JENNIFER M. SUH AND KERRI FULGINITI

5. DEVELOPING MATHEMATICAL POTENTIAL IN UNDERREPRESENTED POPULATIONS THROUGH PROBLEM SOLVING, MATHEMATICAL DISCOURSE AND ALGEBRAIC REASONING

INTRODUCTION

The following study explored strategies for developing mathematical potential and enhancing mathematics instruction for diverse learners from a low socio-economic population identified as "young scholars". The intentional focus on designing and creating opportunities to foster mathematical potential and build collective knowledge influenced many of the pedagogical decisions made by the teacher and researcher in their jointly planned research lessons. The most salient features in developing mathematics potential in these young scholars were giving opportunities to 1) engage in rich mathematical tasks and sequence of related problems, b) use multiple representations to develop representational fluency, and c) develop mathematical communication where reasoning and proof and sense-making became a habit of mind and the focus of classroom discourse. Through encouragement and participation in problem solving, mathematical discourse and algebraic reasoning, students exhibited confidence, competence and more of the behavioural characteristics of mathematically proficient students.

The following research project aimed at developing mathematical potential in under-represented groups by giving access to authentic and rigorous mathematics problem based learning focused on developing mathematical communication and algebraic reasoning. Research indicates that children of color are underrepresented in gifted programs and are less likely to be nominated by teachers as potential candidates for gifted programs when using such traditional measures as IQ and achievement tests for identification (Ford, Harris, Tyson, & Trotman, 2002). The issue of underrepresentation of economically disadvantaged and culturally diverse students in advanced academic programs is multifaceted and complex (see Figure 1). Some major themes in research concerning underrepresentation of minorities and students living in poverty are: a) the biased assessment measures; b) the low self esteem and expectation set by the individual or others; and c) the lack of strong advocacy or referral from parents and guardians (Van Tassel-Baska, Johnson, & Avery, 2002). Some of the recommendations for the biased assessment is to have multidimensional assessments such as portfolios, case studies and anecdotal records which would give more diverse learners opportunity to entry to advanced programs with an advanced curriculum. Some efforts are underway in broadening the identification procedures

B. Sriraman and K.H. Lee (eds.), The Elements of Creativity and Giftedness in Mathematics, 67–79.

The Underrepresentation of Economically Disadvantaged and Culturally Diverse Students in Advanced Academic Programs		
Issues	*Recommendations*	*Outcomes*
Biased assessment measures	Multidimensional assessments (portfolios, case studies, anecdotal records)	Access Entry into advanced programs with advanced curriculum
Low expectations (self and others)	Raise expectations through more rigorous and challenging curriculum	Affirmation Increase expectations and student efficacy
Few parent/guardian referrals	Increase communication/administrator and teacher referrals/parent advocacy training	Advocacy Support students in reaching their fullest potential

Figure 1. Issues related to underrepresentation of diverse students in advanced academics.

to increase the number of minority children in gifted education programs (Van Tassel-Baska, Johnson, & Avery, 2002). Many researchers and practitioners suggest using multiple tests and alternative methods for finding gifted minority students, including performance-based assessment measures based on Gardner's theory of multiple intelligences or other models (Van Tassel-Baska et al., 2002) and nonverbal ability assessments, such as the Naglieri Nonverbal Abilities Tests or Raven's Matrix Analogies Tests (Ford et al., 2002). Raising expectation in academics can begin with raising the bar for teaching the standards of learning and providing more rigorous and challenging curriculum before these students. Finally, parents, guardians and family members from disadvantaged and culturally diverse population need to have frequent and meaningful advocacy training so that collectively they can support, affirm and advocate for their youngsters in reaching their fullest potential.

