Chapter 3

Using the Unique Features of Virtual Manipulatives to Design Lessons

When designing lessons using virtual manipulatives, teachers need to understand what it means to use technology "appropriately" to teach mathematics. The five guidelines for appropriate uses of technology specific to mathematics education are

1. Introduce technology in context
2. Address worthwhile mathematics with appropriate pedagogy
3. Take advantage of technology
4. Connect mathematics topics
5. Incorporate multiple representations (Garofalo et al., 2000, p. 67)

In addition to knowing how to integrate technology appropriately, teachers must focus on worthwhile mathematics and effective pedagogy when using technology. One effective way to do this is to plan mathematics tasks focused on the five process standards (NCTM 2000): representations, communications, connections, reasoning and proof, and problem solving. This chapter focuses on incorporating the guidelines for appropriate uses of technology specific to mathematics and optimizing learning environments with virtual manipulatives and applets.

The template in Figure 1 may be used during planning to guide the activity and classroom discussion so they focus on advancing students' mathematical thinking.

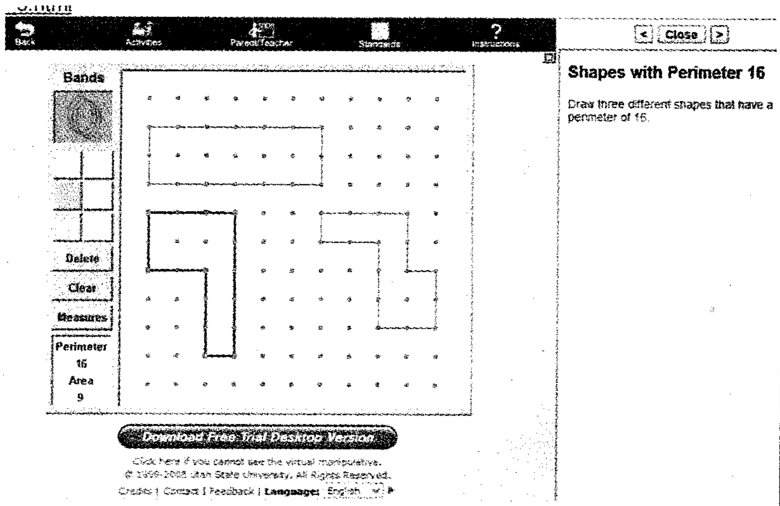
<table>
<tr><td>Website</td><td colspan="3">http://nlvm.usu.edu/en/nav/frames_asid_172_a_2_t_3.html?open=activities&from=category_a_2_t_3.html

</td></tr>
<tr><td>Math Strand</td><td>Geometry</td><td>Grade level 2–5</td><td></td></tr>
<tr><td>Description of mathematical concept (NCTM/SOL)</td><td colspan="3">Properties of plane shapes
Area and perimeter of polygons
Symmetry and congruence</td></tr>
<tr><td colspan="4">Analysis of Mathematical Representations and Models</td></tr>
<tr><td colspan="4">__Concept tutorial/skill practice __ Investigation/problem solving __Open exploration</td></tr>
<tr><td colspan="4">Mathematical thinking opportunities afforded by the mathematics applet</td></tr>
<tr><td colspan="2">Representations:
• Create and use representations to organize, record, and communicate mathematical ideas
• Use mathematical representations to solve problems
• Use representations to model and interpret physical, social, and mathematical ideas</td><td colspan="2">Explain: This virtual geoboard is versatile in that it can model multiple polygons, line segments, and angles, build symmetrical and congruent shapes and be used to find area and perimeter. The screen includes the visual (virtual geoboard) and the numeric (measures).</td></tr>
<tr><td colspan="2">Communication:
• Use the language of mathematics to express mathematical ideas precisely</td><td colspan="2">Explain: Students could construct figures based on descriptions given, or give descriptions of their own shapes for a partner to build. Given names of shapes, students could construct them and then compare. The geoboard offers a great way to develop vocabulary and language.</td></tr>
<tr><td colspan="2">Connections:
• Provides connections among mathematical ideas
• Relates how mathematical ideas interconnect and build on one another to produce a coherent whole
• Applies mathematics in contexts outside of mathematics</td><td colspan="2">Explain: The connection between the created shapes and the perimeter and area through the Measure button allows for students to relate to the meaning of the two forms of measurement concepts. This easily connects to the development of fractions.</td></tr>
<tr><td colspan="2">Reasoning and Proof:
• Develops reasoning and proof
• Opportunities to investigate mathematical conjectures</td><td colspan="2">Explain: Students could explain how the formulas for area and perimeter work for different polygons.</td></tr>
<tr><td colspan="2">Problem Solving:
• Opportunities to apply and adapt a variety of appropriate strategies to solve problems</td><td colspan="2">Explain: Students can click on the Activities button to solve a variety of problems. Students could use different ways to find the area of irregular polygons.</td></tr>
</table>

