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Developing Mathematical Pedagogical Knowledge by Evaluating Instructional Materials

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In this chapter, we describe how artifacts of teaching practice can be used to elicit preservice teachers' understanding of mathematics and mathematics teaching. We gave two assignments to preservice teachers from two mathematics methods courses (elementary and secondary). The elementary assignment asked the preservice teachers to evaluate virtual manipulatives. The secondary assignment asked the preservice teachers to compare different types of curricula. Both assignments asked preservice teachers to explain how the materials could be useful for teaching practice.

A wealth of resources is available for teachers to use and examine online and in print. As a result, developing teachers' ability to evaluate materials is an important aspect of their mathematical pedagogical knowledge for teaching. The analysis of materials combines an understanding of mathematics content, the nature of learning, and the nature of teaching. In this chapter, we present two studies of preservice teachers' analysis of materials for teaching (e.g., curriculum, technology tools). We independently designed tasks for mathematics methods courses we taught at the elementary and secondary levels, and then found that there were shared characteristics of the tasks independent of the course. Analysis of student responses indicated common characteristics of mathematics teacher learning independent of grade level. The purpose of this chapter is to describe a design process for eliciting preservice teachers' mathematical understanding in a pedagogical context.

Research indicates that teacher education programs should include practice-based activities to help teachers learn how to teach mathematics effectively (Ball & Cohen, 1999; Darling-Hammond, 1998; Lampert & Ball, 1998; Wilson & Berne, 1999). Franke and Chan (2006) argue that we need to choose "high-leverage practices," or those aspects of mathematics teaching practice that are central to supporting the development of mathematical understanding, as productive starting places for novice teachers. One of the high-leverage skills for mathematical knowledge for teaching is to make judgments about the mathematical quality of instructional materials and modify them as necessary

(Ball, 2003). Lloyd and Behm (2005) found that preservice teachers' critical analysis of instructional materials provided opportunities to develop mathematical pedagogical knowledge and familiarity with the curriculum. Shulman (1987) described pedagogical content knowledge as the ability to communicate the "most useful forms of representation of these ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others" (p. 9). Yet, novice teachers have difficulty communicating mathematical concepts for teaching because they either lack the mathematics content knowledge or the pedagogical knowledge. By evaluating and selecting best instructional materials or models, preservice teachers can develop this mathematical pedagogical knowledge.

Curricular knowledge includes teachers' understanding of the nature of curriculum and the purposes and functionality of materials for teaching (Shulman, 1986). Teachers' use of curriculum is driven by a variety of factors, including their knowledge, beliefs, and experience (Remillard, 1999). Research related to reform-oriented instructional materials and technology integration indicates that preservice teachers are often challenged because many of them never experienced learning or teaching in that way (Ball & Cohen, 1996; Lloyd, 2002; Remillard, 2000). Lloyd and Behm's (2005) study of preservice elementary teachers' learning with middle-school curriculum materials in undergraduate mathematics courses found that the curriculum materials were different from the mathematics textbooks they previously used. In terms of technology, Battey, Kafai, and Franke (2005) studied preservice teachers' criteria for evaluating and using mathematical software. They found that most preservice teachers focused on surface features, such as clear directions, rather than focusing on the content or pedagogical issues. Statements made by the preservice teachers indicated a concern for general learning, engagement, and motivation, rather than specific mathematical content.

Providing preservice teachers experience with discriminating and evaluating instructional materials can help them become more confident and competent to teach mathematics. This study focused on two tasks designed to provide preservice teachers with opportunities to analyze instructional materials (textbooks and mathematical applets) to develop this high-leverage, practice-based skill. We first describe the students' responses to a textbook analysis task conducted in a secondary mathematics methods course. Next, we discuss a mathematical applet task used in an elementary mathematics course to elicit preservice teachers' understanding of different representations of mathematical content and how to incorporate them into instruction.

Textbook Analysis

The textbook analysis task was designed for a secondary mathematics methods course. Because textbooks play an important role in secondary teaching and because preservice teachers were unfamiliar with available texts, the assignment was designed both to expose them to different textbooks and to elicit

their beliefs about learning mathematics. Traditional textbooks include an explanation of a topic followed by practice problems. Reform textbooks (e.g., *NSF*-supported, *Standards*-based) use an activity-based or problem-based approach for K-12 students to develop understanding. The preservice teachers analyzed units on the same content area from each type of textbook and described the general character of the texts. The preservice teachers responded to the following questions for the analysis: 1) Select a mathematical topic (e.g., addition of fractions) and compare the textbooks' treatment of the topic. How are they similar? How are they different?; 2) How would you, as a teacher, structure your teaching differently with each type of textbook?; and 3) How do the textbooks represent mathematics differently? How do the textbooks represent mathematical problem solving differently?