THEORETICAL FRAMEWORK

Participation Gap among Diverse Learners in Accessing Rigorous Mathematics

Sociocultural approaches emphasize the interdependence of social and individual processes in the co-construction of knowledge (Vygotsky, 1986). "Through participation in activities that require cognitive and communicative functions, children are drawn into the use of these functions in ways that nurture and 'scaffold' them" (pp. 6–7). Vygotsky (1986) described learning as being embedded within social events and occurring as a child interacts with people, objects, and events in the environment. Through socially shared activities learners also internalized processes. Following this approach, researchers have explored sociomathematical norms and how teachers actively guide the development of classroom mathematical practices and individual learning through capitalizing on opportunities that emerge through students' activities and explanations (Ball, 1993; Cobb, Wood & Yackel, 1993; Lampert, 1990; McClain & Cobb, 2001). However, recent research offers some important insight on participation gaps which exist among students from diverse social, cultural, and racial backgrounds in mathematics classrooms and how classrooms can be structured to better afford opportunities to participate in mathematics by a wider range of students (DIME, 2007).

Typically, school systems that serve economically disadvantaged or minority student struggle to meet academic achievement and traditionally reform movements have aimed at remedial models to improve students' achievement. According to this model, students from diverse population get more of the basic skill learning without the opportunity to participate in learning opportunities that develop unique talents, creative thinking and problem solving strategies. According to a report issued by the Partnership for 21[st] Century Skills, today's graduates need to be critical thinkers, problem solvers, and effective communicators who are proficient in both core subjects and new, twenty-first-century content and skills. These include learning and thinking skills, information- and communications-technology literacy skills, and life skills.

In reform oriented approaches like problem based learning, students work in teams to explore real-world problems and create presentations to share what they have learned. Compared to learning primarily from textbooks, this approach has many benefits for students, including deeper knowledge of subject matter, increased self-direction and motivation, improved research and problem-solving skills, and understanding how academics connect to real-life and careers. The study by Boaler (1999) found that students at the problem-based school did better than those at the more traditional school both on math problems requiring analytical or conceptual thought and on those considered rote, that is, those requiring memory of a rule or formula. The focus of the mathematics communication was to place more emphasis in the mathematical reasoning and communication which is reflected through cognitive actions such as justifying, proving, investigating, analyzing and explaining. This increased emphasis on proving and justifying results in the algebraic reasoning and the habits of mind that will prepare students for more advanced studies in mathematics.

This design research focused on developing mathematical potential of diverse learners by building collective mathematical knowledge in the classroom through mathematical discourse, students' reasoning and proof and algebraic reasoning. In addition, the research team was intentional in the use of pedagogical content tools defined as "devices such as graphs, diagrams, equations, or verbal statements that teachers use to connect students' thinking while moving the mathematical agenda forward" (Rasmussen & Marrongelle, 2006, p. 388).

The research questions that guided this design research were:

1) What designed learning opportunities afforded access, affirmation and development of mathematics potential for under-represented students?
2) How do pedagogical content tools promote development of mathematical communication, algebraic connections and generalizations methods among elementary students?

METHODOLOGY

Participants

The participants in this study were sixteen "young scholars" in fourth through sixth grade students who participated in the summer program focused on problem based learning. Many of these students' former classroom teachers recommended them based on their exhibition of mathematically promising traits during the academic year. These students attended a Title I (low socio economic) elementary school in a major metropolitan area with a diverse population of 600 students at the school: 51% Hispanics, 24% Asians, 16% Caucasians, 3% African Americans and 6% others, with over 50% receiving free and reduced lunch. Using multiple instruments, students were selected for the "Young Scholar: MATH 4-1-1" Project which provided supplementary educational experiences for a select group of bright African American, Hispanic, and economically disadvantaged students. The project focused on exposing students to a variety of high-achieving peer groups that would enhance and develop the "scholar identity" of these students. These classes and support sessions began in the summer and continued into the school year through after school meetings. The classes and sessions were designed to enrich and support the mathematics learning. The research and design team for this project consisted of a university mathematics educator and researcher and a gifted and talented specialist at an elementary School. This team co-designed and co-taught the lessons and analyzed student learning to make adjustment each day to the follow up lessons.

