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Leveraging Cognitive Technology Tools to Expand Opportunities for Critical Thinking on Data Analysis and Probability in Elementary Classrooms

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The following case studies describe technology-enhanced mathematics lessons in two diverse fifth and sixth grade classrooms at a Title I elementary school near the metropolitan area. The project's primary goal was to design tasks to both leverage technology and enhance access to critical thinking in mathematics, particularly with data analysis and probability concepts. This paper highlights the opportunities that technology-rich mathematics environments afford the teachers and the students in teaching and learning mathematics through critical thinking. In addition, the case studies illustrate how to design and implement mathematical tasks using technology to provide opportunities for higher mathematical thinking processes as defined by the Process Standards of the National Council of Teachers of Mathematics (NCTM, 2000): problem solving, connections, representations, communication, reasoning and proof.

Currently, an important pedagogical consideration for all mathematics educators is to leverage technology to improve mathematics teaching and learning. Often times the challenge in incorporating technology as a mathematics pedagogical tool is that it requires a reconceptualization of the nature and content of mathematics, goals and materials. Technology enhances the ability to present and explore mathematics in novel and efficient ways. By successfully leveraging technology, students who have not typically had learning opportunities which incorporated technology have the ability to gain access to mathematical concepts using cognitive technology tools for exploring and doing mathematics. The notion of improving opportunities and access to technology has been the focus of bridging the digital divide among groups who have physical access to technology and those who have not, especially among socio-economic, racial and geographic groups. The Diversity in Mathematics Education Center's article (2007), "Culture, Race, Power, and Mathematics Education", provides evidence that opportunity gaps can lead to achievement gaps among student groups. These diverse student groups require more opportunities to access high quality instruction in rigorous mathematics learning environments in order to succeed in bridging the achievement gaps.

The focus of the present study was to examine and describe the process of designing mathematical tasks which used technology to both expand access and provide opportunities to elicit critical thinking and reasoning with diverse learners. This research involved the collaboration of classroom teachers and the researcher who jointly planned technology-enhanced mathematics lessons. Two underlying research questions were: 1) what affordances exist in a technology-filled learning environment that promote mathematical thinking? 2) what mathematical processes become amplified by the use of technology tools? This article presents case studies that document the development, implementation and evaluation of lessons designed to leverage technology as a cognitive technology tool.

REVIEW OF RESEARCH LITERATURE

Cognitive Technology Tools

Students in 21st century schools are growing up in a technology-advanced society where working flexibly and thinking critically with technology to problem solve is an increasingly important skill. Jonassen (1996) defined computers as mind tools that should be used for knowledge construction while learners engage in critical thinking about the content they are studying. According to the Technology Principle in the Principles and Standards of School Mathematics (NCTM, 2000), "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and *enhances* student learning" (p. 24). The word "enhances" is what characterizes technology as a tool with high leveraging power because technology has specific affordances that can enrich learning tasks. Some of the affordances of technology in mathematics include the ability to graph, compute, visualize, simulate, manipulate dynamic objects, and give the user immediate and visual feedback. When used appropriately with a purposeful mathematical task, affordances in technology can be used to innovate teaching. Pea (1987) defined cognitive technologies as “technologies that help transcend the limitation of the mind…in thinking, learning and problem solving activities” (p.91). More recently, Zbiek, Heid, Blume, and Dick (2007) highlighted the importance of technology tools in mathematical activities with its externalization of representations, dynamic actions and linkage among multiple representations that promotes representational fluency. One of the affordances of cognitive technology tools is that these external representations displayed on the screen have the potential to provide students with internal mental representations of specific mathematical ideas. The dynamic nature of the tool allows for students to manipulate the objects in the virtual environment beyond the capabilities of their physical counterparts. For example, when manipulating virtual base ten blocks, the user can actually break apart “flats” into “longs” and “longs” into “units” with a hammer tool so that a clear connection is made to the composition and decomposition of numbers; something that is difficult, if not impossible, to do with plastic base ten blocks. In many ways, the breaking apart through the virtual base ten blocks more closely resembles the mathematical idea of composing and decomposing numbers than the traditional trading model. This resemblance has been termed as *mathematical fidelity* (Zbiek et al., 2007), the degree in which the actions taken on a representation reflect the mathematical behavior. Additionally, there are constraints and support systems that can reinforce the learning processes. For example, the algebra balance applet from the National Library of Virtual Manipulatives (http://matti.usu.edu) has the ability to tilt from one side to another, which facilitates the understanding that equality is represented by the balancing of equations. Constraint systems also exist for virtual manipulatives. Although, one might think of a constraint in a tool as a limiting factor, constraint systems are positive aspects of cognitive technology tools. In fact, many of the constraint systems in the cognitive technology tools eliminate the extraneous cognitive load that may be placed when learning with a physical tool. In Suh’s (2005) study with virtual fraction applets, students were better focused on the concept of finding equivalent fractions when using the virtual fraction applets rather than the physical fraction circles. Instead of being distracted by a multitude of loose plastic fraction pieces, students were specifically focused on looking for patterns and analyzing relationships among equivalent fractions. The fraction applet also allows users to add fractions after renaming the unlike denominators. This built-in two-step process in the applet guides students to recognize both that the renaming of fractions is an important prerequisite when combining fractions and that the constraint system eliminates the common student error of “adding across” unlike fractions. This two-step process is an example of a tool having *cognitive fidelity* (Zbiek et al, 2007), or the degree in which the computer process reflects what takes place in human cognition while solving a problem. Cognitive technology has the potential for broadening the representational tools available to teachers and students. Researchers must recognize that thoughtful incorporation of representations is critical in instructional design. As stated by Hoadly and Kirby (2004), “Seeing the representations work in the learning context and educational environment and trying to optimize those representations is an important part of the design and instruction” (p. 2).