FIGURE 1 *Example of the "Advancing Mathematical Thinking" planning sheet*

Exploring Relationships among Equivalent Fractions (Fourth-Grade Lesson)

The following sections describe how technology was used to teach a mathematics lesson. We describe the task and the tool that supported the learning, the role of the teacher in capitalizing on the learning in the technology-rich environment, and how the technology gave more access and opportunity to diverse learners for learning. The planning sheet presented in Figure 1 was used to think through the instructional elements in the design of the lesson.

The Mathematical Task and the Technology Tools as Learning Supports

In the fourth-grade lesson on fractions, the objective was to rename fractions and find equivalent fractions using the virtual manipulative called Fraction Equivalence found at the National Library of Virtual Manipulatives (NLVM) (http://nlvm.usu.edu/en/nav/vlibrary.html). (See Figure 2.) The lesson was designed to let students explore the relationships among equivalent fractions by analyzing patterns in a list of equivalent fractions. The goal was to help students generalize rational number concepts based on the pattern they identified among the fractions and then construct a rule for themselves.

During the task, students were presented with a fraction circle or square with parts shaded and were directed, "Find a new name for this fraction by using the arrow buttons to set the number of pieces. Enter the new name and check your answer" (as shown in Figure 2). To find the new fraction name, students clicked on arrow buttons below the whole unit, which changed the number of parts. When students had an equivalent fraction, all lines turned red. When a common denominator was identified, students typed in the number for each equivalent fraction in the appropriate box. Students checked their answers by clicking the Check button. Once they were given feedback, they were asked to find several other names for that fraction. Each step of the way, the pictures were linked to numeric symbols that dynamically changed with moves made by the students. To help students explore the relationships among equivalent fractions, the applet prompted students to find several equivalent fractions. This applet was specifically designed to develop the concept of renaming fractions. Although constrained to one specific objective, the tool allowed for more exploration than do physical manipulatives, such as fraction circles or bars, which are usually limited by the number of fractional pieces. This applet

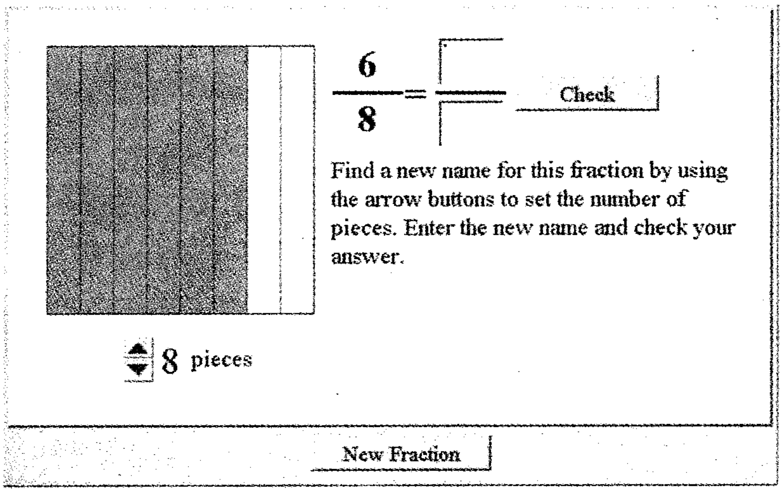


FIGURE 2 *NLVM applet Fractions – Equivalent*

enabled students to equally divide a whole into 99 pieces, which allowed them to generate multiple equivalent fraction names.

In addition to many of the design features that enhance student learning, each applet has language options for Spanish, French, and English. This particular classroom was in a Title I school that was predominately Hispanic, and the opportunity to select the Spanish language option was a great learning support for students (see Figure 3).

