

Ppt and handouts and all strategies are available at
<http://mason.gmu.edu/~jsuh4>

Let's talk math!

Engaging all learners in meaningful mathematical discourse

NCTM Lesson Study Group 2007-2008

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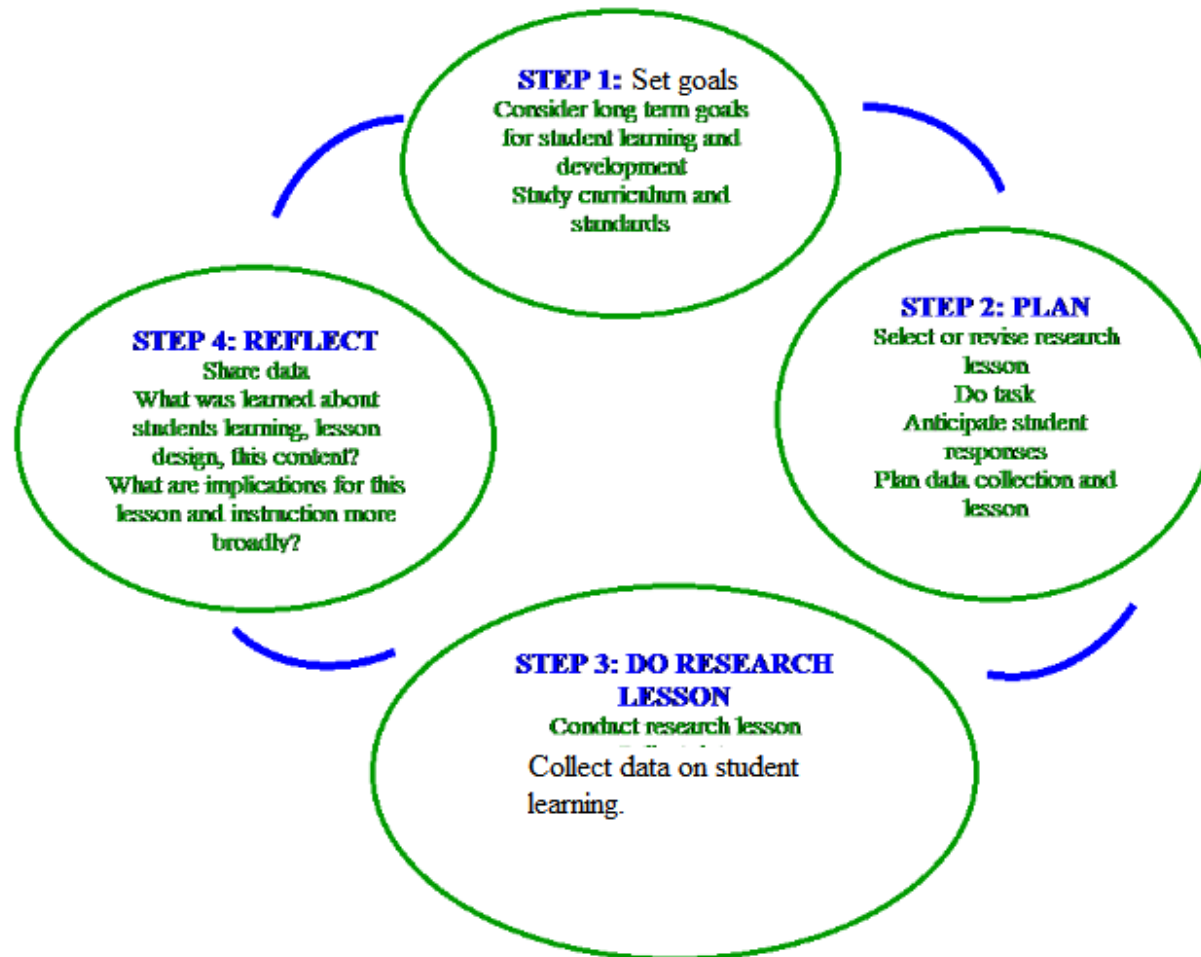
What is Lesson Study?

STEPS IN LESSON STUDY

<http://mason.gmu.edu/~jsuh4/lessonstudy/steps.htm>

[Back to Lesson Study](#)

STEPS IN LESSON STUDY



Overarching Goals for Grades 3-8

- Students will develop meaningful mathematical discourse

Content goal:

- Student will communicate their mathematical ideas clearly and respectfully with precise mathematical language.

Research Driven...

MEANINGFUL MATHEMATICAL DISCOURSE

- ▶ “Mathematics can be thought of as a language that must be meaningful if students are to communicate mathematically and apply mathematics productively. It is important, therefore, to provide **opportunities** for them to “talk mathematics.” (NCTM Curriculum and Evaluation Standards for School Mathematics, p. 26)
- ▶ “Talking about their **thinking clarifies students’ ideas** and gives the teacher valuable information from which to make **instructional decisions.**” (p. 79)
- ▶ **Urban** classrooms with students that are **below grade level** in mathematics can function and learn as a **math-talk** learning community. Ackles, Fuson, and Sherin (2004)

MEANINGFUL MATHEMATICAL DISCOURSE... Research Driven

- ▶ “Mathematics for Language Minority Students: Cultural background or difficulties with the English language must not exclude any student from full participation in the school’s mathematics programs.” (NCTM, 1987)
- ▶ All ESL learners “made tremendous progress in classrooms using intensive talk. Classroom discourse “actually **supports language learning as well as mathematical thinking.**” (Chapin, O’Connor, and Anderson, p. 159)
- ▶ “**Teacher questioning behaviors are very much related to increases in student achievement....**Higher order questions (e.g. those that ask students to apply, analyze, and reason) produce more learning than questions that focus simply on recall of information.” (p. 41)

How do we give access to math discourse to diverse learners?

Give them something to talk about...

- Universal design (borrowed from architecture)

Universal design is a relatively new paradigm that emerged from "barrier-free" or "accessible design"

Examples: Do you use any of these universal design features in your daily life?