Mathematical Tasks that Elicit Critical Thinking and Reasoning

Current reform efforts are focused on developing students with critical thinking processes. Educational tasks that engage students in critical thinking involve posing and solving rich mathematical problems, making and testing conjectures, looking for patterns, and justifying answers through reasoning and proofs. In an analysis of mathematical tasks used in reform classrooms, Stein, Grover and Henningsen (1996) reported that the tasks used in mathematics learning highly corresponds with the type of thinking processes in which students engage in, thereby influencing student learning outcomes. A mathematical task can be defined by: a) the original mathematical task as represented in curricular/instructional material, b) a mathematical task as set up by teacher in the classroom, or c) a mathematical task as implemented by students in the classroom. Mathematical tasks that are defined as high level tasks involve “doing mathematics” or in other words using formulas, algorithms or procedures with connection to concepts, understanding or meaning (p. 467).

The following research involved the collaboration of a fifth grade mathematics teacher, a sixth grade mathematics teacher and the researcher who jointly planned technology-enhanced mathematics lessons for a diverse student population. The participating school was a Title I elementary school in a major metropolitan area with approximately 600 students: 51% Hispanic, 24% Asian, 16% Caucasian, 3% African American and 6% other. Over 50% of the student population receives either free or reduced lunches. With regards to the diversity of the school population, 44% received English for Speakers of Other Languages services and 49% were identified as limited

English proficient. The goal of the project was to design tasks suitable for a highly diverse population, which leveraged technology into elementary mathematics while enhancing access to critical thinking in data analysis and probability.

CASE STUDIES OF TECHNOLOGY -ENHANCED MATHEMATICS LESSONS

The two case studies focused on the design of technology enhanced mathematics lessons and the technology affordances that supported meaningful mathematics teaching and learning. Research was guided from the initial conceptualization of the idea to the enactment of the lesson in an authentic classroom setting by the following continuous cycle: design, enactment, analysis, and redesign. Through collaborative planning, debriefing and reflection, the teachers and researcher refined the key components of using cognitive technology tools for mathematics teaching and learning. Descriptions of the teaching and learning processes were documented through the following data sources: the analysis of students work, the researcher's memos and narrative reports from the teachers. The following sections will describe the design process and the enactment of the mathematical tasks using technology that provided opportunities for both critical thinking and rigorous mathematics.

The Design Process

To begin, the researcher and teachers reviewed the important curricular objectives for the lesson. Multiple curricular and instructional resources were considered. In particular, technology tools that would enhance learning were discussed. Next, the appropriate technology tool that both possessed *mathematical and cognitive fidelity* and would also be effective in eliciting mathematical thinking for students was determined. Through the design processes, a planning sheet was created and utilized that allowed the teachers to focus discussions on the important mathematical processes that were afforded by the technology tool (See Figure 1). In particular, focus was placed on the five Process Standards outlined by NCTM: problem solving, connections, representations, communication, and reasoning and proof. While planning the lesson, questions were developed that would elicit important mathematical ideas and help students make connections and create generalizations while simultaneously using the technology tools. Specific questions were asked in order to get students to make connections between the multiple representations, such as, "How are the pictures related to the numbers that are represented on the screen?" Some questions were utilized to motivate students to search for patterns or relationships, such as, "How are things changing as you input different numbers? Is there a pattern, rule or a relationship here? What steps are you doing over and over?" Other questions were intended to guide students in making and testing out conjectures. For example, "Is there information here that lets me predicts what's going to happen? When I do the same thing with different numbers, what still holds true? What changes? What if, I *(did this)* then what would happen?" These questions, along with lesson-specific questions, allowed for an important discussion of the mathematical concepts and elicited essential mathematical processes such as connections, problem solving, critical thinking and reasoning and proof.

