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Robust Bias Mitigation Algorithm for Localization in Wireless Networks

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Abstract— Non-line-of-sight (NLOS) signal propagation is a major issue in location estimation. Several application oriented approaches have been proposed in recent years to mitigate the errors induced by the biased distance estimates from NLOS propagation. In this paper, we present a new algorithm which reduces the adverse effects of bias by utilizing the topological diversity provided by L unique beacon combinations from the power set of $N(>3)$ beacons. The algorithm consists of two complementary weighted average techniques combined to provide high level of robustness. Our simulation and empirical work shows that the proposed algorithm is effective independent of the bias distributions. Thus, it can be implemented over a variety of existing localization methods in wireless networks to mitigate the adverse effects of NLOS propagation and achieve high accuracy in practical scenarios.

I. INTRODUCTION

Localization of a wireless device or a sensor node has been a problem of great importance in recent years. Location services are becoming an essential part of the traditional wireless networks as well as the wireless sensor networks. Indoor localization in particular has been an issue of interest in recent studies since GPS is not suitable for indoor environments. The localization approaches can be classified based on the received signal metric used to estimate the distance between the fixed nodes (beacons) and the target node. These signal metrics include time of arrival (TOA), time difference of arrival (TDOA) and received signal strength (RSS). Once these distances are estimated, the target location can be computed with various methods, such as trilateration.

The main issue in target node location estimation is the distance estimates affected by the non-line-of-sight (NLOS) signal measurements. In any practical setting, the NLOS errors are inevitable and need to be mitigated in order to achieve acceptable accuracy. The NLOS error component in the distance estimate is often termed as "bias" and is assumed to be a positive quantity in virtually all scenarios. Several methods have been proposed to model and mitigate the effect of bias. The bias can be modeled with different probability distributions such as Exponential [1], Uniform [2] and Gaussian [3]. The effect of bias on the location estimate is arbitrary and depends on multiple factors, such as topology of the beacons, location of the target node as well as the localization technique. One class of bias mitigation techniques includes identification and rejection of the NLOS distance estimates [4] [5]. A major drawback of this approach is the requirement of at least three LOS or unbiased measurements, which may not be possible in many practical cases. Other methods of NLOS error mitigation techniques include addition of a correction factor [6], linear programming [3] and weighted average [1]. Most of these approaches are dependent on the underlying assumption for the bias distribution and thus they are only effective for specific applications.

In this paper, we propose two complementary weighted average methods, in order to mitigate the NLOS errors in a wide range of scenarios. We assume that number of available beacons is four or more. The weighted averages are performed on the location estimates from different beacon combinations which provide topological diversity. First, we propose the Inverse Estimator Weighted Average (IEWA) method, which is similar to Residual weighting algorithm (Rwgh) proposed in [1], with key differences in the error estimation technique. Then we propose another method called the Null Space Weighted Average (NSWA), which works complementary to the IEWA method. Jointly, these two methods provide a highly robust algorithm, and through simulation results we show that the proposed algorithm improves the accuracy of location estimate independent of bias distributions. We also present the experimental results to test our algorithm in a practical scenario. Even though the actual testing is performed using RSS measurements in a wireless network, the algorithm is applicable to other localization techniques such as TOA, and other classes of networks such as wireless sensor networks.

II. SYSTEM MODEL

Consider a wireless network with N beacons. Let $\hat{\boldsymbol{d}}$ and $\boldsymbol{d}$ be the estimated and actual distance vectors from these beacons, respectively. Then

$$\hat{\boldsymbol{d}} = \boldsymbol{d} + \boldsymbol{e} \qquad (1)$$

In an ideal environment, error $\boldsymbol{e}$ would be a zero vector. However, in the presence of error, the following system of inconsistent equations is obtained:

$$(x - x_i)^2 + (y - y_i)^2 = \hat{d}_i^2, \quad i = 1,2 \ldots, N$$

where (x_i, y_i) are the coordinates of the ith beacon, (x, y) is the location of the target node, and $\hat{d}_i \in \hat{\boldsymbol{d}}$. The location of the target can be estimated from these equations using non-linear optimization techniques. Alternatively, the equations can be transformed into a linear system of equations by subtracting the rth equation from the remaining $N-1$ equations. The geometric analysis of this technique is given in [7].