Twenty-two preservice teachers completed the textbook analysis assignment across two semesters (14 men, 8 women). Nine participants were employed as mathematics teachers (five in public high school, one in private high school, three in middle school). Because these nine were practicing teachers, some of their comments related to a textbook they were currently using to teach mathematics.

Textbook Analysis Results

The participants' beliefs about mathematics teaching fell into three categories: real-world applications, skills and algorithms, and a blend of the aforementioned approaches (Hjalmarson, 2005). Some preservice teachers recognized the importance of real world applications in mathematics learning as a venue for making mathematics meaningful and purposeful to their students. The *Standards*-based textbooks often fill the need for units based in real world applications. The second category of responses were from advocates of skill-based or algorithmic approaches because students need to understand the steps and procedures of mathematics. A third group envisioned a blended teaching approach between applications and skills. The third approach was an attempt to mediate the need for conceptual and procedural understanding.

Classroom discussion about the textbook analysis helped teachers see diverse perspectives about curriculum. Part of the in-class discussion focused on the purposes for textbooks and the nature of mathematics learning in each context, and was reflected in the teachers' responses:

> The [reform-based] book focuses on the concepts and how they relate to other mathematical concepts. The primary focus is not on regurgitating a formula to find answers. I believe that the [traditional textbook] has a tendency to promote just finding the answers.

Reform-based textbooks are often designed around problems and real-world contexts and the materials themselves may not contain formulas or definitions because the problems are intended to help students develop such formulas and definitions. However, some of the preservice teachers had used textbooks as reference guides to help them solve problems. Reform-based textbooks are not intended to be reference guides in the sense that a dictionary is a reference. Given their own experiences in using a textbook to learn mathematics, we want our preservice teachers to grapple with two main questions: 1) From a learning perspective, how is instruction balanced between concepts explained in a textbook and concepts explored via inquiry and investigation in real-world contexts?; and 2) How do different types of textbooks support different modes of instruction?

Mathematics Applets Analysis

The mathematics applets analysis was designed for elementary preservice teachers taking an elementary mathematics methods class. The 22 preservice teachers, 20 women and 2 men, were placed in local elementary schools as interns. The mathematics applet analysis involved three processes: 1) evaluating effective representations and affordances for mathematical thinking; 2) evaluating and selecting the appropriate model for teaching using technology; and 3) collaborative lesson planning using technology. We expected these processes to help teachers develop the ability to discriminate among multiple models and make judgments on the clarity, effectiveness, and appropriateness of the representations.

For Process 1, the instructor engaged the preservice teachers in a discussion about selecting effective representations by examining specific criteria. She posed two questions related to the chart in Figure 1: 1) Does the representation have transparency, efficiency, generality, clarity, and precision? (see National Research Council, 2001, pp. 99-101 for definition of terms); and 2) What mathematical thinking opportunities are afforded through the mathematics applets (e.g., understanding, applying, problem solving and reasoning, making and testing conjectures, and/or creating)?

As a homework assignment (Process 2), preservice teachers found at least four mathematics applets that focused on one concept. They were asked to consider the following questions taken from *Young Mathematicians at Work* (Cameron, Dolk, Fosnot, Hersch, & Werner, 2006) as they compared and analyzed the applets.

1. What are the different uses of these mathematical models?
2. How are students' developmental needs supported by the different uses of these models?
3. Is there a developmental progression for these models? Are some uses of the model precursors to others?
4. What role does the model play in helping students visualize different strategies?
5. How might one represent a given strategy with each model and mathematical applet? (p. 27)

Website			
Applet name		Grade level	
Description of mathematical concept			
__Concept tutorial/Skill Practice __ Investigation/problem solving __Open exploration			
Analysis of Mathematical Representations and Models (5 ***** excellent – 1*poor)			
Transparency: How easily can the idea be seen through the representation?			
Efficiency: Does the representation support efficient communication and use?			
Generality: Does the representation apply to broad classes of objects or concepts?			
Clarity: Is the representation unambiguous and easy to use?			
Precision: How close is the representation to the exact value?			
Mathematical thinking opportunities afforded by the mathematics applet			
Connecting: Constructing conceptual connections and multiple representations			
Applying: Carrying out or using procedural knowledge flexibly & efficiently			
Problem Solving and reasoning: Formulating, representing and solving mathematical problems			
Making and testing conjectures: Making conjectures or judgments about mathematical ideas			
Creating: Putting elements together to generate, plan, or produce mathematical ideas			

Figure 1. Checklist for evaluating mathematics applets.