Design Features of the Project

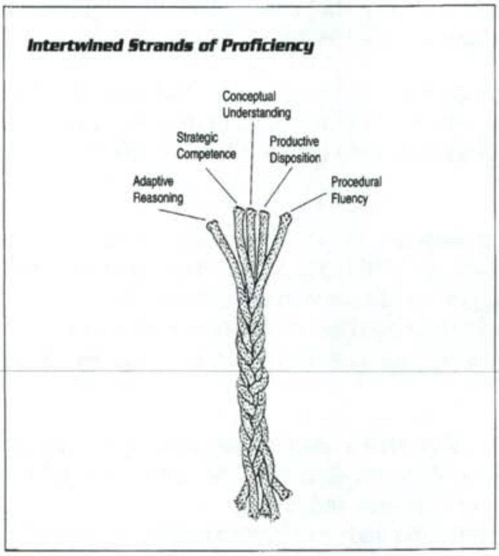
The focus of the summer program was to immerse students in authentic mathematics problem solving while utilizing local community resources such as invited community speakers and culturally relevant and contextualized tasks. The idea was to expose students from diverse backgrounds to new, engaging and rigorous mathematics while fostering their algebraic thinking and positive attitude towards mathematics. The project was called MATH 4-1-1: Young Mathematicians On Call where students solved rich engaging, meaningful and mathematically authentic problems presented by the community. The structure of the enrichment fell into three main categories based on Renzulli's Schoolwide Enrichment Model (SEM). This enrichment cluster design defines authentic learning as *applying* relevant knowledge, thinking skills, and interpersonal skills to the solution of real problems (Renzulli, Gentry & Reis, 2003).The idea was to expose students to new engaging and rigorous mathematics content which was classified as Type I activities. Type II activities were skill training that connected to what practitioners do within their field. Finally during Type III activities, students utilized the developed interest areas (Type I) and newly acquired skills (Type II) to create an authentic project or service for an authentic audience. However, central to enrichment clusters was the idea that students would direct their learning and invest in their interest areas. Some of the enrichment projects included the following:

- *Math 411: Helping the community solve problems with mathematics.* Meet some people in your community with real life problems in their work places or their lives. Together with your teammates, help people in your community solve problems using your math skills.
- *Be a statistician: Survey your community to know what they want.* Do you want to find out what people think about things? Survey your friends, family and community about something you've always wanted to know. Organize your responses in a creative way. Decide how to share this information. Develop and conduct a survey and communicate your results in this exciting enrichment cluster.
- *MATH 4 US: Starting a business.* Research a need in your community and plan a business. Present your idea to investors and determine the resources needed to run your business. Keep a budget of expenses and profit.

The fundamental nature of such authentic high-end learning activities created an environment in which students applied the relevant knowledge and skills to solving real problems (Renzulli et al., 2004). Emphasis was placed on creating authentic learning situations where students were thinking, feeling, and doing what practicing professionals did in solving daily problems (Renzulli, Leppien & Hays, 2000).

Focusing on the Core Traits of Mathematically Proficient Students

The research team used the Rating scale (see table 1) developed by Gavin (2003) to identify scholarly identities and the National Research Council's (2001) Five Strands of mathematical proficiency (NRC, 2001) to focus on the learning goals. The researcher team mapped out the behaviour characteristics from the rating scale and the Five Strands of Mathematics Proficiency (see Figure 2) and found a direct correlation among the behaviour traits and the strands of mathematical proficiency. The mapping exercise allowed both the teacher and the research to agree on a set of core behaviour characteristics that would be the focus of the student outcome for the diverse learners in the *"MATH 4-1-1: Young Mathematicians o*n Call*"* project. For example, conceptual understanding, which is the comprehension of mathematical concepts, operations, and relations matched up with Gavin'*s* (2003) Rating Scale 2, 4: organizes data and information to discover mathematical pattern and understands new math concepts and processes more easily than other students and displays a strong number sense (e.g., makes sense of large and small numbers, estimates easily and appropriately). In addition, strategic competence which is the ability to formulate, represent, and solve mathematical problems, matched up with Rating Scale 5, 7, 9 and 10: has creative (unusual and divergent) ways of solving math problems; frequently solves math problems abstractly, without the need for manipulatives or concrete materials; when solving a math problem; can switch strategies easily, if appropriate or necessary; regularly uses a variety of representations to explain math



