SCAFFOLDING SPECIAL NEEDS STUDENTS' LEARNING OF FRACTION EQUIVALENCE USING VIRTUAL MANIPULATIVES

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This collaborative action research project explored strategies for enhancing mathematics instruction for students with special needs using virtual manipulatives. Teachers and researchers employed qualitative action research methods using memos to systematically record the processes of planning, teaching, observing and reflecting. Results showed that teachers' opportunities for reflection and discussion influenced their instructional strategies. Teachers' observations of their students' learning indicated that affordances in the virtual manipulative applets enabled special needs learners to "off-load" information and focus more on mathematical processes and relationships among equivalent fractions.

THEORETICAL FRAMEWORK

Kaput (1992) noted that the impact of technological tools in mathematics learning and teaching is the ability to "off-load" routine tasks, such as computations, to compact information and providing greater efficiency in learning. More recently, Zbiek, Heid, Blume, and Dick (2007) highlighted affordances of technology tools in mathematical activity, based on their externalized and linked representations, dynamic actions, and built-in constraints. Pea (1987) defines cognitive technologies as "technologies that help transcend the limitation of the mind…in thinking, learning and problem solving activities" (p. 91). Technology tools in mathematics have the capability to graph, compute, visualize, simulate, and manipulate, while providing users with immediate feedback.

When students work with physical manipulatives, one major challenge is that the manipulation of multiple pieces creates an excessive cognitive load for learners. This causes them to lose sight of the mathematics concept that is the intent of the lesson and prevents them from connect the physical manipulations with mathematical ideas. Kaput's (1989) explanation for this disconnect was that the cognitive load imposed during the activities with physical manipulatives was too great, and that students struggled to maintain a record of their actions. Essentially, students are unable to track all of their actions with the physical manipulatives, and additionally, are not capable of connecting multiple actions with mathematical abstraction and symbol manipulation.

Recent research seems to indicate that the built in constraints in virtual manipulatives overcomes some of the limitations of physical manipulatives. For example, research using virtual manipulatives has shown benefits for ESL learners (Moyer, Niezgoda, & Stanley, 2005) and lower ability learners (Moyer & Suh, 2008). The topic of fraction equivalence was chosen for this project because it is an important prerequisite to understanding rational numbers and the addition, subtraction, multiplication and division of fractions. Developing visual models for fractions is crucial in building fraction understanding. Yet conventional instruction on fraction computation tends to be rule based. In particular, special needs learners often receive direct instruction on "how to" perform algorithmic procedures using mnemonic devices or steps to follow without having opportunities to construct conceptual understandings of mathematical processes. The research question that guided this collaborative action research was: How can teacher reflection and discussion be used to enhance instruction using virtual fraction applets with special needs students?

METHODOLOGY

Participants and Data Sources

The participants in this study were 19 fourth-grade students. Ten of the 19 were identified as special needs students and had Individual Education Plans (IEPs). Both a regular education teacher and a special education teacher worked collaboratively in this inclusive classroom. Students attended a Title One designated elementary school in a major metropolitan area with a diverse population of 600 students at the school: 51% Hispanics, 24% Asians, 16% Caucasians, 3% African Americans and 6% others, with over 50% receiving free and reduced lunch.

The qualitative data included teachers' and researchers' memos during the planning, teaching, observing and debriefing processes, students' written work, student interviews and classroom videotapes. Students' written work contained drawings, solution procedures, numeric notations and explanations. These qualitative data were examined and categorized along dimensions of students' strategies and sense making procedures. Student interviews, memos, and classroom videotapes were used to triangulate students' understanding of fraction equivalence.

Research Design

The study used qualitative methods of collaborative action research using memos. According to Miles and Hubermann, memos are essential techniques for qualitative analysis. They do not just report data; they are a powerful "sense-making" tool that ties together different pieces of data into a recognizable form (Miles & Hubermann, 1994, p. 72). Maxwell (1996) recommends regular writing of memos during qualitative analysis to facilitate analytic thinking, stimulate analytic insights, and capture one's thinking. Strauss and Corbin (1990) recommend that memo writing begins from the inception of a research project and continues until the final writing. To follow this design, researchers asked teachers to record their observations and reflections throughout the planning, teaching, observing and debriefing phases of the study. These memos were collected and analyzed for emerging themes.