- Ramps to buildings or sidewalk ramp
- Smooth ground surfaces of entranceways, without stairs
- Wide interior doors and hallways
- Lever handles for opening doors rather than twisting knobs
- Light switches with large flat panels rather than small toggle switches
- Choice of language on speech output
- Ramp access in swimming pools
- Closed captioning on television networks



Universal design in Math Learning

- **Multiple means of engagement** to tap into learners' interests, challenge them appropriately, and motivate them to learn
- **Multiple means of representation** to give learners various ways of acquiring information and knowledge
- **Multiple means of action and expression** to provide learners alternatives for demonstrating what they know, and

Universal design can benefit diverse learners...

- Learning disabilities such as dyslexia
- English language barriers
- Emotional or behavioral problems
- Lack of interest or engagement
- Sensory and physical disabilities



Universal Design in math



Universal Design Techniques for Learning



GAINING ACCESS TO MATHEMATICAL LEARNING

Multiple Methods of Engagement



- Real-life contexts
- Rich math tasks
- Physical manipulatives
- Virtual manipulatives
- Students' interests
- Cross Curricular ties
- Technology integration
- Cooperative learning
- Multiple intelligences
- Games & Simulations
- Academic vocabulary
- Inquiry Based activities
- Background knowledge

Multiple Methods of Representations



- Numeric representations
- Written symbols
- Hands-on math
- Visual/Auditory
- Graphs/Tables
- Diagrams
- Textbooks(modified)
- Computer software
- Physical manipulatives
- Virtual manipulatives
- Concept Mapping
- Graphic Organizers
- Discourse

Multiple Methods of Expression of Assessment



- Mathematicians log
- Formative & summative
- Performance based assessments
- Drawings, graphs, charts
- Computer work
- Presentations
- Verbal and written explanations: Proofs

○ Multiple pathways to learning mathematics

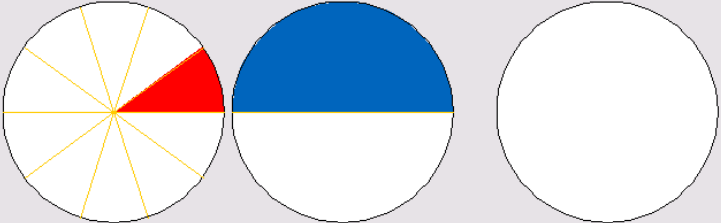
○ Multiple ways to demonstrate understanding

○ Multiple ways to apply and connect ideas

Mathematical knowledge map with related concepts and representations



Technology with special affordances for access to rigorous math



10
 $\frac{1}{10} = \frac{\square}{\square}$

2
 $\frac{1}{2} = \frac{\square}{\square}$

Revisar

Expresa $\frac{1}{10}$ y $\frac{1}{2}$ en fracciones equivalentes, de modo que ambos denominadores sean iguales. Luego, revisa tu respuesta.

Problema Nuevo

Dificultad:
☒ Fácil
☐ Media
☐ Difícil

Discover

Rate

Graph

Function

Apply

Rate (intro)

Rate (intermediate)

Rate (advanced)

Graph (intro)

Graph (intermediate)

Graph (advanced)

Function (intro)

Function (intermediate)

Function (advanced)

Challenge

Healthy Forest

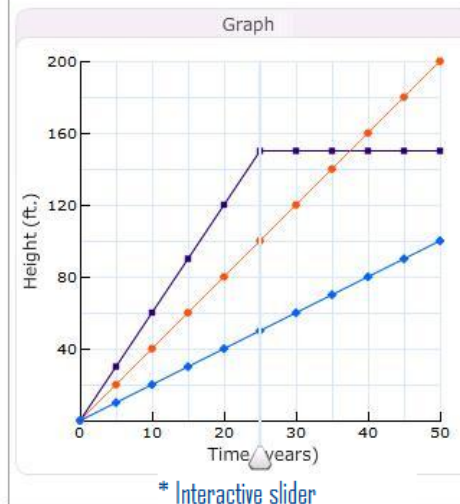


[*Tiered lessons and tasks](#)

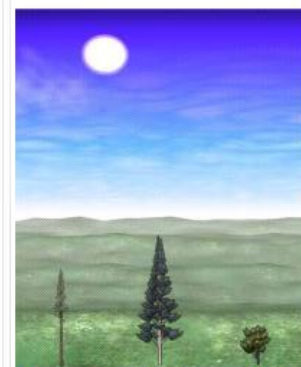
Rate of Change (discover)

Three different trees are shown. Each grows at a different speed and has a different maximum height. Move the slider below the graph to see how each tree's height changes with age. Which tree is the tallest at 10 years? Which tree is tallest at 50 years? Can you explain why the tallest tree at 10 years is not the tallest at 50 years? What do you notice happening to the Pine tree at 25 years?

Rate (discover)



Simulation



Sequoia White Pine Bur Oak

[Vocabulary connections](#)