The Role of the Teacher in Facilitating Learning in a Technology-Rich Environment

The teacher's role in extending students' thinking during this task was to encourage students to record a list of equivalent fractions, and then to determine patterns among the numbers to eventually generate a rule. For example, using the applet on a SMART Board, one student demonstrated $\frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\frac{4}{12}$. As the class recorded this on the board, students' eyes widened and hands went up in the air, with students saying, "Oh, oh, I know the rule!" When students shared their rules, some noticed the additive rule. One student stated, "The denominators are going by a plus 3 pattern." Another student shared, "It's like skip counting." Another voiced, "It's the multiple of 3." To encourage their understanding of the mathematical relationships, the teacher asked students to explore the fraction $\frac{2}{3}$ so that they could see the multiplicative pattern for the numerator and the denominator. For example, students listed $\frac{2}{3}=\frac{4}{6}=\frac{6}{9}$, and

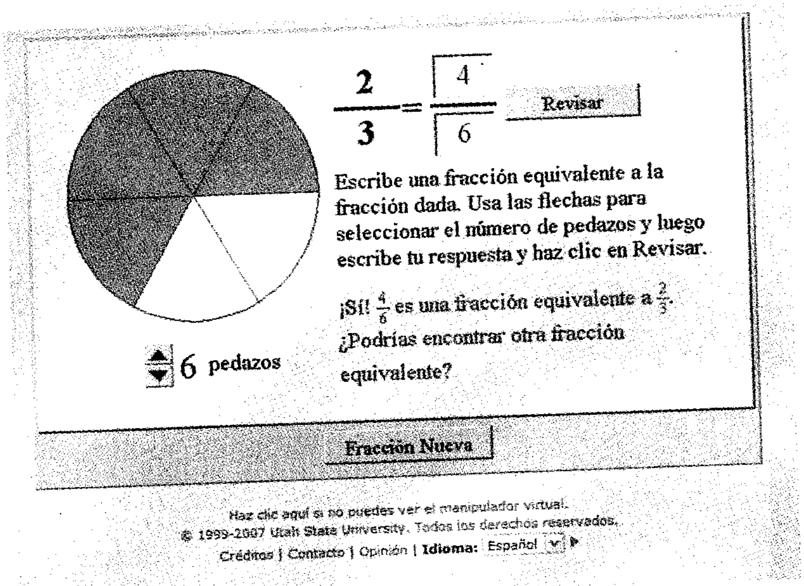


FIGURE 3 *NLVM appletFractions – Equivalent in Spanish*
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again they quickly saw the additive pattern and the multiples of 2 for the numerator and 3 for the denominator.

Next the teacher posed questions to extend their investigation: "Are $\frac{2}{3}$ and $\frac{20}{30}$ equivalent fractions? What about $\frac{2}{3}$ and $\frac{10}{15}$?" Students used the applet and talked with a partner to identify relationships between the given fractions and to explore other fractions to find a rule beyond the additive rule.

When students gathered again as a group, several students shared their discoveries. One student noted, "The fractions $\frac{2}{3}$ and $\frac{20}{30}$ are equivalent because you multiply both numerator and denominator by 10. And in $\frac{2}{3} = \frac{10}{15}$, you multiply both numerator and denominator by 5." This discussion led to a lively conversation on how $\frac{10}{10}$ is one whole and $\frac{5}{5}$ is one whole. The teacher connected this idea to the identity property of multiplication by asking, "What happens when we multiply 1 by any number?" This discussion reinforced the idea that no matter how you rename the fractions, as long as you multiply it by 1 or $\frac{5}{5}$, you will have an equivalent fraction.

To challenge the students, the teacher posed a new question: "What would the equivalent fraction be for $\frac{1}{3}$ if the denominator was divided into 99 parts?" This type of questioning encouraged students to extend their thinking by making conjectures and testing their rule or hypothesis.

A critical part of lesson planning is making sure there is time for mathematical discourse before, during, and after using the technology tool. As students explore patterns and relationships, it is critical that they participate in mathematical discourse with their peers and their teacher.

Opportunities to Learn via Technology: Equity and Access for Diverse Learners

Using the Fraction applet helped students think and reason about the relationships among equivalent fractions, instead of merely mimicking an algorithm demonstrated by the teacher. In this class, the teacher matched student partners so that a student with limited proficiency in English was paired with a student who spoke both English and Spanish proficiently. As the pairs worked together with the applet, they were able to make sense of the mathematics by talking through the processes. In addition, the ability to switch the language to Spanish gave many of the English language learners access to the mathematics, as shown in Figure 3.