For $i = 1,2 \dots, N$ and $i \neq r$, we can write

$$\begin{bmatrix} x_1 - x_r & y_1 - y_r \\ x_2 - x_r & y_2 - y_r \\ \vdots & \\ x_N - x_r & y_N - y_r \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \hat{d}_r^2 - \hat{d}_1^2 - (x_r^2 + y_r^2) + (x_1^2 + y_1^2) \\ \hat{d}_r^2 - \hat{d}_2^2 - (x_r^2 + y_r^2) + (x_2^2 + y_2^2) \\ \vdots \\ \hat{d}_r^2 - \hat{d}_N^2 - (x_r^2 + y_r^2) + (x_N^2 + y_N^2) \end{bmatrix}$$

which is in the form of **Ax** = ½ **B** and can be solved using the Linear Least Squares (LLSQ) method.

$$\hat{\boldsymbol{x}} = \frac{1}{2}\,(\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{A})^{-1}\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{B} \tag{2}$$

The distance errors are embedded in $\boldsymbol{B}$, and they are commonly modeled as follows:

$$e_i = n_i + b_i \tag{3}$$

where noise $n_i \sim N(0, \sigma^2)$ and b_i is the bias in the NLOS distance estimate, and it can be modeled by various probability distributions such as Exponential, Uniform or Gaussian. We will assess the performance of our algorithm in reducing the effect of bias for these three distributions in Section VII.

III. Effect of Bias on Location Error

The effect of bias on the accuracy of location estimate is dependent on several factors. Reference [8] provides a set of closed-form expressions for the mean square error (MSE) in the location estimate calculated using the LLSQ method. We analyze this set of expressions to understand the effect of bias on location error for different beacon combinations. The set up for the analysis is as follows: Beacons ($B_i, i = 1{:}4$) are located at [0, 0], [40, 0], [40, 40] and [0, 40]. Since $N = 4$, there are five possible subsets with three or more beacons. The path of the target node is along the diagonal from B_1 to B_3 with an increment of one unit in both X and Y directions, and its position is denoted by the location index. The noise component is normally distributed as $\mathcal{N}(0, 1)$. For Fig. 1a and Fig. 1b bias is 2, and for Fig. 1c it is uniformly distributed in [0, 2]. In Fig. 1a only B_1 provides a biased distance estimate. Fig. 1b shows the effect of two biased estimates, and Fig. 1c shows when all estimates are biased with a uniformly distributed random value. Fig. 2 shows the effect of different bias configurations on location error for a particular beacon combination. (Bias is 2 for each beacon in the configuration.)

The observations made from these figures are as follows:

1. The effect of bias on MSE of location estimate is highly dependent on the target location. An exclusive use of the combination with all beacons may not provide the most accurate location estimate.
2. The topology of beacons plays an important role. It is possible to have a combination with high amounts of bias in the distance estimates and still provide better location estimate than a combination with relatively small bias.
3. Lastly, as a corollary of the first two observations, it can be seen that symmetry can help reduce the effect of bias. Fig. 2 shows that the MSE in the case of symmetrically biased beacons (B_1, B_3) is lower than the asymmetric case (B_1, B_4). Interestingly, at some locations, it is even lower than the MSE in the case of the single biased beacon (B_1) combination.