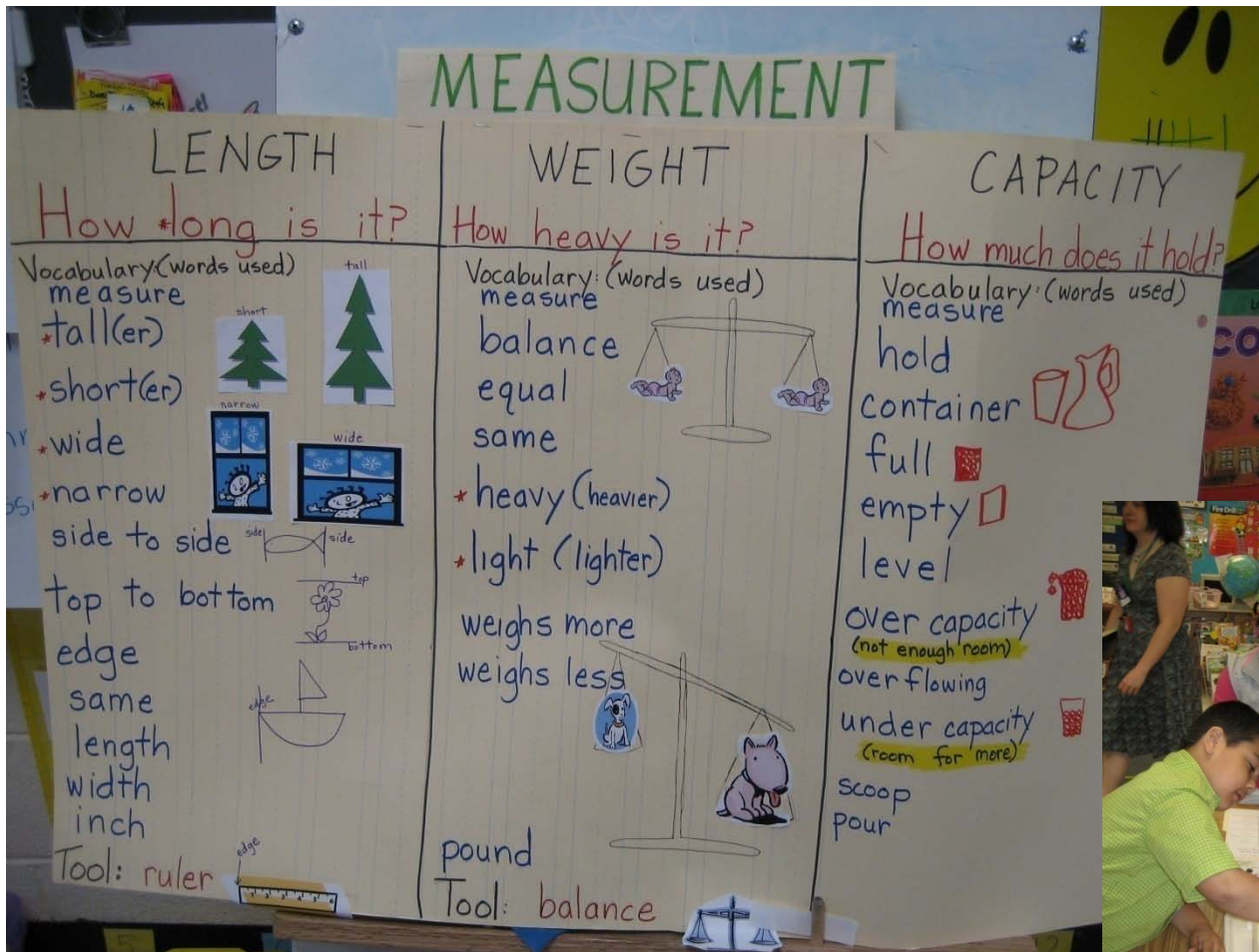
Math Terms

Axes, Equation, Linear Function, Rate, Ratio, Slope, X-intercept, Y-intercept

Visuals for measurement vocabulary to help students discuss their discoveries

MEASUREMENT

LENGTH	WEIGHT	CAPACITY
How long is it?	How heavy is it?	How much does it hold?
Vocabulary: (words used) measure *tall(er) *short(er) *wide *narrow side to side top to bottom edge same length width inch Tool: ruler	Vocabulary: (words used) measure balance equal same *heavy (heavier) *light (lighter) weighs more weighs less pound Tool: balance	Vocabulary: (words used) measure hold container full empty level over capacity (not enough room) overflowing under capacity (room for more) scoop pour



Using manipulatives to discuss the difference between mean, median and mode

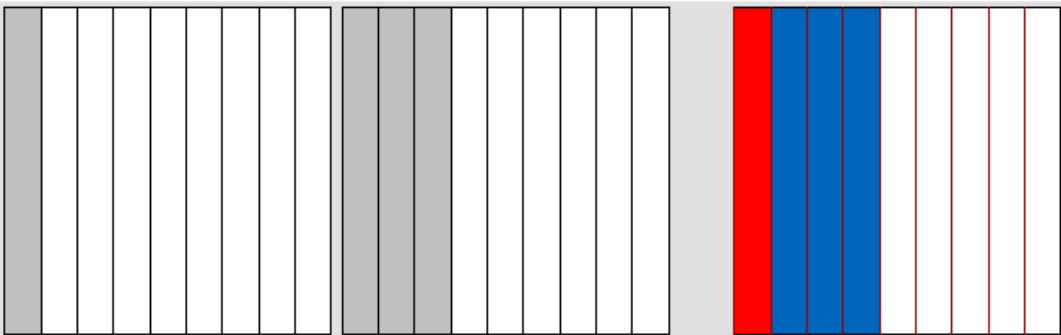


Use of mathematics applets to support procedural and conceptual understanding

Fractions - Adding - NLVM - Microsoft Internet Explorer

Address http://nlvm.usu.edu/en/nav/frames_asid_106_g_2_t_1.html

Back Activities Parent/Teacher Standards Instruction



9 pieces

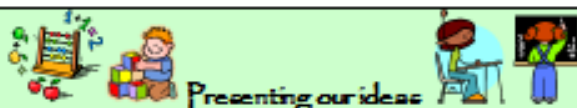
$$\frac{1}{9} + \frac{1}{3} = \frac{1}{9} + \frac{3}{9} = \frac{4}{9}$$

Check

Good work! Click the 'New Problem' button for a new addition problem.

Elicit, support, extend student thinking through

MATH TALK: Building mathematical ideas



Presenting our ideas

Who can give me an example?
 What is another way to solve this problem?
 How can you convince your classmates (partner)?
 Can you compare your thinking with your partner's thinking?
 How can you show your thinking using drawings, manipulatives, numbers, and words?



Adding to other ideas



Do you agree/disagree with _____'s idea? Why?
 What would you like to add to _____'s idea?
 Do you have a question about _____'s idea?
 How can we restate it (question or statement) in our own words?



Making connections



What connection can you make to what we have learned before?
 How would you use this math in the real world?
 How is this related to _____? What if _____?

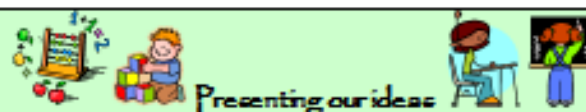


Reflecting on what we learned



What concepts or ideas have you learned?
 How do we use _____?
 How can we answer the "E" essential question of the day?