Procedures

In the first phase of the study, teachers identified topics that were challenging to special needs learners based on previous state assessments. Based on these topics, teachers and researchers chose fraction equivalence as the focus of the study. To begin the collaborative planning, one researcher, a mathematics educator, engaged the classroom teachers and special educators in planning a lesson. This process involved three phases over two 3 hour sessions: 1) Collaborative planning phase, where novice, experienced, and special educators collaborated on planning the lesson; 2) Teaching and observation, where one teacher taught the lesson and the others observed and took notes; 3) Debriefing phase, where teachers reflected on the lesson design, tasks, student engagement and learning and discussed future steps.

Planning of the lesson

To begin the planning, teachers created a concept map that unpacked the concept of equivalent fractions. (See figure 1.)

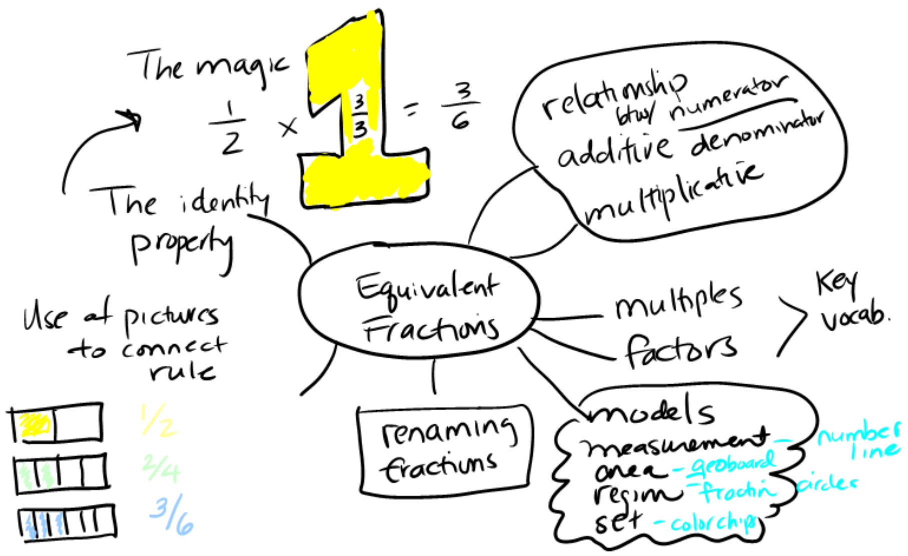

Figure 1: Concept map for fraction equivalence.

Some of the guiding questions crucial to the planning and teaching processes were:

- What is the important mathematical understanding that students need to learn?
- What are potential barriers and anticipated student responses?
- What conceptual supports and instructional strategies can best address our students' learning? How will we respond when students have difficulty?
- How will we know when each student has learned it?

Teachers took active notes and memos to self as they proceeded in the research.

Teaching of the lesson

Researchers used the virtual manipulative tool, Equivalent Fractions, from the National Library of Virtual Manipulatives (at http://matti.usu.edu). (See figure 2.)

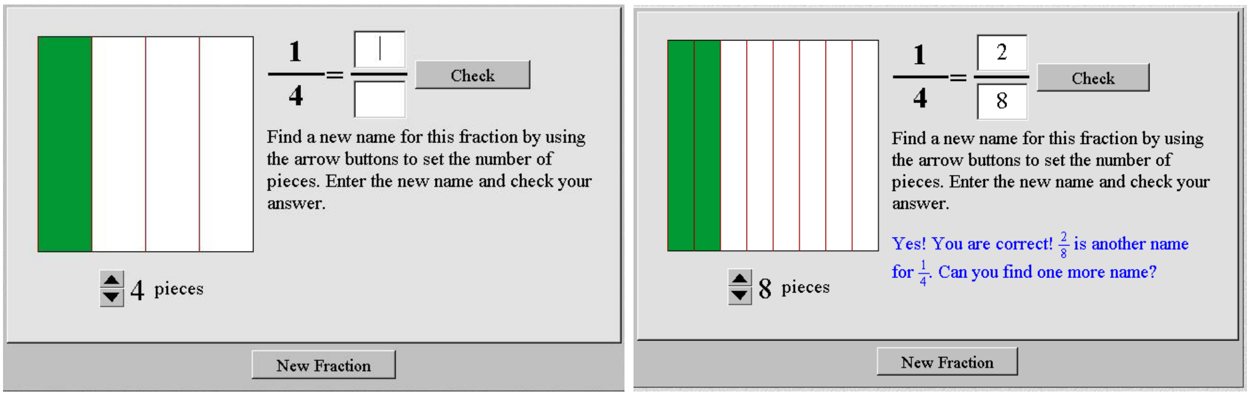

Figure 2: Virtual manipulative applet for fraction equivalence.

