BUILDING COLLECTIVE MATHEMATICAL KNOWLEDGE IN THE ELEMENTARY GRADES USING PROBLEM BASED LEARNING AND PEDAGOGICAL CONTENT TOOLS

Jennifer M. Suh

George Mason University

Kerri Fulginiti

Westlawn Elementary School

This design research explored strategies for enhancing mathematics instruction for students from a low socio-economic diverse population by developing collective mathematical knowledge using problem based learning and pedagogical content tools. Results showed that teachers' opportunities for reflection and discussion focused on developing collective knowledge influenced many pedagogical decisions. The most prominent pedagogical decisions influenced were a) selecting tasks and sequence of problems, b) integrating pedagogical content tools, and c) orchestrating classroom discourse through pedagogical moves and questioning. As a result the teacher researchers developed a working framework to guide their efforts in building collective knowledge to enhance students' learning.

THEORETICAL FRAMEWORK

Sociocultural approaches emphasize the interdependence of social and individual processes in the co-construction of knowledge (Vygotsky,1986). Through participation in activities that require cognitive and communicative functions, children are drawn into the use of these functions in ways that nurture and 'scaffold' them" (pp. 6-7). Vygotsky (1986) described learning as being embedded within social events and occurring as a child interacts with people, objects, and events in the environment. Through socially shared activities learners also internalized processes. Following this approach, researchers have explored sociomathematical norms and how teachers actively guide the development of classroom mathematical practices and individual learning through capitalizing on opportunities that emerge through students' activities and explanations (Ball, 1993; Cobb, Wood & Yackel, 1993; Lampert, 1990; McClain & Cobb, 2001).

However, recent research offers some important insight on participation gaps which exist among students from diverse social, cultural, and racial backgrounds in mathematics classrooms and how classrooms can be structured to better afford opportunities to participate in mathematics by a wider range of students (DIME, 2007). Typically, school systems that serve economically disadvantaged or minority student struggle to meet academic achievement and traditionally reform movements

2009. In Tzekaki, M., Kaldrimidou, M. & Sakonidis, C. (Eds.). *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 177-184. Thessaloniki, Greece: PME.

have aimed at remedial models to improve students' achievement. According to this model, students from diverse populations get more of the basic skill learning without the opportunity to participate in learning opportunities that develop unique talents, creative thinking and problem solving strategies. For students to be prepared for the 21st century they need to be critical thinkers, problem solvers, and effective communicators who are proficient in core subjects, technology literacy skills, and life skills.

In reform oriented approaches like problem based learning, students work in teams to explore real-world problems and create presentations to share what they have learned. Compared with learning primarily from textbooks, this approach has many benefits for students, including deeper knowledge of subject matter, increased self-direction and motivation, improved research and problem-solving skills, and understanding how academics connect to real-life and careers. The study by Boaler (1999) found that students at the problem-based school did better than those at the more traditional school both on math problems requiring analytical or conceptual thought and on those considered rote, that is, those requiring memory of a rule or formula.

This following collaborative design research focused on building collective mathematical knowledge in the classroom by integrating students' reasoning and proof and designing meaningful 'pedagogical content tools'. Pedagogical content tools have been defined as "devices such as graphs, diagrams, equations, or verbal statements that teachers intentionally use to connect students thinking while moving the mathematical agenda forward" (Rasmussen & Marrongelle, 2006, p.388).

The research questions that guided this design research were:

- What influences do teachers' deliberate reflections and discussions focused on developing collective knowledge in the classroom and making connections and generalizations have on pedagogical decisions?
- What design features of the project and pedagogical content tools promote development of collective knowledge, algebraic connections and generalizations methods among elementary students?

METHODOLOGY

The participants in this study were sixteen fourth through sixth grade students who participated in the summer program focused on problem based learning. These students attended a Title One designated elementary school in a major metropolitan area with a diverse population of 600 students at the school: 51% Hispanics, 24% Asians, 16% Caucasians, 3% African Americans and 6% others, with over 50% receiving free and reduced lunch. Many of these students' former classroom teachers recommended them based on their exhibition of mathematically promising traits during the academic year. The focus of the summer program was to immerse students in authentic mathematics problem solving while utilizing local community resources such as invited community speakers. The idea was to expose students from diverse backgrounds to challenging mathematics while fostering their algebraic thinking and positive attitude towards mathematics. The project called *MATH 4-1-1: Young Mathematicians on Call* had students solving rich engaging, meaningful and mathematically complex problems presented by the community.