MATH TALK: Building mathematical ideas



Presenting our ideas

I have an idea...
 I have an example...
 I have another way...
 I can prove my thinking by...
 I can show you what I am thinking using...
 (drawings, manipulatives, numbers, words)



Adding to other ideas



I agree/disagree with _____'s idea...
 I'd like to add to _____'s idea...
 I have a question about _____'s idea...
 I am not sure if I understand _____'s idea...



Making connections



I can make a connection to what we learned before...
 I use this math in the real world when I...
 This is related to...



Reflecting on what we learned



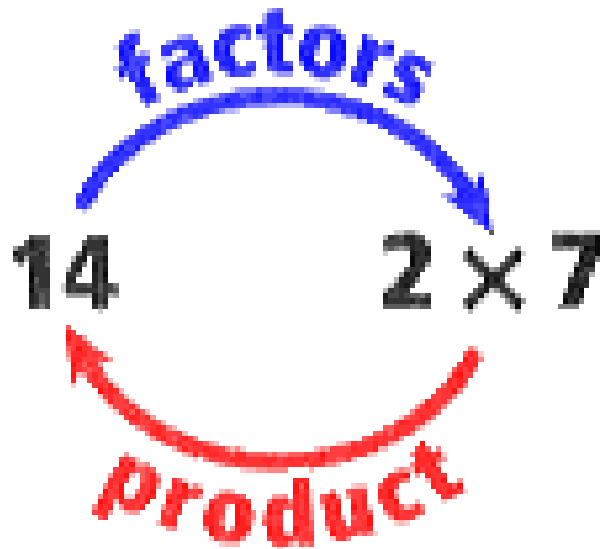
I learned that...
 I know how to...
 I can answer the "E" essential question of the day...

Meaningful mathematics discourse

Standard 2, The Teacher's Role in Discourse, recommends that teachers orchestrate discourse by "posing questions and tasks that elicit, engage and challenge each student's thinking; listening carefully to students' ideas; asking students to clarify and justify their ideas orally and in writing; deciding what to pursue in depth from among the ideas that students bring up during a discussion; deciding when and how to attach mathematical notation and language to students' ideas; deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty; monitoring students' participation in discussions and deciding when and how to encourage each student to participate" (NCTM, 1991, p. 35).

Playing the Game –

Action: playing the game



The Product Game

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

Factors:

1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---

Questions to consider:

- 1. What are the factors of numbers and how do you find them?
2. What are the multiples of numbers and how do you find them?
3. What did you learn about prime and composite numbers while you were playing the Factor Game?
4. Is it possible to get every product with the factors given?
5. Are any products missing for these factors?
6. Suppose that the game is in progress and you want to cover the number 12 on the grid. Describe one way this can happen? Can you get 12 in more than one way?
7. Suppose the game is in progress and one of the paper clips is on the 5. What products can you make by moving the other paper clip?

The Product Game

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

Factors:

1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---

Making your own Game Board

Part 2

1. Choose factors.
2. Determine all the products you need to include on game board.
3. Find a game board that will accommodate all products.

Making your own Game Board

- What challenges did students have in creating their own game boards?
- What kinds of interactions and/or conversations did you observe as students made their own game boards?
- What insights did you learn about your students from their game boards?



How did you group your students to play the game?



What kinds of interactions and/or conversations did you observe when students played?



How did students of different ability levels respond to the game?



What challenges did students have in creating their own game boards?

2 7 2007



What kinds of interactions and/or conversations did you observe as students made their own game boards?

2 7 2007

What insights did you learn about your students from their game boards?



Key elements

- Setting the norms in the classroom
 - Clarifying one's own thinking or classmates
 - Making and testing conjectures
 - Asking questions
- Rich engaging and rigorous math tasks
- Math Tools for Discourse
(manipulatives , technology, games, graphs, tables, representations)
- Building academic vocabulary
- Using different group structures

Creating Classrooms that Encourage Discourse

A factor in developing classrooms discourse is creating a social, and mathematical environment in which students:

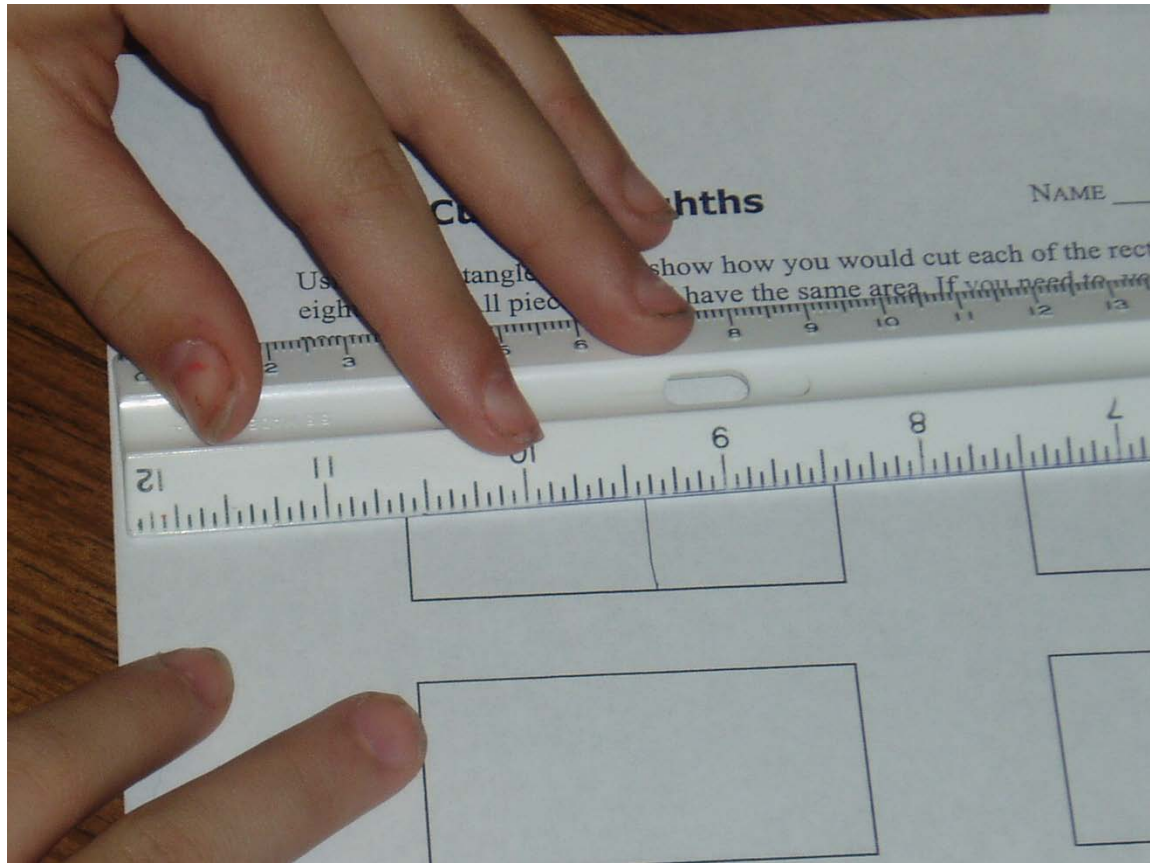
- ▶ Listen to one another
- ▶ Respect one another and themselves
- ▶ Accept opposing views,
- ▶ Participate in a genuine give-and-take of ideas and thoughts;
- ▶ Collaboration and active learning are emphasized, valued, and celebrated by teachers and students, and
- ▶ The teacher's role has changed and is not seen as the only expert and within the classroom.