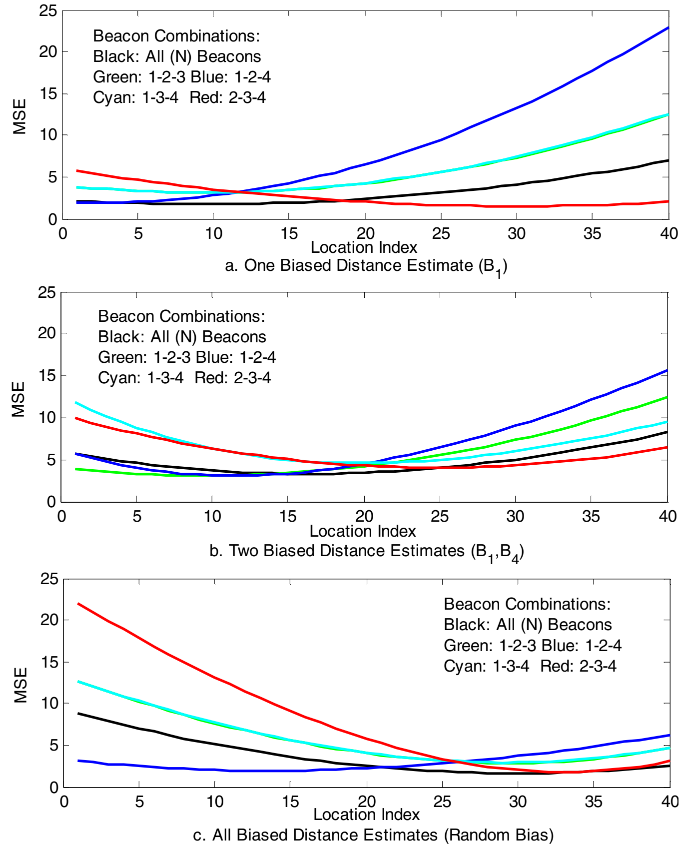

Fig. 1. MSE Comparison of Location Estimates from Various Beacon Combinations in the Presence of Bias in Distance Estimates

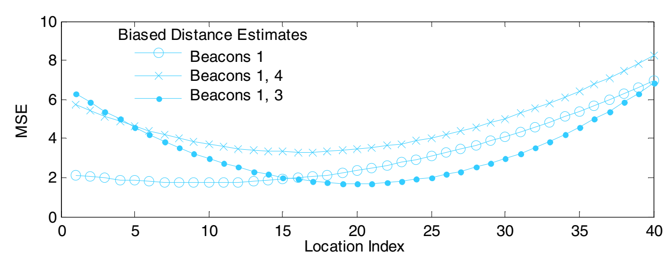

Fig. 2. MSE Comparison for a Particular Beacon Combination with Various Bias Configurations

Based on these observations we define an estimator that is dependent on the beacon topology and captures the effect of symmetry. This estimator is described in the next section.

IV. Location Error Estimator

Let $\hat{\boldsymbol{x}}$ be the target location estimate from some combination of M beacons. Then we define a vector estimator

$$\boldsymbol{\Gamma} = \sum_{j=1}^{M} \frac{\hat{d}_j - \left\|\hat{\boldsymbol{x}} - \boldsymbol{x}_j\right\|}{\left\|\hat{\boldsymbol{x}} - \boldsymbol{x}_j\right\|} \left(\hat{\boldsymbol{x}} - \boldsymbol{x}_j\right) \tag{4}$$

where $\boldsymbol{x}_j$ is the jth beacon location for j = 1,2…,M; and $\hat{d}_j$ is the distance estimate from the jth beacon, obtained from signal measurements such as RSS or TOA. Now we can write the location error expression in terms of $\boldsymbol{\Gamma}$.

Denote

$$r_j = \frac{\hat{d}_j - \|\hat{\boldsymbol{x}} - \boldsymbol{x_j}\|}{\|\hat{\boldsymbol{x}} - \boldsymbol{x_j}\|}$$

and substitute $\hat{\boldsymbol{x}} = \boldsymbol{x} + \boldsymbol{\xi}$. Then

$$\boldsymbol{\Gamma} = \sum_{j=1}^{M} r_j\,(\boldsymbol{x} + \boldsymbol{\xi} - \boldsymbol{x_j})$$

$$\boldsymbol{\xi} = \frac{\boldsymbol{\Gamma} + \sum_{j=1}^{M} r_j\,(\boldsymbol{x_j} - \boldsymbol{x})}{\sum_{j=1}^{M} r_j}$$

$$\boldsymbol{\xi} = a\boldsymbol{\Gamma} + \boldsymbol{G} \qquad (5)$$

where $\boldsymbol{\xi}$ is the error in the location estimate, and

$a = \frac{1}{\sum_{j=1}^{M} r_j}$

$\boldsymbol{G} = \frac{\sum_{j=1}^{M} r_j(x_j - x)}{\sum_{j=1}^{M} r_j}$,

Now for any given beacon combination, location estimate error can be decomposed into two vector components, $a\boldsymbol{\Gamma}$ and $\boldsymbol{G}$. Note that $\boldsymbol{G}$ is a weighted vector sum of the actual distances from beacons to target node. We assume the x and y components of $\boldsymbol{G}$ for any beacon combination are as follows:

$$G_{i_x} \sim \mathcal{N}\left(\mu_{Gi_x}, \sigma^2\right) \text{ and } G_{i_y} \sim \mathcal{N}\left(\mu_{Gi_y}, \sigma^2\right) \qquad (6)$$

for the ith combination ($i = 1,2,\ldots,L$).