The study used qualitative methods, specifically, the design research approach and research memos. The design-based research method is aimed at improving educational practices through systematic, flexible, and iterative review, based upon collaboration among researchers and practitioners in real-world settings, which leads to design principles or theories (Brown, 1992; The Design-Based Research Collective, 2003). Using design research which emphasizes the processes of iteration, feedback loops and narrative reports, we refined the key components of our shared learning activities. As teacher researchers, we took active notes and memos as we proceeded with the research exploring the interplay between the individual and collective knowledge in the mathematics classroom through discourse and the development of new ideas through records of students' thinking. We collected video recordings of each class session, retained copies of students' work and work displayed in the "collective workspace" and recorded observations and reflections throughout the planning, teaching, and debriefing phases of the study. Daily debriefing meetings allowed for formative analysis which focused on what was revealed during the class session and how to plan for subsequent classes by modifying the task, tools and teaching methods based on the feedback. In addition, these memos and artifacts (i.e. students' written work contained drawings, solution procedures, numeric notations and explanations) were analyzed for emerging themes at the end of the project for a summative analysis.

RESULTS

Qualitative analysis from the teacher-researchers' memos indicated that the process of collaborative planning, teaching and debriefing focused on developing collective knowledge impacted many pedagogical decisions. The most prominent pedagogical decisions influenced were a) selecting tasks and sequences of problems, b) integrating pedagogical content tools, and c) orchestrating classroom discourse through pedagogical moves and questioning.

a) Selecting tasks and sequences of problems. In selecting the problems, the teacher researchers were deliberate in developing and posing problems that were related yet increasingly more complex. This allowed for students to naturally make connections to previous problems solved in class and to build upon the knowledge they had acquired. By having the previous problems displayed on the *Generalization Posters* and readily accessible, students had entry to problems and to solution strategies and built new knowledge based on previous knowledge. The problems were embedded in a real-life service project for the school to raise money for a natural habitat in the school courtyard. Through this, we used a business theme to work on many classes of real life math problems such as budgeting, analysing cost and revenue, maximizing profits, figuring out combination problems, using discounts and comparing prices using unit pricing. Another design feature that allowed for students to extend their thinking was through *Thinking Connection Cards*, which presented related problems with increasing complexity. Due to the nature of the multi-age and grades of the students, *Thinking Connection Cards* allowed for differentiation and extension for students who were ready.

b) Integrating pedagogical content tools to extend students' reasoning. In order to promote students' mathematical reasoning in the classroom, we implemented a design feature called *Collective Workspace* and *Generalization Posters*, sometimes referred to as *Poster Proofs*. *Collective Workspace* was a method for students to bring their individual work to their group and discuss different solution strategies and compare each others' strategies specifically looking for connections, efficiency, multiple representations and generality. This collective space and time allowed for students to connect their way of knowing to other strategies. In addition, it allowed students a chance to debate on which strategy was most efficient and effective to broader classes of problems.

Figure 1. "Collective Workspace" to build collective mathematical knowledge

Generalization Posters were created as a class to summarize the essential mathematical learning. Pictures of individual strategies were attached to these posters so that students could use them as a reference for future problems. It also allowed for students to build on each other's ideas so that every student has ownership of the collective thinking. Just as mathematicians over centuries built on conjectures and theorems, these young mathematicians were given the same opportunity to engage in building collective knowledge