In the final phase (Process 3), the preservice teachers worked in groups planning a lesson for a specific concept (e.g., subtraction with regrouping). The lesson planning allowed the preservice teachers to discuss the salient aspects of different mathematics applets as well as their selection criteria and analysis, using the previous five questions to plan an effective lesson.

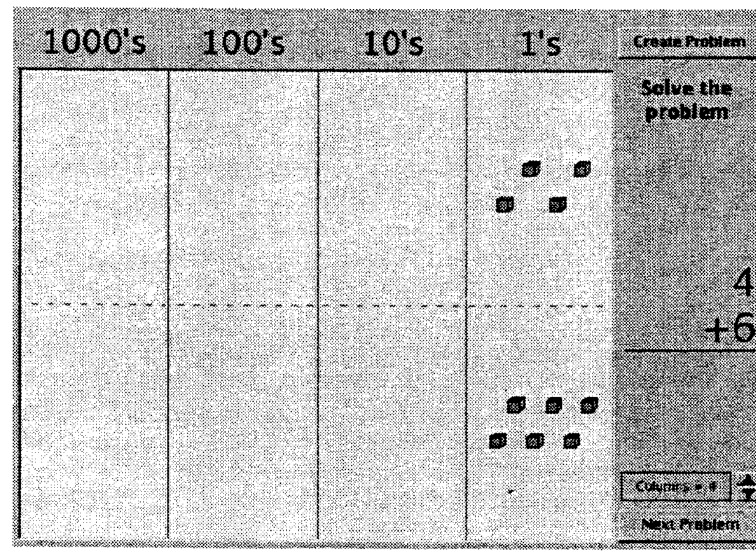
Mathematics Applet Analysis Results

The preservice teachers found a variety of mathematical models and representations available on the web and identified the uses of these models using their methods textbook. They critically examined each model using the effective representations criteria and determined the developmental needs of the learners when using these models. In doing so, they referred to the state and national standards to familiarize themselves with the objectives at each grade level.

Preservice teachers' analysis of models using technology showed the use of their mathematical knowledge for teaching. The major themes that emerged were a) the ability to identify, represent, and explain a concept with multiple models, and b) an understanding of the developmental levels for different mathematical models. They commented that the mathematics applets helped them visualize abstract mathematical ideas with interactive and dynamic pictorial models and new, nontraditional ways to model math ideas. By considering the prerequisite knowledge necessary for students to engage with each applet, the preservice teachers had to consider students' developmental levels. This consideration helped the preservice teachers determine which mathematics applet would be appropriate for different grade levels and led to important discussions about differentiation and tiered learning for different learning styles. They could readily see how these materials could be used for visual learners, students with special needs, and English Language Learners, all of whom may need the support of pictorial representations.

For example, one teacher described in her written analysis how the different technology applets represented four different models of the base ten system: proportional vs. non-proportional models; number line (measurement model) vs. chip trading model (composing and decomposing numbers). In addition, this teacher commented on how the proportional model, *Base Block Addition* (National Library of Virtual Manipulatives, 2008a), is the easiest type of model to use when thinking about values (see Figure 2). However, non-proportional models are also essential to understanding the complexity of our number system. For example, money applets (Arcytech, 2003) can be used to assist students in learning to compute with money. In considering which model to use when teaching skip-counting, the preservice teacher felt that number line manipulatives (e.g., Freudenthal Institute, 2008) were easiest to use. For modeling composing and decomposing numbers, Base block addition and Chip Abacus (National Library of Virtual Manipulatives, 2008a, 2008b) illustrated the concept explicitly.