Five Strands of Mathematical Proficiency
National Research Council's (2001)

– conceptual understanding - comprehension of mathematical concepts, operations, and relations
– procedural fluency - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
– strategic competence - ability to formulate, represent, and solve mathematical problems
– adaptive reasoning - capacity for logical thought, reflection, explanation, and justification
– productive disposition - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Figure 2. Five strands of mathematical proficiency, NRC (2001).

Table 1. Rating the behavioral & mathematical characteristics (Gavin, 2003)

Rating the Behavioral & Mathematical Characteristics (Gavin, 2003)

1. is eager to solve challenging math problems (A problem is defined as a task for which the solution is not known in advance).
2. organizes data and information to discover mathematical patterns.
3. enjoys challenging math puzzles, games, and logic problems.
4. understands new math concepts and processes more easily than other students.
5. has creative (unusual and divergent) ways of solving math problems.
6. displays a strong number sense (e.g., makes sense of large and small numbers, estimates easily and appropriately).
7. frequently solves math problems abstractly, without the need for manipulatives or concrete materials.
8. has an interest in analyzing the mathematical structure of a problem.
9. when solving a math problem, can switch strategies easily, if appropriate or necessary.
10. regularly uses a variety of representations to explain math concepts (written explanations, pictorial, graphic, equations).

concepts (written explanations, pictorial, graphic, equations). Adaptive reasoning, which is capacity for logical thought, reflection, explanation, and justification (matched up with Rating Scale #8, 10: has an interest in analyzing the mathematical structure of a problem; regularly uses a variety of representations to explain math concepts (written explanations, pictorial, graphic, equations). Most importantly, having productive disposition which is the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy aligned with the behavioural characteristic of 1 & 3: is eager to solve challenging math problems and enjoys challenging math puzzles, games, and logic problems.

Procedures

The study used qualitative methods of collaborative action research using the design research approach and research memos. Design-based research method is a research methodology aimed to improve educational practices through systematic, flexible, and iterative review, based upon collaboration among researchers and practitioners in real-world settings, and leading to design principles or theories (Brown, 1992; The Design-Based Research Collective, 2003). Using design research which emphasizes the processes of iteration, feedback loops and narrative reports, we refined the key components of our design. As teacher researchers, we took active notes and memos as we proceeded with the research exploring the interplay between the individual and collective knowledge in the mathematics classroom through discourse and the development of new ideas through records of students' thinking. We collected video recordings of each class sessions, retained copies of students work and work displayed in the "collective workspace" and recorded observations and reflections throughout the planning, teaching, and debriefing phases of the study. Daily debriefing meetings allowed for formative analysis of the design process by focusing on what was revealed during each class session and to plan for subsequent classes while modifying the task, tools and teaching methods based on feedback. In addition, these memos and artifacts (i.e. students' written work contained drawings, solution procedures, numeric notations and explanations) were analyzed for emerging themes at the end of the project for a summative analysis. Student responses, memos, and classroom videotapes were used to triangulate students' understanding.

RESULTS

Qualitative analysis from the teacher-researchers' memos indicated that the process of collaborative planning, teaching and debriefing focused on fostering mathematical potential and building collective knowledge impacted many pedagogical decisions. The most salient features in developing mathematics potential in these young scholars were giving opportunities by a) selecting tasks and sequences of related problems, b) integrating pedagogical content tools to develop representational fluency, and c) orchestrating classroom discourse through questioning that focused on reasoning, proof and sens.