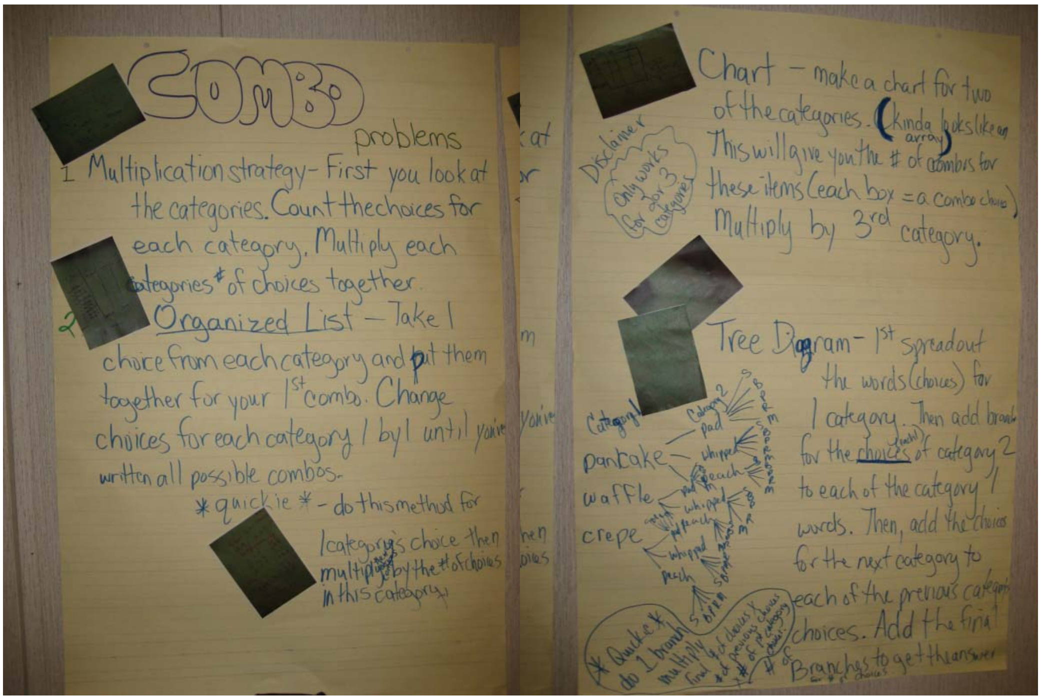

Figure 2. Generalization Posters with strategy photos

c) Orchestrating classroom discourse through pedagogical moves and questioning. The teacher's role in extending students' thinking during this task was in engaging students to share strategies and to look for an efficient way to solve problems and to generate a rule. To analyse the mathematical discourse, teacher researchers used these codes to make sense of pedagogical moves and questioning strategies.

PEDAGOGICAL MOVES and QUESTIONING

Zooming in and zooming out: making generalization
Connecting: making connections among representations or algebraic concepts
Marking: marking critical features which the students should pay attention to.
Directing: keeps the students on task and encouraged to persist;
Extending: Pressing on for justification
Scaffolding: simplifying or clarifying

In one of the combination problems, students were asked to determine the number of sundae choices an ice cream shop owner could offer her customers. Some of the students began by drawing the ice cream with different flavors and toppings, but quickly found that drawing a picture was not an efficient strategy. Below is an excerpt from the discussion that took place during the collective workspace.

> Teacher: Let's look closely at how your classmates solved this problem. *(PMQ: Zooming In)*
>
> Lana: I drew a picture of the ice cream with its topping and syrup, but it was not easy so I decided to list all the different combinations. Then I noticed my partner was using the first letter of the flavor, topping and syrup and it seemed like a short cut then writing out the whole word, like strawberry.
>
> Teacher: So you decided to use S to stand for strawberry and C for chocolate and V for vanilla. I see that you have listed the possible sundaes. How did you know you had all the possible combinations? *(PMQ: Connecting and Marking)*
>
> Jose: I decided to create a chart with the flavors going down and the topping going across and had a 3 by 4 table. But then I realized for each I also had to decide it I wanted caramel or chocolate syrup. So I had to take the 12 types of ice creams and double them for the syrup and got 24 different combinations.
>
> Mariam: I used a tree. I started with the 3 flavors and each flavor had 4 topping choices and then from there I had 2 syrup choices, so I knew that it would be 3 x 4x 2 = 24 different kinds.
>
> Teacher: I see that Mariam used multiplication to help her see how many combinations she had. Do others see how this equation may appear in your solution? *(PMQ: Connecting)* So how are your different strategies similar or different from each other? Take a few minute to look at your own and turn to a partner and share. *(PMQ: Zooming in and zooming out)*
>
> Frances: I noticed that Lana's list was done in a similar fashion as Mariam's tree. She seemed to start with one flavor and go to the next topping and then to the syrup. She wrote it each time making sure she did not double it up.
>
> Teacher: Frances mentioned flavors, toppings, and syrups. What were in each category? *(PMQ: Marking and extending)*
>
> Brandon: There were many choices, for example, there were four topping Choco chips(CC), oreo cookies(OC), rainbow sprinkles(RS) and fresh berries(FB).
>
> Teacher: So what can we write on our Generalization Poster about combination problems? *(PMQ: Zooming out)*