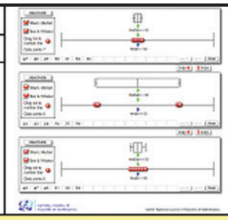
Case Study 1: Teaching and Learning Data Analysis Using Technology

The ability to make sense of data and understand probability is an important skill in today's technological society where data and information are both exchanged and produced in high volume. The challenge for many elementary teachers is to go beyond the basic level of constructing, organizing, representing and reading data and graphs and delve deeper into the concepts by interpreting, experimenting and conjecturing data representations.

In a 5^{th} grade classroom, students experimented with statistical data to understand mean, median and mode. In previous years, the classroom teacher reported that students would often confuse the three measures of central tendency. Many students were able to calculate these measures of central tendency but were unable to differentiate the use of one over the other. The teacher reported, "They know the definition [of] and the procedures for finding mean, median and mode but have no idea the significance of each and how they are used in real life."

The goal of the lesson was to design a task that gave students the opportunity to investigate the significance of the three measures of central tendency: mean, median, and mode. To begin the lesson, the class collected information about the number of letters in their names. Their initial data showed that their results ranged from 3 to 11. The class used the box plot applet from the National Library of Virtual Manipulatives (http://nlvm.usu.edu/) to record their data and then discussed the various aspects of the box plot and the measures of central tendency that were displayed on the screen.

Use of Technology to Enhance Mathematical Thinking

Website	http://illuminations.nctm.org/activitydetail.aspx?ID=160			
Math Strand	Data Analysis	Grade level	6th	
Description of mathematical concept (NCTM)	Can you create three data sets, all of which have 6 data points, a mean of 50, a median of 50, and meet the following criteria? Set A: Every data point is between 35 and 65. Set B: Every data point is either less than 25 or greater than 75. Set C: The difference between every pair of two consecutive data points is the same. (from Illuminations lesson http://illuminations.nctm.org			

Analysis of Mathematical Representations and Models	
__Concept tutorial/Skill Practice _X_ Investigation/problem solving __Open exploration	
Mathematical thinking opportunities afforded by the mathematics applet	
Representations: Gives students opportunity to ... ▪ Create and use representations to organize, record, and communicate mathematical ideas ▪ Select, apply, and translate among mathematical representations to solve problems ▪ Use representations to model and interpret physical, social, and mathematical phenomena	This tool allowed students to create three box plots on the screen at one time and displayed the numeric data with the mean and the median.
Communication: Gives students opportunity to ... ▪ Organize and consolidate their mathematical thinking through communication ▪ Communicate their mathematical thinking coherently and clearly to peers, teachers, and others ▪ Analyze and evaluate the mathematical thinking and strategies of others; ▪ Use the language of mathematics to express mathematical ideas	Tool allowed students generate several possible solutions data sets which promoted discussions in small group. Ease of the display helped communication.
Connections Gives students opportunity to ... ▪ Recognize and use connections among mathematical ideas ▪ Understand how mathematical ideas interconnect and build on one another to produce a coherent whole	Students could see how the data set affected the mean, median and mode through data manipulation.
Reasoning and proof: Gives students opportunity to ... ▪ Recognize reasoning and proof as fundamental aspects of mathematics ▪ Make and investigate mathematical conjectures ▪ Develop and evaluate mathematical arguments and proofs	Students made and tested their conjectures about how they generated data sets that met the criteria. They had to prove their thinking to their partner.
Problem Solving: Gives students opportunity to ... Build new mathematical knowledge through problem solving ▪ Solve problems that arise in mathematics and in other contexts ▪ Apply and adapt a variety of appropriate strategies to solve problems ▪ Monitor and reflect on the process of mathematical problem solving	Problem solving context allowed students to think critically about the relationship between mean, median and mode.

