.



What does all this mean for our 2x2 economy? The utility maximization process represents the demand side, while the PPF represents the supply side. The two of these interact, and a tangency of an indifference curve and the PPF determine the relative price ratio P_1/P_2 . It is important to recognize that, in general, we work with a single household for the entire economy, known as a *representative household*. Doing this sweeps all distributional issues under an analytical rug. That said, it is possible to redeem ourselves on this issue by disaggregating the household in applied general equilibrium (AGE) models of trade policy analysis as describe in Khan (1997).

The determination of relative goods prices in the 2x2 model is as shown below.



We now have all the pieces to represent the 2x2 model as a system of equations. We need to recall that we have not yet said anything about trade. Indeed, we are still working in a closed economy without any trading relations. For this reason, we will first consider a set of equations for such a closed economy and then consider a similar set of equations for an open, trading economy in the following chapter. In doing this, it will be helpful to introduce a bit of additional notion for our quantities pertaining to our goods, namely Q_1 and Q_2 . We will use an “s” superscript to denote supply-side and a “d” superscript to denote demand-side. Doing so allows us to represent the closed-market or autarky equilibrium using the following ten equations:

$$Q_1^s = f_1(L_1, K_1) \tag{1}$$

$$Q_2^s = f_2(L_2, K_2) \tag{2}$$

$$MRT = \frac{P_1}{P_2} \tag{3}$$

$$MRS = \frac{P_1}{P_2} \tag{4}$$

$$MRTS_1 = \frac{w}{r} \quad (5)$$

$$MRTS_2 = \frac{w}{r} \quad (6)$$

$$L_1 + L_2 = L \quad (7)$$

$$K_1 + K_2 = K \quad (8)$$

$$Q_1^d = Q_1^s \quad (9)$$

$$Q_2^d = Q_2^s \quad (10)$$

A system of ten equations allows us to determine ten endogenous variables. These are P_1/P_2 , w/r , L_1 , L_2 , K_1 , K_2 , Q_1^s , Q_2^s , Q_1^d , and Q_2^d . The total resource endowments L and K are exogenous. Graphically, Equations 1 to 10 describe the closed economy represented by the box diagram and the PPF diagram.

Towards an Open Economy

The above system of equations describes a closed economy that does not engage in any trade relationships. That is hardly our purpose here, so it is time to bring in the rest of the world. The key thing here is that Equations 9 and 10 are no longer relevant when trade becomes possible; domestic demand and domestic supply can diverge in the presence of imports and exports. That is, it is possible that $Q_1^d \neq Q_1^s$ and $Q_2^d \neq Q_2^s$. Instead, what must be true is that *the value of imports in world prices equals the value of exports in world prices*. This is the *trade balance constraint*, and it can be brought into the model in three different ways.

First, there is the general or excess demand form:

$$P_1^W (Q_1^d - Q_1^s) + P_2^W (Q_2^d - Q_2^s) = 0$$

Second, there is the trade balance form:

$$P_1^W (Q_1^d - Q_1^s) = P_2^W (Q_2^s - Q_2^d)$$

Or:

$$P_1^W Z_1 = P_2^W E_2$$

Third, there is the income-expenditure form:

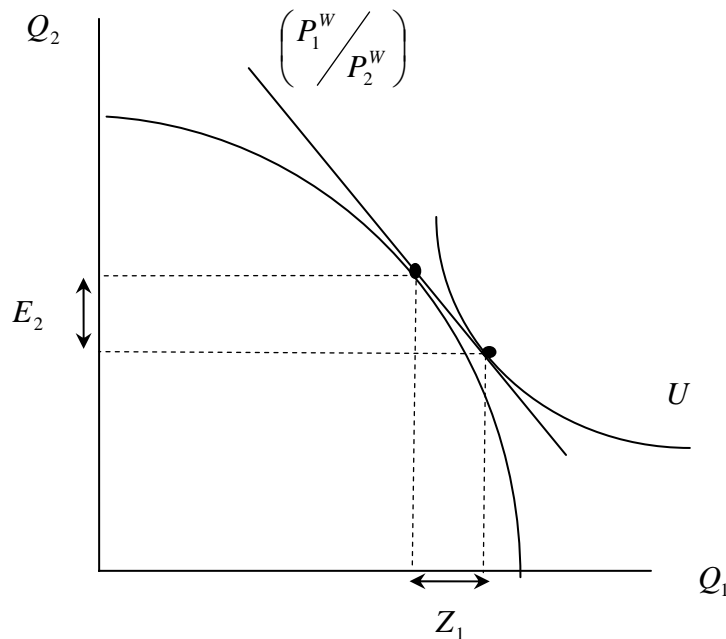
$$P_1^W Q_1^s + P_2^W Q_2^s = P_1^W Q_1^d + P_2^W Q_2^d$$

Each of these is equivalent. Note that, comparing the second and third forms, we see that trade balance is another way of saying that there is no (dis)savings. The use of the no (dis)savings assumption by trade theory is one characteristic that often distinguishes it from open-economy macroeconomics and international finance where divergences in income and expenditures among the countries of the world drive both trade imbalances and global capital flows.