Learners with special needs, including language needs, are often given direct instruction on how to perform an algorithm using mnemonic devices or steps to follow without an opportunity to construct a conceptual understanding of the procedures they are performing. One challenge when students work with physical manipulatives, such as fractions circles, is that the manipulation of multiple pieces creates too much of a cognitive load on students' thinking processes and students lose sight of the mathematics concept. In this classroom example, when students worked with the virtual applet, the applet off-loaded some of the physical manipulation so that students could focus more on the mathematical processes and the relationships among the equivalent fractions. An additional support for special needs students was provided by the instructor, who worked with a small group of students on the SMART Board while the other students explored at individual computers with their partners.

Overall, this applet gave students with special needs access to the mathematics without creating a cognitive overload. Having the visual and numeric representations closely tied together and displayed on the screen helped students make direct connections in the relationships among equivalent fractions. Throughout the lesson the teacher worked with a small group of English language learners and special needs students on the SMART Board. These students required more teacher support and benefited from the small group interaction. Also, the kinesthetic/tactile advantages of the SMART Board enabled these students a greater understanding of the concept. The teacher was able to reteach and reinforce skills as needed. Students took turns manipulating the SMART Board and guiding each other through the given task.

Leveraging Technology to Enhance the Mathematical Learning

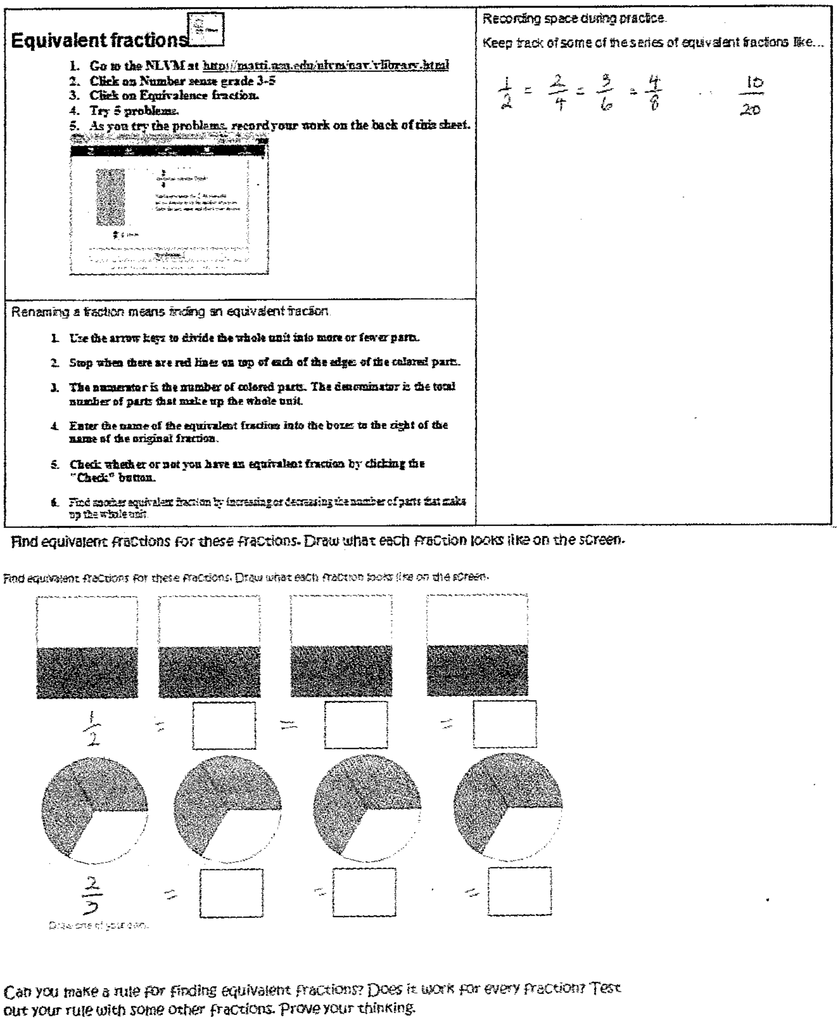
Learning environments that take advantage of virtual manipulatives offer a number of affordances for students as they develop their mathematical understanding. The example presented in this chapter identifies five primary benefits of virtual manipulatives:

1. Linked representations, providing connections and visualization between numeric and visual representations
2. Immediate feedback, which allows students to check their understanding throughout the learning process and prevents misconceptions
3. Interactive and dynamic objects that take mathematics from a noun to a verb (from *mathematics* to *mathematize*)
4. Opportunities to teach and represent mathematical ideas in nontraditional ways
5. Ease of differentiation and scaffolding to meet the needs of diverse learners

Recording Students' Mathematical Thinking and Learning

As teachers structure their learning environments using technology, the primary focus should be on supporting mathematical understanding. There are a number of design and assessment issues that are unique to using technology. For example, teachers should consider having students use a task sheet to record their work and thoughts, and asking students to record examples from the virtual manipulative by printing their work. By writing and recording their work, students can reflect on their own experiences, a metacognitive process that is essential in problem solving. The task sheet (like the one shown in Figure 4) can facilitate this recording process and provides a permanent record, which can be used for assessment purposes by the teacher.