V. Analysis of Weighted Average Techniques

A. Computation of Initial Location Estimations

Let S be the set of N beacons. Then the set of all possible combinations of N beacons is the power set of S. The number of total combinations is given by:

$$|\wp(S)| = 2^N$$

where $\wp(S)$ is the power set of S. However, since we are considering combinations of three or more beacons, the resultant set is the subset of the power set, and the number of elements in this subset is given by:

$$L = |\wp(S)| - \binom{N}{2} - \binom{N}{1} - \binom{N}{0} = 2^N - \frac{N^2 + N + 2}{2}$$

Hence L increases exponentially as a function of N. For N beacons we have L unique combinations and each of them provides one raw location estimate using LLSQ method. The final location is computed based on a weighted average of these raw estimates. We will describe the two methods of determining the weights in Section VI. But first, we provide an analysis of the error in the final location estimate computed using the weighted average method.

B. Error in the Location Estimation using Weighted Average

Let $\tilde{\boldsymbol{x}}$ be the location estimate from a weighted average of L initial location estimates. Then $\tilde{\boldsymbol{x}}$ can be written as:

$$\tilde{\boldsymbol{x}} = \sum_{i=1}^{L} w_i\,\hat{\boldsymbol{x}}_{\boldsymbol{i}} \qquad (7)$$

where w_i is the weight of the ith combination, and $\sum w_i = 1$. Then the error vector for the final location can be computed as follows:

$$\tilde{\boldsymbol{\xi}} = \tilde{\boldsymbol{x}} - \boldsymbol{x} = \textstyle\sum_{i=1}^{L} w_i\,\hat{\boldsymbol{x}}_{\boldsymbol{i}} - \sum_{i=1}^{L} w_i\,\boldsymbol{x}$$

$$\tilde{\boldsymbol{\xi}} = \textstyle\sum_{i=1}^{L} w_i\,\boldsymbol{\xi_i}$$

From (5)

$$\boldsymbol{\xi_i} = a_i\boldsymbol{\Gamma_i} + \boldsymbol{G_i}$$

Therefore from (6)

$$\boldsymbol{\xi_i} \sim \mathcal{N}(a_i\boldsymbol{\Gamma_i} + \boldsymbol{\mu_{Gi}}, \boldsymbol{\Sigma}) \qquad (8)$$

$$\tilde{\boldsymbol{\xi}} \sim \mathcal{N}\left(\textstyle\sum_{i=1}^{L} w_i\,(a_i\boldsymbol{\Gamma_i} + \boldsymbol{\mu_{Gi}}), \tilde{\boldsymbol{\Sigma}}\right) \qquad (9)$$

where the parameters are as follows:
$\boldsymbol{\mu_{Gi}} = [\mu_{Gi_x}, \mu_{Gi_y}]$
$\tilde{\sigma}^2 = \sum_{i=1}^{L} {w_i}^2\,\sigma^2 < \sum_{i=1}^{L} w_i\,\sigma^2 = \sigma^2$
(assuming $\boldsymbol{\xi_i}$ are independent)
and $\boldsymbol{\Sigma}, \tilde{\boldsymbol{\Sigma}}$ are Covariance Matrix of $\boldsymbol{\xi_i}, \tilde{\boldsymbol{\xi}}$ respectively.

Consequently, $\|\boldsymbol{\xi_i}\|$ and $\|\tilde{\boldsymbol{\xi}}\|$ follow a Rician distribution with parameters (v_i, σ) and $(\tilde{v}, \tilde{\sigma})$, respectively; where v_i and $\tilde{v}$ are given by:

$$v_i = \|a_i\boldsymbol{\Gamma_i} + \boldsymbol{\mu_{Gi}}\|$$

$$\tilde{v} = \|\textstyle\sum_{i=1}^{L} w_i\,(a_i\boldsymbol{\Gamma_i} + \boldsymbol{\mu_{Gi}})\|$$