Figure 1. Example of the MATH-Technology Integration Planning Sheet

The mean appeared as 5.41. A quick glance at the data chart showed that 5 was the mode. The box also indicated that the 50% of the class names were between 4 and 6 by looking at the lower and upper quartiles. Based on this information, students agreed that the best way to measure the central tendency was to use the mode of 5 since having 5.41 letters was not logical. After interpreting and discussing the results of the box plot, the teacher posed the following problem.

> "Class, today the registrar told me that we have a new student from China. And he has a great long name, Tikki tikki tembo-no sa rembo-chari bari ruchi-pip peri pembo. His name means "the most wonderful thing in the whole wide world." (Tikki Tikki Tembo is a well-known Chinese folklore and children's book). "What do you think this name with 50 letters does to our mean, median and mode? Talk with your group about how this will affect our box plot and central tendencies."

Students began to think critically about the relationships between each point of data and the mean, median, and mode. After allowing students to make conjectures and list their ideas on the board, one of the students entered the number 50 into the data set and clicked the UPDATE BOX-PLOT button (See Figure 2). Students looked intently at the new box plot and confirmed and refuted their prior conjectures. The discussion led to the extremely important idea of what happens with the presence of outliers; a term introduced after much discussion about the "extreme number". Students engaged in a lively discussion about which was a better measure of tendency when extreme outliers were present. Many students stated that the new mean, which was 7.2, was misleading since 50% of the data centered around 4 and 6. Additionally, students agreed that the mode of 5 and the median were the measures of central tendency that best represented the data.

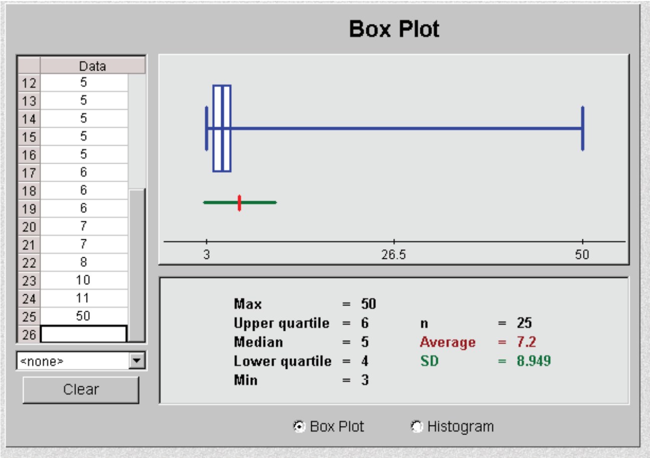


Figure 2. Data analysis when outlier added to data set

Another example of using technology to enhance critical thinking with data analysis, specifically box plots, was a problem posed to a 6th grade class also working on the topics of mean, median and mode. This time, box plots from Illuminations that allowed three box plots to be displayed simultaneously were used (See Figure 3). As a challenge, students were given the following problem:

Can you create three data sets, all of which have 6 data points, a mean of 50, a median of 50, and meet the following criteria?

- *Set A: Every data point is between 35 and 65.*
- *Set B: Every data point is either less than 25 or greater than 75.*
- *Set C: The difference between every pair of two consecutive data points is the same.*

(from Illuminations lesson http://illuminations.nctm.org/activitydetail.aspx?ID=160)