Manouchehri, A., Enderson, M. (1999). Promoting mathematical discourse: learning from classroom examples. *Mathematics Teaching in the Middle School* 4(4).

How did you create Classrooms that Encourage Discourse?

- Revoicing
- Restating someone else's reasoning
- Applying their own reasoning to someone else's reasoning
- Prompting students for further participation
- Using Wait Time

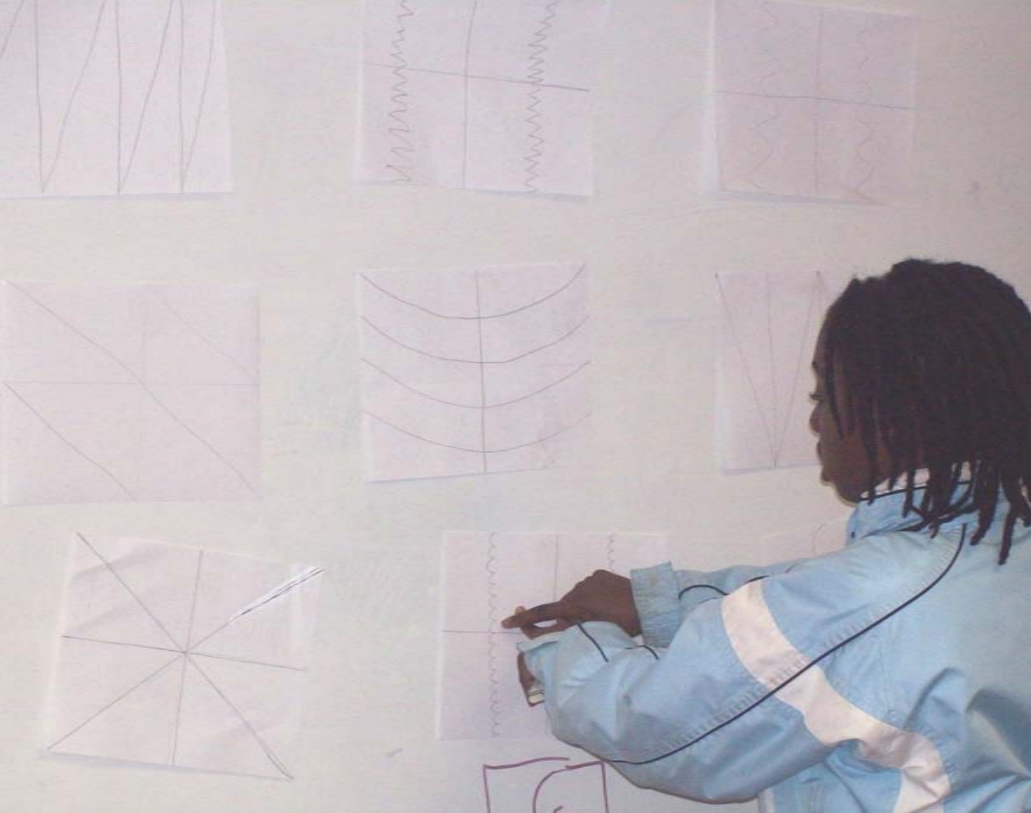
Understanding Rational numbers and proportions (jean)



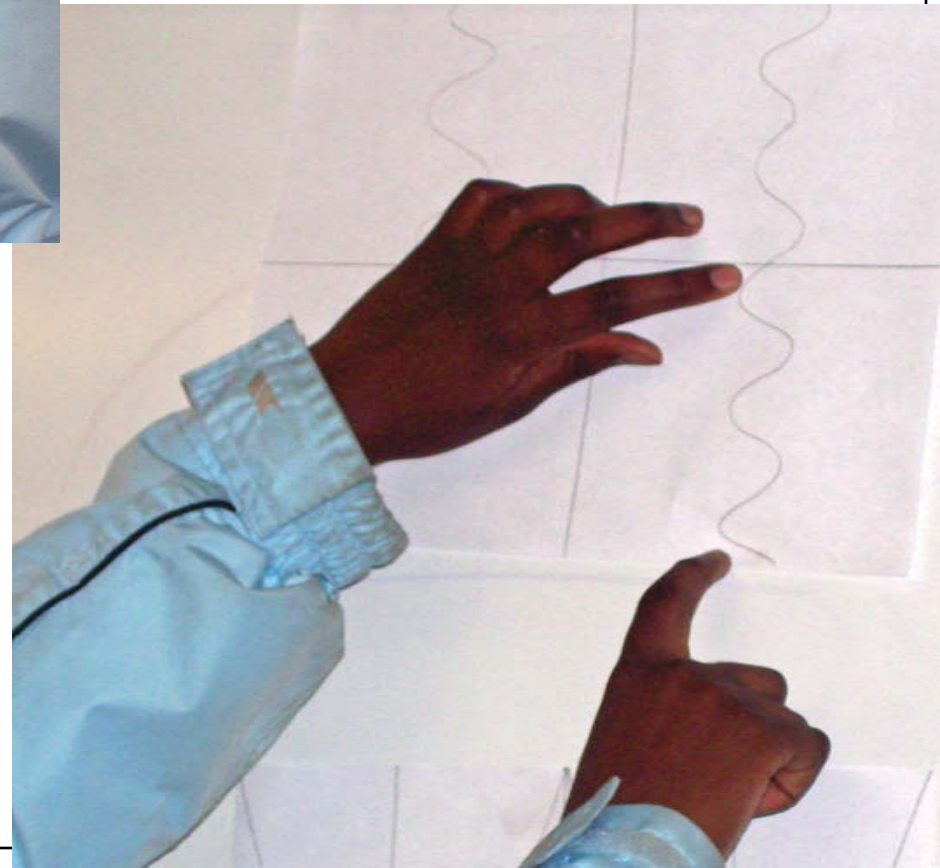
Customers Cut the Cake”