Selecting Tasks and Sequences of Related Problems

In selecting the problems, the teacher researchers were deliberate in developing and posing problems that were related yet increasingly more complex. This allowed for students to naturally make connections to previous problems solved in class and to build upon the knowledge they had acquired. By having the previous problems displayed on the generalization posters and readily accessible, students had entry to problems and to solution strategies and built new knowledge based on previous knowledge. Through a real life service project for the school to raise money for a natural habitat for the school courtyard, we used a business theme to work on many classes of real life math problems such as budgeting, analysing cost and revenue, maximizing profits, figuring out combination problems, using discounts and comparing prices using unit pricing. Another design feature that allowed for students to extend their thinking by exploring related problems with increasing complexity was through Thinking Connection cards. Due to the nature of the multi-age and grades of the students, Thinking Connection cards allowed for differentiation and extension for students who were ready to go beyond.

Integrating Pedagogical Content Tools to Extend Students' Reasoning

Pedagogical content tools such as "devices such as graphs, diagrams, equations, or verbal statements" were used "intentionally to connect students thinking while moving the mathematical agenda forward" (Rasmussen & Marrongelle, 2006, p. 388). Representational fluency, the ability to use multiple representations and translate among these models, has shown to be critical in building students' mathematical understanding (Goldin & Shteingold, 2001; Lamon, 2001). Therefore, the research team agreed that it was important to develop students' abilities to represent mathematical ideas in multiple ways including manipulatives, real life situations, pictures, verbal symbols and written symbols. Translations among the different representations assess whether a student conceptually understands a problem. Some of the ways to demonstrate translation among representations in mathematics is to ask students to restate a problem in their own words, draw a diagram to illustrate the problem, or act it out.

In order to promote representational fluency, the research team focused on developing students' mathematical reasoning in the classroom by implementing a design feature called Collective Workspace, Poster Proofs, and Generalization Posters. Collective Workspace (see Figure 3) was a method for students to bring their individual work to their group and discuss different solution strategies and compare each others' strategies specifically looking for connections, efficiency, multiple representations and generality. This collective space and time allowed for students to connect their way of knowing to others strategies, in addition, it allowed students a chance to debate on which strategy was most efficient and effective to broader classes of problems.

Generalization Posters (see Figure 4) were created as a class to summarize the essential mathematical learning. Pictures of individual strategies were attached to these posters so that students use them as a reference for future problems. It also allowed for students to build on each other's ideas as a class so that every student can have ownership of the collective thinking. Just as mathematicians over centuries built on conjectures and theorems, these young mathematicians were given the same opportunity to engage in this mathematical activity.

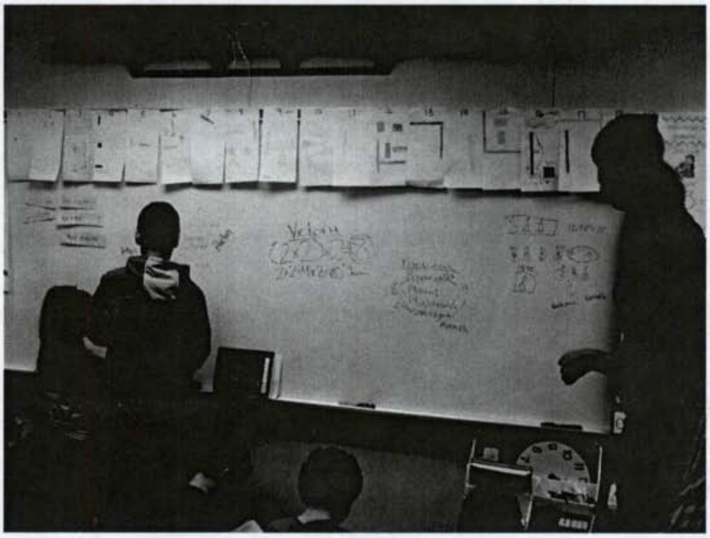


Figure 3. "Collective workspace" to build collective mathematical knowledge.