Now

$$E\{\|\tilde{\boldsymbol{\xi}}\|\} < E\{\|\boldsymbol{\xi_i}\|\}$$

if

$$\tilde{\sigma}\sqrt{\frac{\pi}{2}}\,L_{1/2}\left(-\frac{\tilde{v}^2}{2\tilde{\sigma}^2}\right) < \sigma\sqrt{\frac{\pi}{2}}\,L_{1/2}\left(-\frac{{v_i}^2}{2\sigma^2}\right)$$

where $L_{1/2}(t) = e^{t/2}[(1-t)I_0\left(-\frac{t}{2}\right) - t\,I_1\left(-\frac{t}{2}\right)$ and I_0, I_1 denote the Bessel Functions.
Let

$$\alpha = \frac{v_i}{\tilde{v}}, \beta = \frac{\sigma^2}{\tilde{\sigma}^2} \text{ and } \gamma = \frac{E\{\|\boldsymbol{e_i}\|\}}{E\{\|\tilde{\boldsymbol{e}}\|\}}$$

Then Fig. 3 shows the interrelationship between these three parameters. Note that $\gamma > 1$ implies that there is an improvement in the location accuracy. Since $\tilde{\sigma}^2 < \sigma^2$, therefore $\beta > 1$. Thus the only crucial criterion for the improvement is $\alpha \geq 1$. Consequently, the optimal solution is achieved when $\tilde{v} = 0$. In that case, $\|\tilde{\boldsymbol{\xi}}\|$ reduces to a Rayleigh distribution, which provides optimum results as seen in Fig. 3.

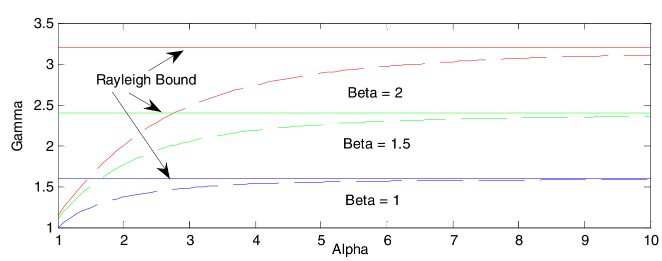

Fig. 3. Accuracy Improvement vs. Alpha

VI. Proposed Algorithm

A. Inverse Estimator Weighted Average (IEWA)

The first weighted average technique is a heuristic method based on the assumption that $\mathbf{\Gamma_i}$ is a good estimator of the error. Therefore, the weights are assigned using the following inverse relation solely based on $\mathbf{\Gamma_i}$:

$$w_i^{IE} = \frac{1/\|\mathbf{\Gamma_i}\|}{\sum_{i=1}^{L} 1/\|\mathbf{\Gamma_i}\|} \qquad (10)$$

Thus,

$$\tilde{\boldsymbol{x}}_{IE} = \sum_{i=1}^{L} w_i^{IE}\, \hat{\boldsymbol{x}}_{\boldsymbol{i}}$$

and

$$\tilde{v}_{IE} = \left\| \frac{1}{NF}\sum_{i=1}^{L} a_i \mathbf{u_i} + \sum_{i=1}^{L} w_i^{IE}\, \boldsymbol{\mu_{Gi}} \right\| = \|\tilde{\mathbf{\Gamma}} + \tilde{\mathbf{G}}\|$$

where the parameters are as follows:

$\mathbf{u_i}$: Unit vector in the direction of $\mathbf{\Gamma_i}$

$NF = \sum_{i=1}^{L} 1/\|\mathbf{\Gamma_i}\|$

$\tilde{\mathbf{\Gamma}} = \frac{1}{NF}\sum_{i=1}^{L} a_i \mathbf{u_i}$

$\tilde{\mathbf{G}} = \sum_{i=1}^{L} w_i^{IE}\, \boldsymbol{\mu_{Gi}}$

$\|\tilde{\mathbf{\Gamma}}\|$ is a known parameter and it is bounded. The upper bound becomes very small as $L \to \infty$. In contrast, $\|\tilde{\mathbf{G}}\|$ is an unknown parameter and for higher values of $\|\tilde{\mathbf{G}}\|$, the IEWA performance degrades. Note that for a particular location, the value of $\tilde{\mathbf{G}}$ is dependent on the weights and can be written as follows:

$$\tilde{\mathbf{G}} = \sum_{i=1}^{L} w_i^{IE}\, \boldsymbol{\mu_{Gi}} = \left[\langle \boldsymbol{\omega^{IE}}, \boldsymbol{\mu_{G_x}} \rangle, \langle \boldsymbol{\omega^{IE}}, \boldsymbol{\mu_{G_y}} \rangle\right] \qquad (11)$$

where the parameters are as follows:

$\boldsymbol{\omega^{IE}} = [w_1^{IE}, w_2^{IE}, \dots, w_L^{IE}]^{\boldsymbol{T}}$

$\boldsymbol{\mu_{G_x}} = [\mu_{G1_x}, \mu_{G2_x}, \dots, \mu_{GL_x}]^{\boldsymbol{T}}$

$\boldsymbol{\mu_{G_y}} = [\mu_{G1_y}, \mu_{G2_y}, \dots, \mu_{GL_y}]^{\boldsymbol{T}}$

$\langle .,. \rangle$: *Inner Product Operator*

Therefore $\|\tilde{\mathbf{G}}\|$ and consequently $\tilde{v}_{IE}$ increase as $\boldsymbol{\mu_{G_x}}, \boldsymbol{\mu_{G_y}}$ become co-linear with $\boldsymbol{\omega^{IE}}$. To mitigate this issue we propose another weighted average method which will act complementary to the IEWA technique, and reduce the arbitrary adverse effects of unknown parameters $\boldsymbol{\mu_{G_x}}, \boldsymbol{\mu_{G_y}}$.

B. Null Space Weighted Average (NSWA)

As explained in Section V, for the maximum improvement ($i.e.\ \min E\{\|\tilde{\boldsymbol{\xi}}\|\}$):

$$\tilde{v} = \|\textstyle\sum_{i=1}^{L} w_i\,(a_i \mathbf{\Gamma_i} + \boldsymbol{\mu_{Gi}})\| = 0$$

$$\Rightarrow \quad \boldsymbol{A\omega^{NS}} = \mathbf{0}$$

where $\boldsymbol{A}$ and $\boldsymbol{\omega^{NS}}$ are given by:

$\boldsymbol{A} = [a_1\mathbf{\Gamma_1} + \boldsymbol{\mu_{G1}} \quad a_2\mathbf{\Gamma_2} + \boldsymbol{\mu_{G2}} \quad \dots \quad a_L\mathbf{\Gamma_L} + \boldsymbol{\mu_{GL}}]$

$\boldsymbol{\omega^{NS}} = [w_1^{NS}, w_2^{NS}, \dots, w_L^{NS}]^{\boldsymbol{T}}$

Theoretically, this can be achieved by selecting $\boldsymbol{\omega^{NS}}$ from the null space of $\boldsymbol{A}$. However since $\boldsymbol{\mu_G}$ is unknown, we use the null space of $\boldsymbol{A_D} = [a_1\mathbf{\Gamma_1} \quad a_2\mathbf{\Gamma_2} \quad \dots \quad a_L\mathbf{\Gamma_L}]$ to compute $\boldsymbol{\omega}$. In addition, to compensate the adverse effects as $\boldsymbol{\mu_{G_x}}, \boldsymbol{\mu_{G_y}}$ become co-linear with $\boldsymbol{\omega^{IE}}$, we propose the following weight selection method.

Let $\mathbf{\Omega}$ and $\mathbf{\Psi}$ denote the null spaces of $\boldsymbol{A_D}$ and $\boldsymbol{\omega^{IE}}$, respectively. Then select $\boldsymbol{\omega^{NS}}$ such that

$$\boldsymbol{\omega^{NS}} \in \mathbf{\Omega} \cap \mathbf{\Psi} \qquad (12)$$

Thus,

$$\tilde{\boldsymbol{x}}_{NS} = \sum_{i=1}^{L} w_i^{NS}\, \hat{\boldsymbol{x}}_{\boldsymbol{i}}$$

This selection process has two important advantages. First, by choosing $\boldsymbol{\omega^{NS}}$ from the null space of $\boldsymbol{A_D}$, we force $\sum_{i=1}^{L} w_i^{NS}\, a_i \mathbf{\Gamma_i}$ to be zero. Therefore,

$$\tilde{v}_{NS} = \|\textstyle\sum_{i=1}^{L} w_i^{NS}\, \boldsymbol{\mu_{Gi}}\| = \left\|\langle \boldsymbol{\omega^{NS}}, \boldsymbol{\mu_{G_x}} \rangle, \langle \boldsymbol{\omega^{NS}}, \boldsymbol{\mu_{G_y}} \rangle\right\|$$