At the end of class, the Generalization Poster read, IN GENERAL, when solving a combination problem with categories and choices, you can find the number of possible combination by multiplying the choices in each category, for example: Number of flavors x number of toppings x number of syrups=Number of possible combinations. F x T x S= total

But multiplying will only tell you the total number, not the different types of combinations. For a list of combinations, the tree method works quite well and keeps the list organized. A table is easy if you have two categories but when you have more, you might have to make another table. A smart way to save time is to use a shorten form or just the first letter of the choice so that you are not wasting time writing it all out.

It was during the conversation that took place in the collective workspace that students negotiated the meaning of solving combination problems and concretized the learning for the individual and for the collective group. As evidenced by the excerpt, the advancement of ideas that resulted from students' reasoning became a collective record through the *Generalization Poster*. In addition, student generated representations, such as, the table, tree diagram, equation and verbal explanation became important pedagogical content tools for scaffolding questions for algebraic connections, explanations and generalizations and for students to compare, connect and extend their thinking.

CONCLUSION

Through this research, we developed a working framework called Building Collective Knowledge to Enhance Students' Learning. Principles to this framework included, a) *adhering to the authenticity of problems*, which proved to be motivating for students. The teacher researchers ensured that the task required students to use higher ordered thinking skills, to consider alternate solutions, and to think like a mathematician; b) *making connections and generalizations* to important mathematical ideas that go beyond application of algorithms by elaborating on definitions and making connections to other mathematical concepts, which led to; c) *navigating through guided reinvention,* (Gravemeijer & Galen, 2003) where students go through similar processes as mathematicians so that they see the mathematical knowledge as a product of their own mathematical activity (p. 117); d) *elaborating and communication through justification***,** where students demonstrate a concise, logical, and well-articulated explanation or argument that justifies mathematical work; e) *participating in shared learning* and the interdependence of social and individual processes in the co-construction of knowledge.

In this study, we benefited from the opportunity to plan and debrief together which allowed us to determine when, what kind and how to use tools such as graphs, diagrams, equations, spreadsheets, or verbal statement to connect students thinking and to build collective mathematical knowledge in the classroom. This process required the combination of pedagogical and mathematical knowledge. This study suggests that integrating reflective planning with effective mathematical tools such as representations, notations and explanations and the use of critical pedagogical moves and questioning can help build collective mathematical knowledge in the classroom.

References

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal*, *90*, 449-466.

Boaler, J. (1999). *Mathematics for the moment, or the millennium? Education Week.* Downloaded from www.edweek.org/ew/ewstory.cfm?slug=29boaler.h18&keywords=boaler*.

Brown, A.L. (1992) Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *Journal of the Learning Sciences*, *2*(2), 141-178.

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking and classroom practice. In N. Minick E. Forman & A. Stone (Eds.) *Education and mind* (pp.91-119). New York: Oxford University Press.

Design-based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher*, *32*(1), 5–8.

Diversity in Mathematics Education Center for Learning and Teaching.(2007). Culture, race, power and mathematics education. In F. Lester (Ed.), *The second handbook of research on mathematics teaching and learning* (pp. 405-434). Reston, VA:NCTM.

Gravemeijer, K., & van Galen, F. (2003). Facts and algorithms as products of students' own mathematical activity. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 114-122). Reston, VA: NCTM.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal* *27*(1), 29-63.

McClain, K. & Cobb, P. (2001). An analysis of development of sociomathematical norms in one first grade classroom. *Journal for Research in Mathematics Education*, *32*(3), 238-266.

Rasmussen, C., & Marrongelle, K. (2006). Pedagogical content tools: Integrating student reasoning and mathematics in instruction. *Journal for Research in Mathematics Education*, *37*(5), 388-420.

Vygotsky, L. (1986). Thought and language. Cambridge, MA: The MIT Press.