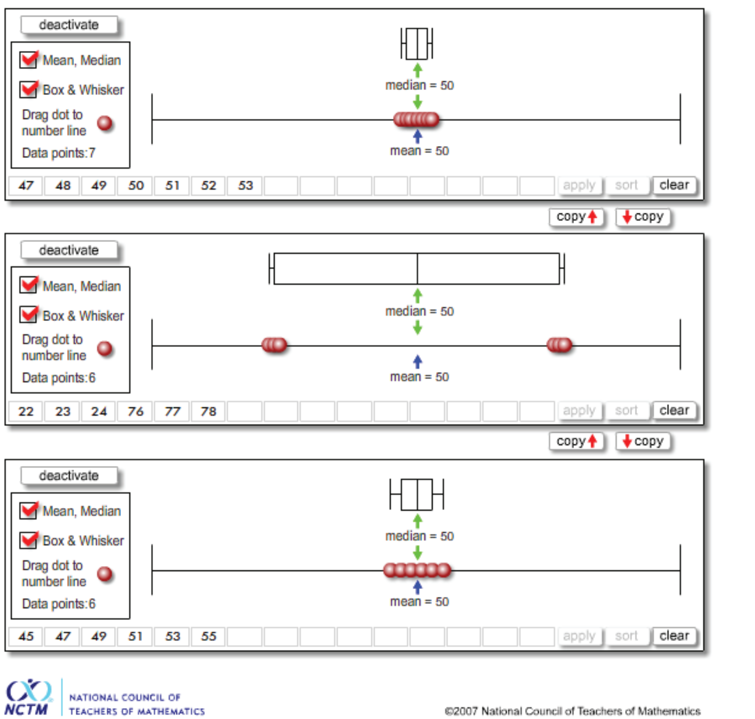


Figure 3. Problem Solving through display of 3 box plots

Opportunities for rich mathematical discourse about both the mean and median and the connections between the procedures of calculating and the concepts of central tendencies were elicited through the use of follow up questions such as: "How are these sets different from one another? How are they alike? Are there other data sets with six points, a mean of 50, and a median of 50 that look different from the three you've created?" One student remarked, "I discovered that as long as pairs of data equaled 100, I had a mean of 50. I used this strategy to meet the statements [criteria]." As students compared different sets of numbers, they were given the opportunities to reason and prove how multiple solutions were possible as long as the given criteria were met.

Both examples illustrate how the use of technology tools amplify the opportunities available for students to interact with the experimental data, make and test conjectures, and confirm or reject hypotheses. The ease of representing different box plots with such efficiency allows for a higher mathematical complexity than merely creating box plots. With the use of technology tools, an increased amount of experimentations and additional time for deeper analysis are incorporated into the learning environment. As Kaput (1992) noted, the impact of technology tools in mathematics learning and teaching is the ability to "off-load" some routine tasks such as computations. In this case creating box plots for each experimental scenario provided learning efficiency in terms of compacting and enriching experiences. This example is not to discount the importance of students' ability to represent data in different graphical forms but many times, student have less opportunities to utilize their interpretative and critical thinking skills due to the curriculum being so densely packed and insufficient time allocated to developing the depth and complexity of mathematical ideas.

Case Study 2: Probability Experiments Via Technology Tools

The overarching themes in probability for the elementary grades according to our national curriculum is threefold: the initial recognition of the nature of random processes, the exploration of the concept of chance through games and experiments, and the comparison of the likelihood of theoretical and experimental events (NCTM, 2000). The latter is an idea that poses common misconceptions for students in earlier grades. In fact, research shows that students have difficulty understanding the "bidirectional relationship between empirical (experimental) and theoretical probability and the role of sample size in that relationship" (Stohl & Tarr, 2002, p. 314).

However, research shows that experimentation using large sample sizes with the Probability Explorer Microworld technology allows students to begin to appreciate the power of the law of large numbers. An example of this research-in-practice is illustrated by the following case study.

A 5th grade classroom studying the concept of probability engaged in an experimentation using handmade spinners and virtual spinners. The class activity was called "Mystery Spinners", ten spinners were distributed, one to each pair of students. Students were asked to independently look at their spinners, predict the outcome, spin their spinner 30 times and finally record the results as a bar graph. Once the teams of students finished conducting their experiment, their bar graphs were displayed for all to see. The teacher then mixed up the ten spinners and posted them on the board. Students were challenged to match the spinners displayed on the board with the bar graph that they thought was the most likely outcome of the spinner. In the case of many of the spinners, students were able to make accurate matches. However, two spinners resulted in similar outcomes in which each of three colors, green, blue and red had approximately equal amounts. Students argued that spinner A should have had 25% (7 or 8 out of 30) red, 25% blue (7 or 8 out of 30), and 50% green (15 out of 30) because it was ¼ red and ¼ blue and ½ green. The students also felt that spinner B should have had 33.3% red (about 10/30), 33.3% (10/30) blue and 33.3% (10/30) green since it was 1/3 red, 1/3 blue and 1/3 green. This led to a great discussion about theoretical and experimental probability.

The next day, students went to the computer lab and were introduced to the adjustable spinner applet from NCTM's Illuminations website. Through experimentations, students were able to see that with a minute number of trials, an individual could be easily misled as to the composition of the spinner; as in the following illustration with only 10 spins (see Figure 4).

The technology affordances of the spinner applet amplified the mathematical learning opportunities available by allowing the students the ability to adjust the number of spins, to create, and test conjectures, and also appreciate the power of the law of large numbers. One student sat in front of the computer, first testing out the outcomes of ten trials, then 100 trials and then finally 1,000 trials. With each click the student noticed how the experimental probability became closer and closer to the theoretical probability. In addition to the ease of experimentation, the power to click on the third grey button OPEN RESULT FRAME and see the experimental graph and the experimental trials simultaneously was a visual connection to how the outcomes were reflecting the original spinner.