The objective for the 4th grade lesson on fractions was renaming fractions to find equivalent fractions. A subsequent lesson focused on adding and subtracting fractions with unlike denominators. The Fraction Equivalence applet allowed students to explore relationships among equivalent fractions. The applet presents students with a fraction region (circle or square) with parts shaded. The text on the applet directs students to: "Find a new name for this fraction by using the arrow buttons to set the number of pieces. Enter the new name and check your answer." To do this, students click on arrow buttons below the region, which changes the number of parts. When students find an equivalent fraction, all lines turn red. Students then record a common denominator and corresponding numerator in the appropriate boxes, and check their answers by clicking the "check" button. Throughout this process, pictures are linked to numeric symbols that dynamically change with moves made by the student. To help students explore relationships among equivalent fractions, the applet prompts students to find several equivalent fractions. This applet was specifically designed to develop the concept of renaming fractions. Unlike physical manipulative fraction pieces, the virtual fraction applet allows students to equally divide a whole into 99 pieces, thereby generating multiple equivalent fraction names. In the final task, students were asked to create a rule for finding equivalent fractions.

RESULTS

Analysis of the Memos from Planning

In their memos on planning, teachers wrote that the concept map was extremely helpful in planning the lesson for the special needs students.

> Teacher 1: I have used concept maps to teach writing, but never thought of using them in mathematics. It helped me unpack the math and see the complex nature of fractions. It made me see how all of the other skills were linked to fraction equivalence. By unpacking the mathematics, I was able to see what

> skills I may have to remediate with my special needs students in order for them to grasp this concept.

Another teacher was excited to see how the computer based virtual manipulative might give students more guidance with the concept.

> Teacher 2: Using the virtual manipulatives as a learning support will be interesting to see, since many times, students get so distracted by the physical manipulatives. But with this virtual manipulative, students can't play around as much with the tools. They will have a chance to experiment without really goofing off.

A common theme from the memos was the importance of the mathematical discourse among teachers about sequencing the lesson, anticipating students' misconceptions, and preparing teaching strategies to overcome those misconceptions.

> Teacher 3: Discussing the common misconceptions and errors that student made on previous assessments allowed for us to pinpoint what the problem was with our past teaching strategies.

Analysis of the Memos from Teaching and Observing

The teacher's role in extending students' thinking during this task was in engaging students to record a list of equivalent fractions, to determine patterns among the numbers, and to generate a rule. For example, using the applet on a SMARTBOARD, one student showed the class that 1/3=2/6=3/9=4/12. As this was recorded on the board, students' eyes started to widen and hands raised saying:

> Student 1: Oh, oh, I know the rule! The denominators are going by a plus 3 pattern.

> Student 2: It's like skip counting.

> Student 3: It's the multiple of 3 like 3, 6, 9, 12…yeah.

As students shared their observations, some noticed the additive rule. To encourage students to explore the relationship more deeply, the teacher asked students to think about equivalent fractions for 2/3, so that they could see the multiplicative patterns in the numerators and the denominators. Students listed 2/3=4/6=6/9 and again they quickly saw the additive pattern and the multiples of two for the numerator and three for the denominator. Then the teacher posed the question: Are 2/3 and 20/30 equivalent fractions? What about 2/3 and 10/15?

Students used the fraction applet to explore this question with a partner. Other fractions were provided to encourage students to determine relationships beyond the additive rule. When students returned to the whole class discussion, several of them shared their discoveries. One student noted:

> Student: The fractions 2/3 and 20/30 are equivalent because you multiply both numerator and denominator by ten. And in 2/3 = 10/15, you multiply both numerator and denominator by 5.

The teacher extended the previous student’s comment by taking out a learning tool she called “the magic 1” and placed it next to the fraction 2/3. With an erasable marker, the teacher wrote 10/10 on the laminated “magic 1” next to 2/3 and said,

> Teacher: So are you saying that we are multiplying it by 10/10, otherwise know as 1? Turn to your partner and talk about this.