In addition, since $\boldsymbol{\omega^{NS}}$ is also from the null space of $\boldsymbol{\omega^{IE}}$, NSWA works complementary to IEWA in the following sense. For particular $\boldsymbol{\mu_{G_x}}, \boldsymbol{\mu_{G_y}}$, if

$$\langle \boldsymbol{\omega^{IE}}, \boldsymbol{\mu_{G_x}} \rangle, \langle \boldsymbol{\omega^{IE}}, \boldsymbol{\mu_{G_y}} \rangle \to \max \qquad (\because \textit{colinearity})$$

$$\Rightarrow \quad \langle \boldsymbol{\omega^{NS}}, \boldsymbol{\mu_{G_x}} \rangle, \langle \boldsymbol{\omega^{NS}}, \boldsymbol{\mu_{G_y}} \rangle \to 0 \qquad (\because \langle \boldsymbol{\omega^{NS}}, \boldsymbol{\omega^{IE}} \rangle \to 0)$$

And for obvious reasons the converse holds true as well. Thus, as $\tilde{v}_{IE}$ increases due to co-linearity, $\tilde{v}_{NS}$ decreases. In the worst case when both $\boldsymbol{\mu_{G_x}}$ and $\boldsymbol{\mu_{G_y}}$ are co-linear with $\boldsymbol{\omega^{IE}}$, $\tilde{v}_{NS}$ reduces to zero and NSWA provides the optimal solution. The null spaces $\mathbf{\Omega}$, $\mathbf{\Psi}$ can be found by singular value decomposition. The vector $\boldsymbol{\omega^{NS}}$ can be chosen from a linear combination of the obtained basis, and should contain as many positive elements as possible. This can be achieved using methods such as linear programming. Since $\boldsymbol{\omega^{IE}}$ consists of all positive elements, $\boldsymbol{\omega^{NS}}$ will have a few negative weights. The effect of these negative weights becomes negligible as $L \to \infty$. The only case when the effect is noticeable is $N = 4$, where the dimension of $\mathbf{\Omega} \cap \mathbf{\Psi}$ reduces to 2, which is smaller than the rank of the augmented matrix $[\boldsymbol{A_D}, \boldsymbol{\omega^{IE}}]$. In that particular case $\boldsymbol{\omega^{NS}}$ can be chosen exclusively from the null space of $\boldsymbol{A_D}$.

C. Final Location Estimate

The final location estimate is a simple average of estimates from the IEWA and NSWA methods. This is given by:

$$\tilde{\boldsymbol{x}}_{final} = \frac{\tilde{\boldsymbol{x}}_{IE} + \tilde{\boldsymbol{x}}_{NS}}{2} \qquad (13)$$

VII. Simulation and Experimental Results

We compared the performance of the proposed algorithm to two other algorithms and results are shown below. The first is the "Original" algorithm which considers only one combination of all available (N) beacons. The second is the Rwgh algorithm from [1]. All simulations and experiments are performed for 20 randomly chosen target locations and the results are averaged out. In the experiment, RSS to distance conversion was performed using the simplified path loss model. Fig. 4 shows the location error in meters as the number of beacons increases for various mean bias values. It can be seen that the proposed algorithm is more stable and provides rapid increase in accuracy compared to other algorithms. Fig. 5 shows the probability distribution of the error for the case of $N = 5$. Both the figures show that the proposed algorithm yields higher accuracy and robustness independent of the bias distribution. In contrast, the Rwgh method is only effective for Exponential bias distribution. Lastly, Fig. 6 shows the experimental results in a wireless network with $N = 5$, where the proposed algorithm performs better as well.

VIII. CONCLUSION

In this paper, we proposed a dual weighted average algorithm to mitigate the effects of bias. The complementary weighted average methods provide two-fold robustness, making it applicable to a wide range of scenarios independent of the bias distributions. The simulations show an error reduction up to 40% and 25% while the actual experiments show a reduction of 56% and 28% compared to the "Original" and Rwgh methods, respectively. A significant reduction in the variance of error is also observed. Even though the actual experiment was carried out in a wireless network, the algorithm can be applied to other types of networks as well.