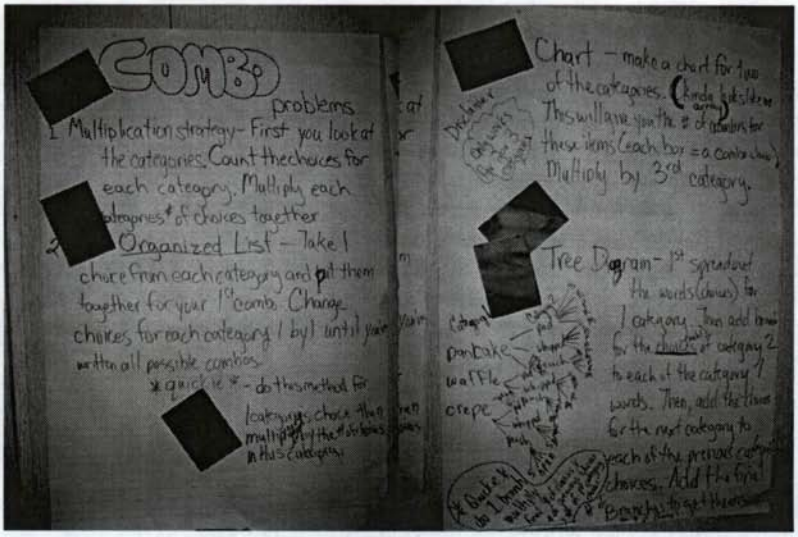


Figure 4. Generalization posters with strategy photos.

Orchestrating Classroom Discourse through Pedagogical Moves and Questioning

The teacher's role in extending students' thinking during this task was in engaging students to determine share strategies and to look for an efficient way to solve

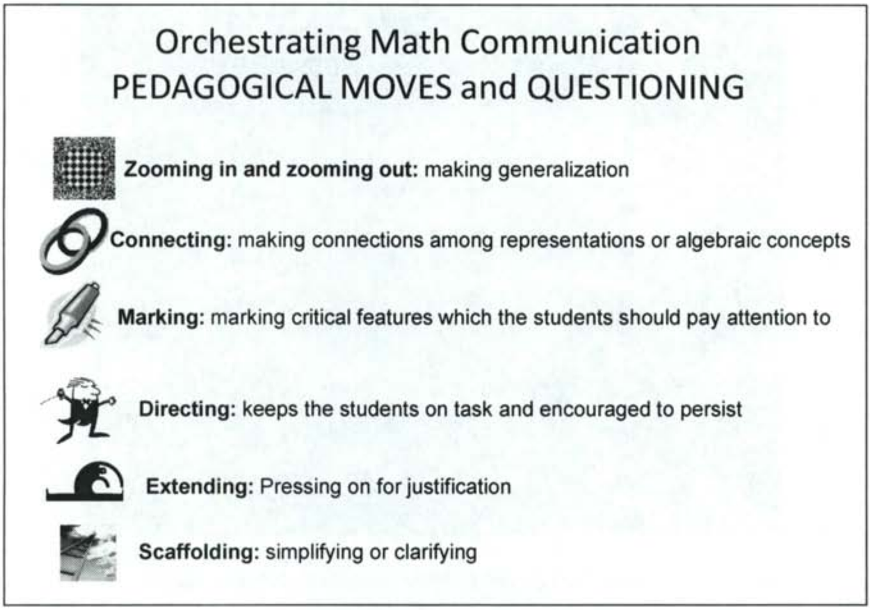


Figure 5. Orchestrating mathematics communication through pedagogical moves and questioning.

combination problem and to generate a rule. To analyse the mathematical discourse, teacher researchers used these codes and instructional strategies to make sense of pedagogical moves and questioning strategies.

In one of the combination problem, students were asked to determine the number of flavors that an ice cream shop owner could offer her customers. Some of the students began by drawing the ice cream cones with different flavors but quickly found that drawing a picture was not an efficient strategy. Below is an excerpt from the discussion that took place during the collective workspace.

Teacher: Let's look closely how your classmates solved this problem. *(PMQ: Zooming In)*

Lana: I drew a picture of the icecream with its topping and syrup but it was not easy so I decided to list all the different combinations. Then I noticed my partner was using the first letter of the flavor, topping and syrup and it seemed like a short cut then writing out the whole word, like strawberry.