As we move to an open economy, we reduce Equations 1-10 above to a set of only nine. Equations 1 through 8 remain the same, and Equation 9 becomes:

$$P_1^W (Q_1^d - Q_1^s) = P_2^W (Q_2^s - Q_2^d) \quad (9b)$$

We now have a system of nine equations allows us to determine nine endogenous variables: w/r , L_1 , L_2 , K_1 , K_2 , Q_1^s , Q_2^s , Q_1^d , and Q_2^d . This new relationship modifies our PPF diagram so that consumption and production still occur along a single price line (the world price line) but not necessarily at the same point.

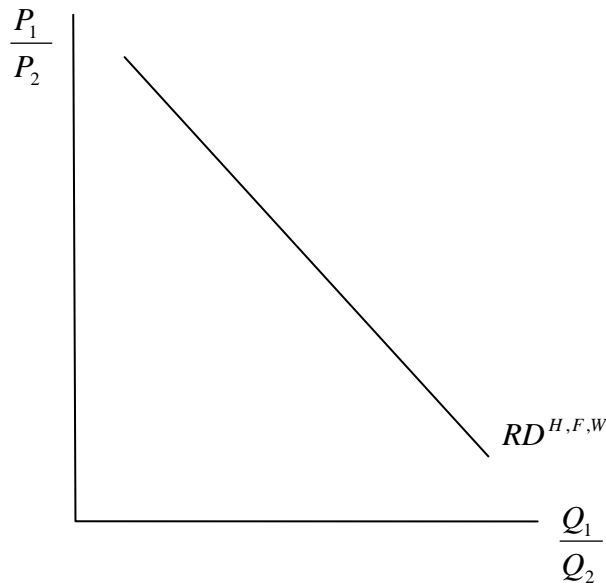


Home and Foreign: A 2x2x2 Model

Our final task here is to explicitly account for the presence of two countries in a two-country, two-sector, two-resource model or a 2x2x2 model. Our two countries will be

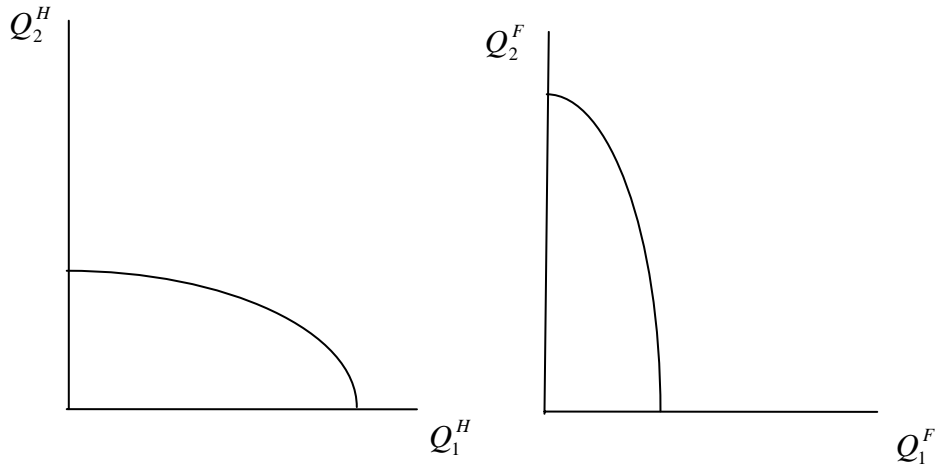
Home (H) and Foreign (F). The purpose of this is to identify $\left(\frac{P_1^W}{P_2^W}\right)$ so that it is an endogenous rather than an exogenous variable as above in Equations 9b. More

specifically, we are going to determine $\left(\frac{P_1^W}{P_2^W}\right)$ in a *relative supply/relative demand diagram*. In order to simplify things and focus on the supply side of the Home and Foreign countries, we will assume that both countries have identical tastes and preferences. Then the relative demand curve for Home, Foreign and the World is the following:



The downward slope of $RD^{H,F,W}$ reflects the fact that, as $\frac{P_1}{P_2}$ increases, demand shifts away from good 1 and towards good 2. This is just a reflection of substitution effects.

Next, we are going to assume that Home's PPF is biased towards good 1 and that Foreign's PPF is biased towards good 2. *We will not yet say why this could be the case.* That we will leave for subsequent chapters. For now, these relative shapes are only by assumption as a way of getting started.



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