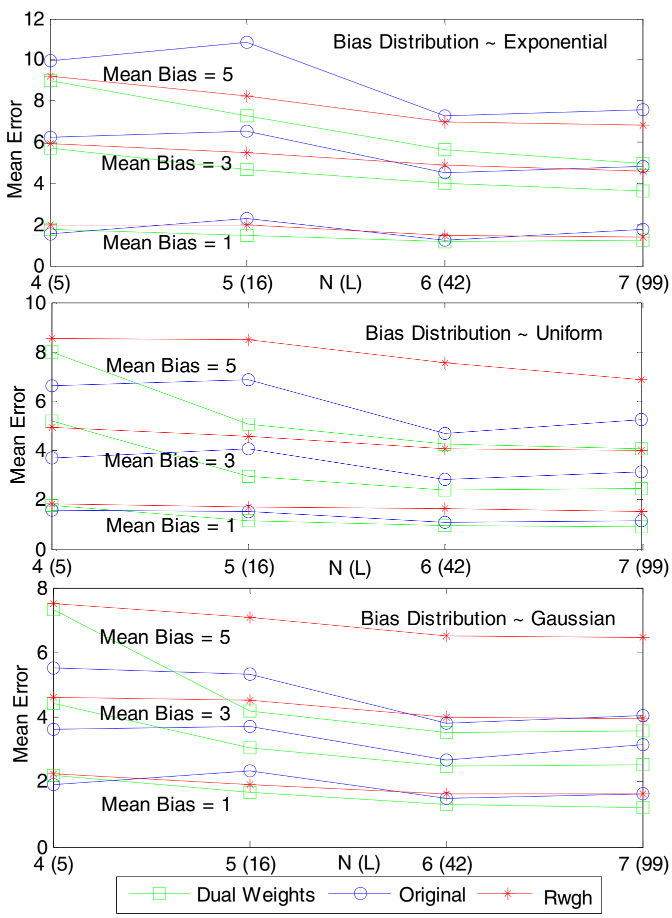

Fig. 4. Location Error vs. Number of Beacons

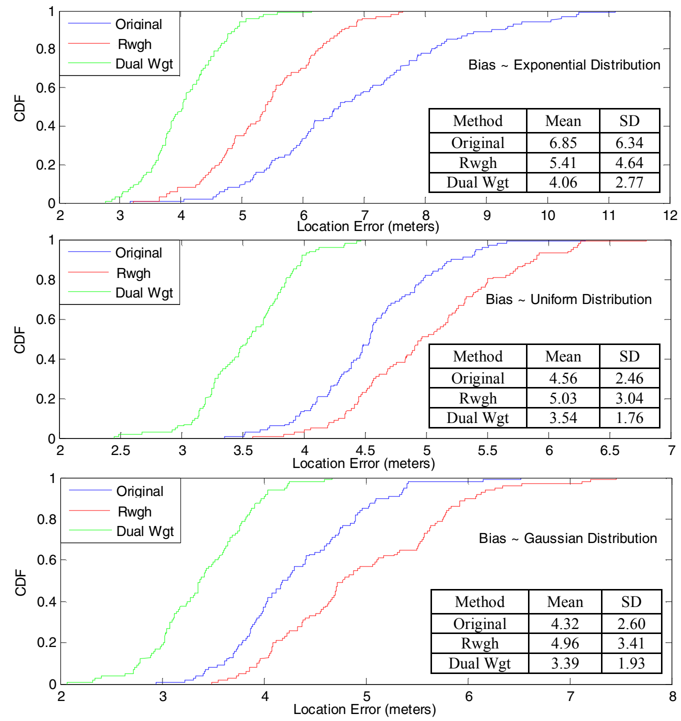

Fig. 5. CDF of Error for Various Bias Distributions ($\mu_{Bias} = 3m$)

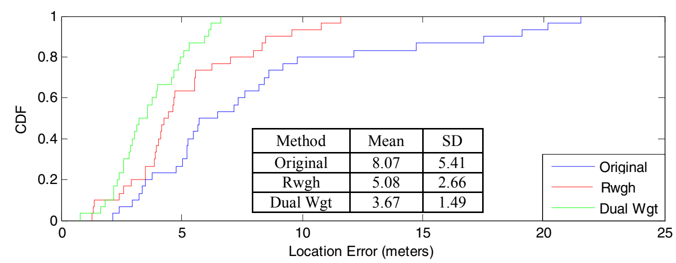

Fig. 6. Error Statistics from Experimental Testing

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