This discussion led to a lively conversation on how 10/10 and 5/5 equal one whole. The teacher connected this idea to the identity property of multiplication by asking:

> Teacher: What happens when we multiply any number by one?

This discussion reinforced the idea that, no matter how you rename the fractions, as long as you multiply it by one or *n/n*, the result will be an equivalent fraction. To challenge the students, the teacher posed the question:

> Teacher: What would the equivalent fraction be for 1/3 if the denominator was divided into 99 parts?”

This type of questioning encouraged students to extend their thinking by making conjectures and testing their rule or hypothesis.

Analysis of the Memos from the Debriefing

During the debriefing, teachers noted that students were able to generate a rule and stick with that rule to test other equivalent fractions. Among the different rules were:

1) The additive rule: Students tended to list a sequence of equivalent fractions:

> 1/2 = 2/4 = 3/6 = 4/8
>
> Each denominator is increasing by 2s and the numerator is increasing by one.
>
> 2/3 = 4/6 = 6/9 = 8/12
>
> Each denominator is increasing by 3s and the numerator is increasing by two

2) Doubling numerator and denominator

> 1/2= 2/4=4/8=8/16 : 2/3= 4/6= 8/12=16/24
>
> Students doubled the numerator and denominator

Only a few students noticed the multiplicative relationship prior to the group discussion. Teachers noted, during the debriefing, that perhaps listing the fractions horizontally led students to seek a pattern going across the series of fractions leading to the additive rule. In a later class discussion, the teacher listed some of the equivalent factions in pairs to extend their discussion of the relationships. For example, she listed:

Teachers reported that the use of the virtual manipulatives seemed to provide special needs students with greater access to the mathematics by allowing them the flexibility to make and test conjectures with the applet. Teachers agreed that the lesson gave students a better understanding of using “one” to find an equivalent fraction.

> Teacher 1: In the past years, we have used the idea of the “magic number 1” to show students that you can find equivalent fractions when you multiply both numerator and denominator by the same number such as, 3/3=1. So if you multiply 3/3 by 1/2 that equals 3/6, which is equivalent to ½, but before I think we taught it like a procedure to follow. I know for sure, my special education students just learned “the trick” without really understanding it and without having a mental image of it. After having used the virtual manipulatives and having to record the list and draw the pictures that they saw on the screen, I feel that students have a better understanding of the idea that multiplying it by *n*/*n* is not changing the fraction but simply renaming it.

The applet seemed to benefit special needs learners by giving them built in supports for the mathematical ideas that reduced their cognitive overload. Having the visual and numeric representations closely tied together on the screen allowed students to make direct connections between the images of fractions and the fraction symbols. The kinesthetic/tactile advantages of using the SMARTBOARD also enabled special needs students to be more involved in the manipulation of the fractions on the screen.

CONCLUSION

In this study teachers benefited through their collaborative reflection which impacted their teaching strategies for fraction concept development. Students benefited as their teachers’ reflective actions translated into instruction, and from the unique affordances of the virtual manipulative tools which were a particular support for the special needs students in the class. The use of the fraction applets allowed students to think and reason about relationships among equivalent fractions. Opportunities to work with student partners encouraged mathematical discourse. Unique features of the virtual tools enabled special needs students to off load the task of maintaining both pictorial images and symbolic notations as the images and notations changed in response to the students’ input. This allowed students to focus more on mathematical processes and relationships, enabling them to formulate a rule that made sense. Kaput (1992) stated that constraint-support structures built in to computer based learning environments “frees the student to focus on the connections between the actions on the two systems [notation and visuals], actions which otherwise have a tendency to consume all of the students cognitive resources even before translation can be carried out” (p.529). The potential of these tools, used in lessons where teachers and students are engaged in meaningful discussions about the mathematics, is important to explore for special needs learners. The linked representations in the virtual fraction environment offer meta-cognitive support, such as keeping record of users’ actions and numeric notations. This allows learners to use their cognitive capacity to observe and reflect on connections and relationships among the representations. This study suggests that integrating reflective planning with effective mathematical tools can benefit special needs students, especially when they are exploring concepts where stored images and notations are necessary for student learning.

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