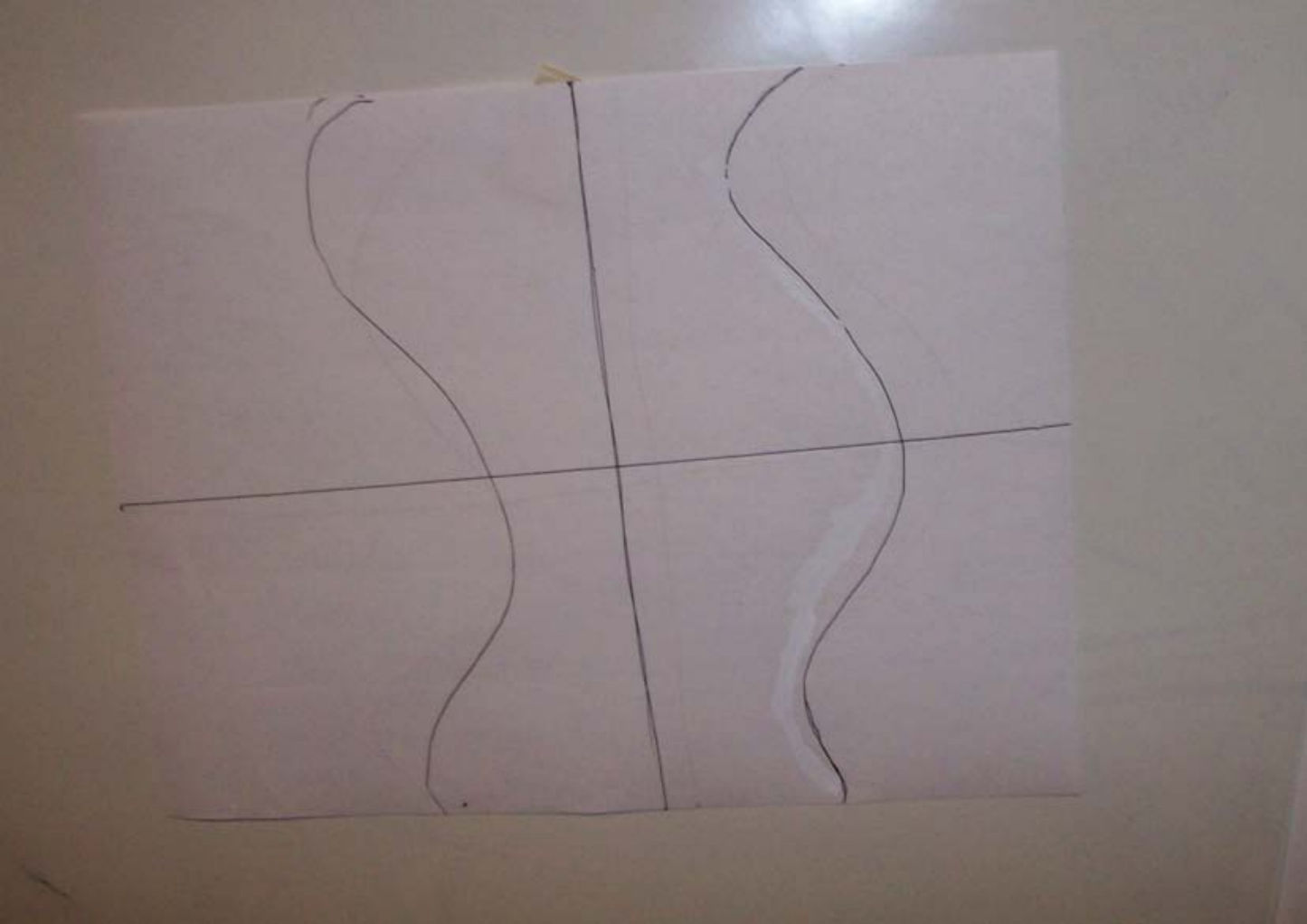
Each day the local baker makes several rectangular sheet cakes, which he cuts into eighths. He sells $\frac{1}{8}$ of a sheet cake for \$1.59. As part of a new promotional campaign for his store, he wants to cut his sheet cakes into eighths a different way each day. Customers who suggest a new way to cut the cakes into eighths win a free piece of cake each day for a week. What are some of the different ways to cut the cake?

What do we have to remember about parts with curved lines?



Are these parts really equal?





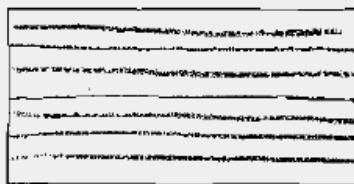
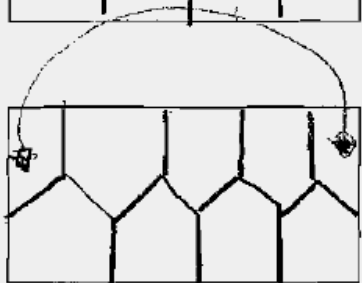
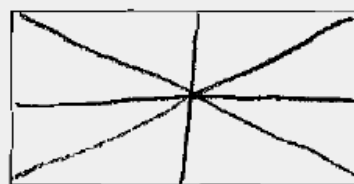
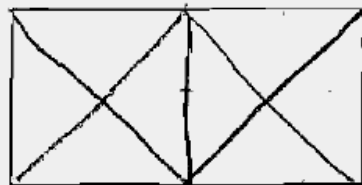
How can we prove that the eight pieces are equal parts?