Teacher: So you decided to use S to stand for strawberry and C for chocolate and V for vanilla. I see that you have listed the possible icecream. How did you know you had all the possible combinations? *(PMQ: Connecting and Marking)*

Jose: I decided to create a chart with the flavors going down and the topping going across and had a 3 by 4 table. But then I realized for each I also had to decide it I wanted caramel or chocolate syrup. So I had to take the 12 types of ice creams and double them for the syrup and got 24 different combinations.

Mariam: I used a tree. I started with the 3 flavors and each flavor had 4 topping choices and then from there I had 2 syrup choices, so I knew that it would be 3 x 4 x 2 = 24 different kinds.

Teacher: I see that Mariam used multiplication to help her see how many combinations she had. Do others see how this equation may appear in your solution? *(PMQ: Connecting)* So how are your different strategies similar or different from each other? Take a few minute to look at your own and turn to a partner and share. *(PMQ: Zooming in and zooming out)*

Frances: I noticed that Lana's list was done in a similar fashion as Mariam's tree. She seemed to start with one flavour and go to the next topping and then to the syrup. She wrote it each time making sure she did not double it up.

Teacher: Frances said categories for the flavor, topping, and syrup. What were in each category? *(PMQ: revoicing and extending)*

Brandon: There were many choices, for example, there were four topping Choco chip(CC), oreo cookies(OC), rainbow sprinkles(RS) and fresh berries(FB).

Teacher: So what can we write on our Generalization Poster about combination problems? *(PMQ: Zooming out)*

At the end of class, the Generalization Poster read,

IN GENERAL, when solving a combination problem with categories and choices, you can find the number of possible combination by multiplying the choices in each category, for example: Number of flavors x number of toppings x number of syrups = Number of possible combinations. F x T x S = total.

But multiplying will only tell you the total number, not the different types of combinations. For a list of combinations, the tree method works quite well and keeps the list organized. A table is easy if you have two categories but when you have more, you might have to make another table. A smart way to save time is to use a shorten form or just the first letter of the choice so that you are not wasting time writing it all out.

It was during the conversation that took place in the collective workspace that students negotiated the meaning of solving combination problems and concretized the learning for individuals and for the collective group. As evidenced by the excerpt, the advancement of ideas that resulted from students' reasoning became a collective record through the Generalization Poster. In addition, student generated representations, such as, the table, tree diagram, equation and verbal explanation became an important pedagogical content tool for scaffolding questions for algebraic connections, explanations and generalizations and for students to compare, connect and extend their thinking.

CONCLUSIONS

Through this study, the research team developed a guiding framework called *Building Collective Knowledge to Develop Mathematical Proficiency.* Principles to this framework included: a) *adhering to the authenticity of problems*, which proved to be motivating for students. The research team ensured that the task required students to use higher ordered thinking skills, to consider alternate solutions, and to think like a mathematician; b) *making connections and generalizations* to important mathematical ideas that goes beyond application of algorithms by elaborating on definitions and making connections to other mathematical concepts, which led to; c) *navigating through guided reinvention,* (Gravemeijer & Galen, 2003) where students go through similar processes as mathematicians so that they see the mathematical knowledge as a product of their own mathematical activity (p. 117); d) *elaborating and communication through justification*, where students demonstrate a concise, logical, and well-articulated explanation or argument that justifies mathematical work; e) *participating in shared learning* and the interdependence of social and individual processes in the co-construction of knowledge.

In this study, the research team made up of the university mathematics educator and gifted and talented specialist benefited from the opportunity to plan and debrief together which allowed them to determine when, what kind and how to use tools such as graphs, diagrams, equations, spreadsheets, or verbal statement to connect students thinking and to build collective mathematical knowledge in the classroom. This process required the combination of pedagogical and mathematical knowledge. This study suggests that well-intentional and purposeful integration of mathematical tools such as representations, notations and explanations and the use of critical pedagogical moves and questioning can help build collective mathematical knowledge in the classroom and build mathematical potential in diverse learners. In addition, having a shared understanding of the core traits of mathematically proficient students allowed for the research team to provide frequent opportunities for "young scholars" to exhibit those traits and encouraged those behaviour characteristics when students demonstrated them in class. Developing mathematical potential and enhancing mathematics instruction for diverse learners from a low socio-economic population is a critical need in education. Through encouragement and participation in problem solving, mathematical discourse and algebraic reasoning, these students began to exhibit more confidence and competence in exhibiting the traits of mathematically proficient students and developed these as habits of young mathematicians.