A. Base Block Addition (National Library of Virtual Manipulatives, 2008a)



B. Chip Trading (National Library of Virtual Manipulatives, 2008b)

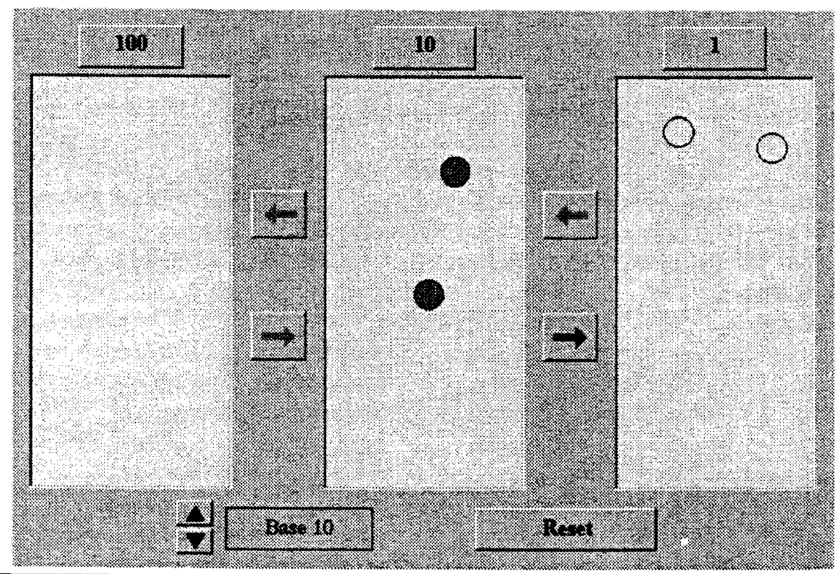


Figure 2. Examples of virtual manipulatives.

Through this task, the preservice teachers recognized that there are various models for illustrating a mathematical concept. In addition, they began to understand that different models supported different developmental progressions for students. Furthermore, by considering how to represent a given strategy with each model in a lesson, preservice teachers applied mathematical knowledge for teaching.

Common Design Characteristics for Promoting Pedagogical Content Knowledge

The central characteristic for both activities required that the preservice teachers think about mathematics from a pedagogical perspective. Although one task used secondary textbooks and the other used technological mathematical models, both elicited preservice teachers' beliefs about mathematics as well as the development of curricular knowledge. The textbook analysis task elicited preservice teachers' beliefs about mathematics content and the framing of that content for students. For the mathematics applets task, the preservice teachers used knowledge of mathematics and student development to evaluate representations of mathematical models. In both tasks, we asked teachers to consider mathematics pedagogically and pedagogy mathematically.

The design considerations for the tasks could be included in other tasks for preservice teachers. First, we placed a premium on experiences for the preservice teachers that would be practice-based – the task should represent a typical aspect of a teacher's job. In many mathematics methods courses, a major focus is often on lesson planning. However, the work of teachers goes beyond lesson planning and includes not only the creation of materials but also the use of existing materials. Curricular knowledge includes the analysis, adaptation, and understanding of the characteristics of instructional materials (Remillard, 1999; Shulman, 1986).

Second, both tasks required that the preservice teachers discriminate between materials by comparing them and describing their potential use in the classroom. We focused on readily-available materials to increase the meaningfulness of the task. The evaluation of materials, particularly in light of a growing variety of both paper-based and computer-based materials, is a task teachers at all levels undertake. Curricular considerations should include analyses of influence on student learning, the nature of mathematics represented in the materials, and how the materials integrate with other products. The evaluation process for both types of materials we described here elicited the preservice teachers' beliefs about the role of such materials in the classroom and their vision for mathematics teaching.

Third, these tasks supported the preservice teachers as they engaged in discourse about instructional materials, a necessary skill for effective teaching practice. Secondary teachers collaborate within a department or with a group of teachers teaching the same course. Elementary teachers collaborate with other teachers in their grade level, mathematics specialists, special education teachers, and ELL specialists. As a result of these assignments, the preservice teachers discussed their individual analyses of the materials in whole-class discussion. This process allowed them to share and to defend the reasoning they use in evaluating curricular materials. This process included both revealing their understanding of the purposes for the curricular materials as well as how they would implement them with students.

Finally, these tasks required the preservice teachers to identify the mathematics in the textbooks and the applets. This skill is critical for developing mathematics knowledge for teaching and analyzing how different materials represent fundamental aspects of mathematics in different ways. The mathematical analyses moved the discussion beyond the aesthetic, surface features of the materials toward student learning and instruction.

Conclusion

These analyses of curricular materials provided our preservice teachers with tangible tools for teaching. Due to the wealth of on-line materials and the regular changes in state and local curriculum guides, the analysis of curricular materials is a part of every teacher's job. Other items that preservice teachers could analyze for similar assignments include state and local standards documents and curriculum frameworks, lesson planning resources, and assessment tools. Commonalities found in the two tasks we described in this chapter indicate that both the textbook analysis and the mathematics applets analysis are practice-based tasks that can be implemented in both elementary and secondary preservice methods courses to promote mathematical pedagogical knowledge.

Knowledge of curriculum and curriculum materials, as well as effective integration of technology in learning mathematics, is necessary for teachers across the grade bands. Teachers at all levels must be able to make judgments about the mathematical quality of instructional materials and represent ideas carefully using multiple models, which are often technologically-based, in order to maximize student learning. Through explicitly designed tasks that elicited preservice teachers' beliefs and supported their understanding of mathematical content knowledge, our preservice teachers developed specialized curricular and pedagogical mathematics knowledge.

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