~~Cakes Cut Into Fourths~~ ^{Eighths}

NAME Emily Heck

Using the rectangles below, show how you would cut each of the "cakes" into four pieces. Each piece does **not** have to have the same area as the other pieces in that cake. If you need to, you may use a piece of printer paper to practice cutting before making your drawings below.

Once you have made your cuts, find out the cost of each piece of cake, assuming the bakery sells 1/8 of a sheet cake for \$1.50



Let's Revisit Problem 2

- "You Can Eat Your Cake and Have It, Too!"

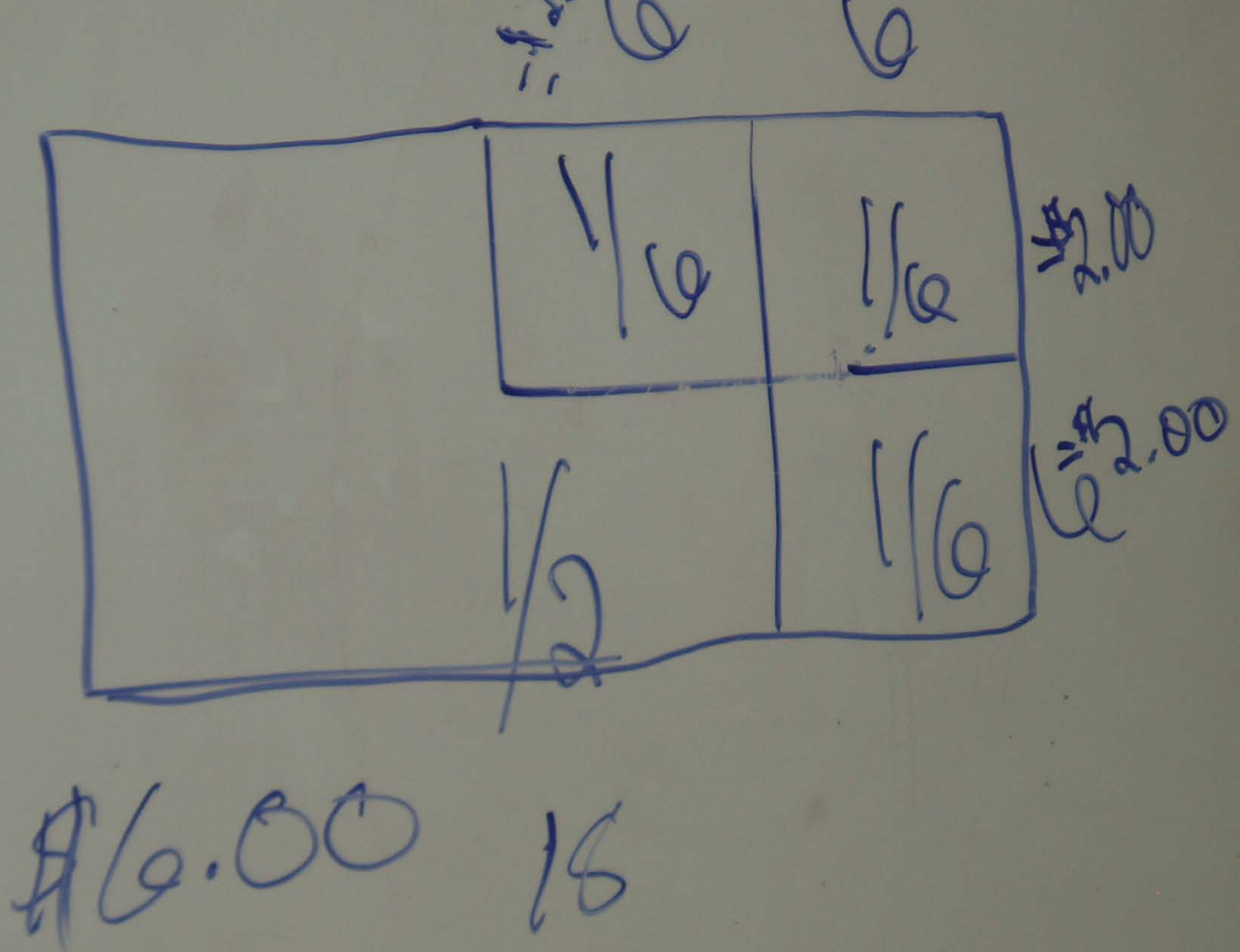
The baker is conducting a second contest, this time for his employees. As part of a new promotional campaign for his store, each day he wants to feature sheet cakes that have been cut into four pieces in a different way. The pieces do not have to be equal for this promotion. The baker has challenged his employees to suggest interesting ways to cut the cakes into four pieces. The employees must also determine the price for each piece. The bakery sells $1/8$ of a sheet cake for \$1.59. What are some of the different ways the cakes can be cut, and how much should each piece cost?

How does the fractional part reflect the price for each of the four pieces of cake?



Discussion

- Could your students find the total cost of the cake given the cost of $\frac{1}{8}$ piece of cake was equal to \$1.59?
- Were your students able to partition the cake into four unequal parts where they could identify the fractional part of the whole?
- Did your students use equivalent fractions to name their four parts – for example going from $\frac{2}{8}$ to $\frac{1}{4}$ or $\frac{4}{8}$ to $\frac{1}{2}$.



This group of students divided their cake using grid paper and determined the fractional cost of each piece based on $\frac{1}{8}$ piece of cake is equal to \$1.50.

Apple pi



Objectives

- 1. Students will measure the circumference and diameter of various circular objects.
- 2. Students will calculate the ratio of the circumference to the diameter.
- 3. Students will discover the formula for the circumference.