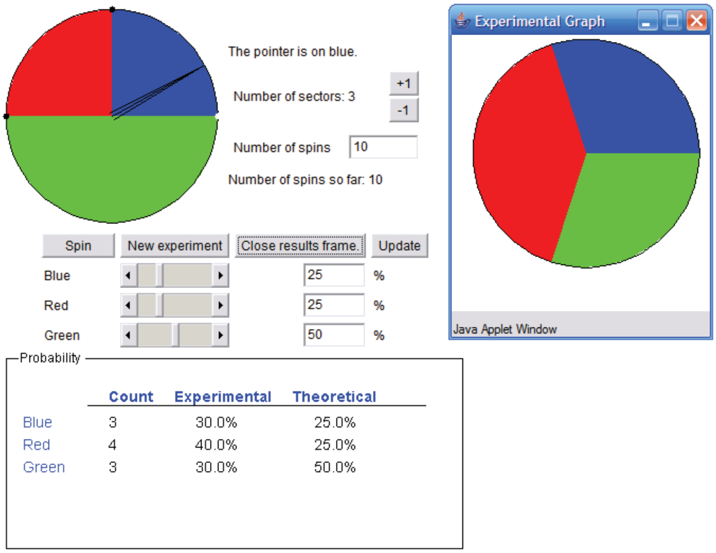


Figure 4. Adjusting spinners with only 10 trial spins

AFFORDANCES OF TECHNOLGY-RICH MATHEMATICS ENVIRONMENTS

Specific opportunities that technology rich mathematics environments afford teachers and students are the abilities to: a) build representational fluency by making connections among multiple representations; b) experiment and test out conjectures which efficiently develop reasoning and proof; and c) facilitate the communication of mathematical ideas through problem solving. Technology applets with dynamic objects and visual tools offer learners multiple representations to consider while learning mathematical concepts. For instance, in the case of the classroom using the spinner applet, the visual representation of the spinner with the experimental graph was linked to the multiple numeric representations of the outcomes, percentages, theoretic probability and experimental probability. Zbiek, Heid, Blume and Dick (2007) stated that "cognitive tools can constrain the possible actions on an external representation to be ones that are potentially mathematically meaningful and it can enforce mathematical rules of behavior of objects on which it acts" and make mathematical consequences more overtly apparent. The dynamic spinner "enforced the mathematical rule of behavior" of the mathematical concept of the law of large numbers. By setting the spin function to take larger trials into consideration, students were able to see how the experimental graph became more like the theoretical graph. These tasks were specifically designed to provide students opportunities to draw logical conclusions and justify both answers and solution processes by explaining why, as well as how they were achieved. In this way, the two classroom case studies demonstrated how the teacher and students actively engaged in the sense making processes. Students were asked to go beyond the obvious in their interpretations and communicate their thoughts clearly and concisely during classroom discourse. As shown in the example with the box plots, the focus of the class was not at the entry level of creating box and whisker plots. Instead the goal was at a more complex level of interpreting each number along the box plot. Additionally, the ability to manipulate the data to see how the mean, median and mode were affected allowed for students to gain a deeper understanding of and differentiate among the central measures of tendency.

Emerging technology in mathematics allows teachers and students to not only easily represent abstract mathematics concepts that are often difficult to illustrate but also gives access to opportunities for rich learning experiences and meaningful class discourse. Mathematics educators and instructional designers need to harness technology's affordances with its constraints and supports systems to design meaningful cognitive technology tools that can help in the teaching and learning of challenging mathematics concepts. Important consideration should be made on characterizing how learning is different or the same in the technological and nontechnological environment. In particular, research should continue to look at the cognitive load that is present in these different environments to see how some extraneous cognitive load posed through mathematical activities can be "off –loaded" so that more of the learner's cognitive capacity can be allocated to the important mathematical learning.

School mathematics curriculum must reflect both the changing societal needs and the demands of schools to growing knowledge about learning and teaching, and capitalize on new developments in order to keep up with the rapidly changing technology. Providing access and opportunity to rich mathematics through technology may be an important catalyst to closing the achievement gaps which are often created due to opportunity gaps that exist in our society. In sum, the mathematics community shares an important responsibility to leverage technology to expand the academic access and opportunities in preparing all students for the skill sets they will need to be successful in the 21^{st} century.

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