Appropriate use of technology in teaching and learning should make the learning environment qualitatively different than teaching without it. That is, integration of technology should not merely add a virtual representation to the lesson but should enhance the mathematics teaching and learning by providing opportunities for



Equivalent fractions

1. Go to the NLVM at http://matti.usu.edu/nlvm/nav/vlibrary.html
2. Click on Number sense grade 3-5
3. Click on Equivalence fraction.
4. Try 5 problems.
5. As you try the problems, record your work on the back of this sheet.

Recording space during practice.

Keep track of some of the series of equivalent fractions like…

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} \quad \ldots \quad \frac{10}{20}$

Renaming a fraction means finding an equivalent fraction.

1. Use the arrow keys to divide the whole unit into more or fewer parts.
2. Stop when there are red lines on top of each of the edges of the colored parts.
3. The numerator is the number of colored parts. The denominator is the total number of parts that make up the whole unit.
4. Enter the name of the equivalent fraction into the boxes to the right of the name of the original fraction.
5. Check whether or not you have an equivalent fraction by clicking the "Check" button.
6. Find another equivalent fraction by increasing or decreasing the number of parts that make up the whole unit.

Find equivalent fractions for these fractions. Draw what each fraction looks like on the screen.

Find equivalent fractions for these fractions. Draw what each fraction looks like on the screen.

$\frac{1}{2}$ = ☐ = ☐ = ☐

$\frac{2}{3}$ = ☐ = ☐ = ☐

Can you make a rule for finding equivalent fractions? Does it work for every fraction? Test out your rule with some other fractions. Prove your thinking.

FIGURE 4 *Tasksheet to record students' thinking while using the tool*

rich mathematical thinking and discussion. This means that there are specific pedagogical considerations for teachers. The classroom example illustrates how using the NCTM process standards along with the unique affordances of the technology tools allows for meaningful learning to take place while meeting the needs of diverse learners.

References

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National Council of Teachers of Mathematics. 2000. Principles and standards for school mathematics. Reston, VA: Author.

showed that students had little prior knowledge; posttest scores indicated that they had made significant gains at the end of the unit. Students' work and responses in the task-based interviews indicated that they were generally able to work with integers using various representational forms (symbols, words, and pictures), particularly for addition. For example, when given an integer addition problem in the form of a story, students could both write a number sentence and draw a picture to represent it. Students also used the different representational forms to help them reason through a given task, explain their thinking, and detect and correct errors in their work.

However, there was evidence that students had more difficulty translating between actions describing or representing subtraction (either in a story or in a picture) and the symbolic form of the problem. For example, when working with a story scenario that described a situation such as (–9) – (–3), students interpreted this either as subtraction (–9 – –3) or as an equivalent addition scenario (–9 + 3) and used these interpretations to evaluate the story and draw a corresponding picture. However, when it came time to write the number sentence, several students who had interpreted the problem as –9 + 3 wrote the number sentence –9 – 3. They understood the action of the story (that subtracting a negative number is equivalent to adding its opposite), but they were unable to translate this to symbols, focusing only on the value of the integers and not on how the operation sign determined that value. This indicates that, while students made gains in their understanding of integer addition and subtraction after using the virtual models, additional work needed to be done to solidify the connection between stories and pictures representing subtraction situations and the role of the subtraction sign in the number sentence that corresponds to those scenarios.

Summary

As these classroom research projects show, teachers can gain a great deal of information from examining their own practices while using virtual manipulatives in a mathematics classroom. These teaching experiments can be useful, not only for the teachers conducting the action research themselves, but also for other educators who are making choices about how to integrate virtual manipulatives and other technology tools into their classrooms.

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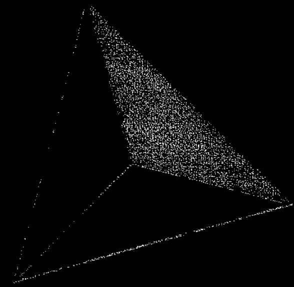

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