Generalization posters
Math vocabulary visuals



WASLEY

CIRCUMFERENCE OF CIRCULAR SHAPES

Directions: Using the circular shape you have been given, lay the string around it to find the circumference. Next, measure the string with your ruler to find the length of the circumference (as accurately as possible). Next, measure the diameter of your circular shape. Finally, using your calculator, divide: circumference ÷ diameter. Record your result. We will then take a class survey to record each team's results. Then we will add them together to find the average.

	Circumference(=)	Diameter (=)	Quotient
Team 1	23.3	7.7	3.025974
	21.3	6.7	3.177272727
Team 2	3.8	1.1	3.5
	1.9	6.5	3.1
Team 3	23.2	4	2.9
	22.1	7	3.2
Team 4	2.2	7	3.14
	28.3	9	3.14
Team 5	1.6	6	3.3
	28.5	9	3.16
Team 6	27.5	9	3.05
	32.4	10	3.24
Team 7	23	7	3.27
	36	11.5	3.13
Team 8	16.5	5.3	3.11
	21.5	7.1	3.03
Team 9	32.2	13	3.26
	17	5	2.83
Sum of Quotients:			56.36
Average of quotients:			3.1317

CIRCUMFERENCE OF (semi-circular) SOAP BUBBLES

Directions: Using a small amount of soap bubble mixture, pour it onto your tabletop. Dip the end of the straw into the remaining bubble mixture in the cup to wet the end of it. Hold your straw at about a 45° angle in the "puddle" of soap on the tabletop. Blow gently into the straw to blow your bubble. **DO NOT SUCK UP ON THE STRAW OR YOU WILL END UP WITH A MOUTHFUL OF SOAP!** Yuk!! When the bubble pops, use your ruler to measure the diameter of the circular impression left on your desktop. Then calculate the circumference. Repeat this three times filling in the data below.

	Diameter	Calculation $C = \pi \cdot d$	Circumference
BUBBLE #1	5 in.	$C = \pi \cdot d$	15.7
BUBBLE#2	6.5 in.	$C = \pi \cdot d$	20.4
BUBBLE #3	7 in.	$C = \pi \cdot d$	22
Bubble #4	11 in.	$C = \pi \cdot d$	34.6

Strategies to support math discourse with diverse populations

Build math discourse language

Modeling Math “Four corner math”

Convince Me

Problem solving strategies

Building Math Vocabulary

Generalization Posters

COMBO

problems

1 Multiplication strategy - First you look at the categories. Count the choices for each category. Multiply each categories # of choices together.

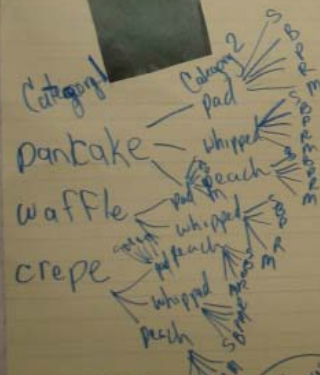
2 Organized List - Take 1 choice from each category and put them together for your 1st combo. Change choices for each category 1 by 1 until you've written all possible combos.

* quickie * - do this method for 1 category's choice then multiply by the # of choices in this category.

Chart - make a chart for two of the categories. (kinda looks like an array) This will give you the # of combos for these items (each box = a combo choice). Multiply by 3rd category.

Disclaimer
Only works for 3 categories

Tree Diagram - 1st spread out the words (choices) for 1 category. Then add branches for the choices of category 2 to each of the category 1 words. Then, add the choices for the next category to each of the previous category's choices. Add the final branches to get the answer.



* Quickie *
do 1 branch
multiply
final # of choices by
of previous category
of choices

What role did each have in promoting meaningful mathematics discourse...

Teacher's role

Students' role

Teachers' role

Table 1: Advancing Children's Thinking framework summary

ELICITING (Eliciting children's solution methods)	SUPPORTING (Supporting children's conceptual understanding)	EXTENDING (Extending children's mathematical thinking)
Describes ways in which teachers can provide students with opportunities and necessary encouragement to express their ideas about mathematics.	Describes instructional techniques that support children in carrying out their mathematical solutions.	Describes ways in which teachers can challenge and extend what children do with their current mathematics thinking.

Fraivilig, J., Murphy, L.A., & Fuson, K. (1999). Advancing Children's Mathematical Thinking in Everyday Mathematics Classrooms. *Journal for Research in Mathematics Education*, 30 (2), 148-170.

Students' Role in math discourse

Piggyback
on an
Idea



Ask
Good
Questions

Explain my
Thinking



$$2 \times 2 = 4$$

Listen
and
Make Sense
of Math

Restate
What my

$$3 + 2 = 5$$

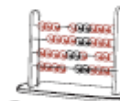
Friend Said



$$5 \text{ is equal to } 3 + 2$$



Model
with Math Tools
or by
Drawing



Agree
to
Disagree



Look for
a Pattern and
Use a Strategy



Make
a
Connection

Universal Design Techniques for Learning

GAINING ACCESS TO MATHEMATICAL LEARNING

Multiple Methods of Engagement



Real-life contexts
Rich math tasks
Physical Manipulatives
Virtual Manipulatives
Students' interests
Music & Literature
Videos clips
Art & architecture
History/Current events
Science and Tech
Multiple intelligences
Pictures
Games
Inquiry Based activities

Multiple Methods of Representations



Numeric
Written symbols
Hands-on
Demonstration
Visual/Auditory
Graphs/Tables
Diagrams
Textbooks
Computer programs
Class Discourse
Physical Manipulatives
Virtual Manipulatives

Multiple Methods of Expression & Assessment



Mathematicians log
Formative & summative
Performance based
assessments
Drawings, graphs,
charts
Computer work
Presentations
Verbal and written
explanations: Proofs

○ Multiple pathways to learning mathematics

○ Multiple ways to demonstrate understanding

○ Multiple ways to apply and connect ideas

Final thoughts

- Website : <http://mason.gmu.edu/~jsuh4/> under lesson study and <http://mason.gmu.edu/~jsuh4/publications.htm>

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