REFERENCES

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal, 90*, 449–466.

Boaler, J. (1999). Mathematics for the moment, or the millennium? *Education Week.* Downloaded from www.edweek.org/ew/ewstory.cfm?slug=29boaler.h18&keywords=boaler*

Brown, A. L. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *Journal of the Learning Sciences, 2*(2), 141–178.

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking and classroom practice. In N. Minick, E. Forman, & A. Stone (Eds.), *Education and mind* (pp. 91–119). New York: Oxford University Press.

Design-based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher*, *32*(1), 5–8.

Diversity in Mathematics Education Center for Learning and Teaching. (2007). Culture, race, power and mathematics education. In F. Lester (Ed.), *The second handbook of research on mathematics teaching and learning* (pp. 405–434). Reston, VA: NCTM.

Ford, D. Y., Harris, J. J., III, Tyson, C. A., & Frazier Trotman, M. (2002). Beyond deficit thinking: Providing access for gifted African American students. Beyond self-concept and self-esteem for African American students: Improving racial identity improves achievement. *The High School Journal*, *87*(1), 18–29.

Gavin, S. (2003). *Scales for rating the behavioral characteristics of superior students*. Mansfield Center, CT: Creative Learning Press.

Goldin, G., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representations in school mathematics* (pp. 1–23). Reston, VA: National Council of Teachers of Mathematics.

Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical Thinking and Learning*, *I*(2), 155–177.

Gravemeijer, K., & van Galen, F. (2003). Facts and algorithms as products of students' own mathematical activity. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 114–122). Reston, VA: NCTM.

Lamon, S. (2001). Presenting and representating from fractions to rational numbers. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in school mathematics, 2001 yearbook* (pp. 146–165). Reston, VA: National Council of Teachers of Mathematics.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, *27*(1), 29–63.

Lesh, R., Cramer, K., Doerr, H., Post, T., & Zawojewski, J. (2003). Using a translation model for curriculum development and classroom instruction. In Lesh, R. & Doerr, H. (Eds.), *Beyond constructivism. Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

McClain, K., & Cobb, P. (2001). An analysis of development of sociomathematical norms in one first grade classroom. *Journal for Research in Mathematics Education*, *32*(3), 238–266.

National Research Council. (2001). Adding it up: Helping children learn mathematics. In J. Kilpatrick, J. Swafford, & B. Findell (Eds.), *Mathematics learning study committee, center for education, division of behavioral and social sciences and education*. Washington, DC: National Academy Press.

Rasmussen, C., & Marrongelle, K. (2006). Pedagogical content tools: Integrating student reasoning and mathematics in instruction. *Journal for Research in Mathematics Education*, *37*(5), 388–420.

Renzulli, J., Leppien, J., & Hays, T. (2000). *The multiple menu model: A critical guide for developing differentiated curriculum*. Manfield Center, CT: Creative Learning Press.

Renzulli, J. S., Gentry, M., & Reis, S. M. (2003). *Enrichment clusters: A practical plan for real-world, student-driven learning*. Mansfield Center, CT: Creative Learning Press.

VanTassel-Baska, J., Johnson, D., & Avery, L. D. (2002). Using performance tasks in the identification of economically disadvantaged and minority gifted learners: Findings from Project STAR. *Gifted Child Quarterly*, *46*(2), 110–123.

Vygotsky, L. (1986). *Thought and language*. Cambridge, MA: The MIT Press.

Jennifer Suh
George Mason University
USA

Kerri Fulginiti
Fairfax County Public School
USA