Transforming Teachers' Beliefs and Practice by Developing Algebraic Connections Through Problem solving

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Abstract:

This research involved forty elementary and middle school teachers in a summer institute called ACT NOW: Algebraic Connections and Technology with follow- up Lesson Study focused on developing algebraic connections and generalization strategies.

The research examined the use of algebraic problem solving to develop teachers' integrated pedagogical content knowledge through analysis of teachers' reflections in problem solving tasks, the use of pedagogical strategies, tools and technology, and teachers' beliefs.

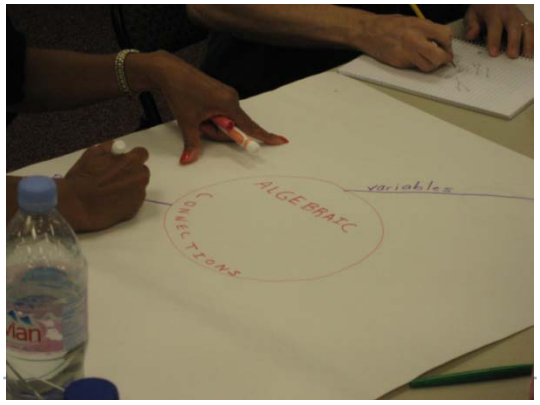
Results revealed three critical areas necessary in transforming teachers' beliefs and practice: 1) building pedagogical content knowledge; 2) developing productive dispositions towards mathematics; 3) translating the knowledge into teaching contexts.

Theoretic Framework

- Research and initiatives emphasize the importance of fostering algebraic reasoning through problem solving and laying the critical foundations before students encounter formal algebra (Blanton, 2008; Driscoll, 1999; NCTM, 2000; NMAP, 2008).
- Educators must have pedagogical content knowledge including the interconnectedness among conceptual understanding, procedural proficiency and problem solving (Shulman, 1986; Ball, 2003) "mathematical knowledge for teaching"

The goal of ACT NOW in MATH: Algebraic Connections and Technology in Middle Grades Math

- To support teachers to transform their practices by bridging more algebraic connections to the existing mathematics curriculum through problem solving and technology.
- To help teachers to reconceptualize algebra in the elementary and middle grades by re-examining the existing curriculum and bringing out the algebraic reasoning that was embedded in the arithmetic already being taught.



Research Questions

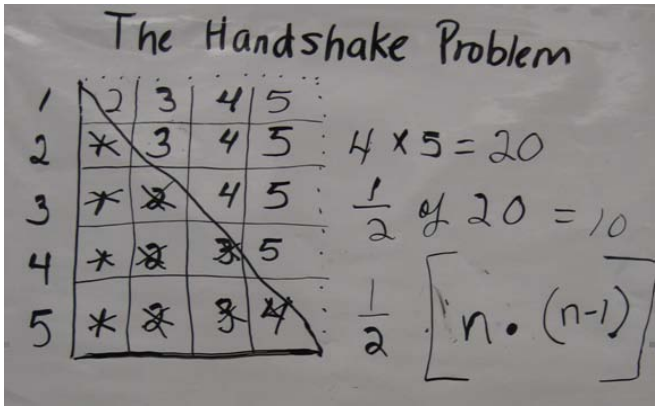
- What opportunities and experiences are most effective in developing teachers to acquire this deep integrated pedagogical content knowledge in early algebra?
- How are teachers' pedagogical content knowledge and beliefs revealed in their problem solving reflections as they take on the role of teachers as learners?

Methods

- Forty elementary and middle grades teachers from grades 3rd -8th met for a *2-week summer institute*. Daily activities included research-based practices and model lessons using a variety of mathematics tools and technology. Participants engaged in mathematically rich activities that connected algebraic content with pedagogical strategies through problem solving.
- In addition, teachers met during the academic year in small groups of 6-7 teachers with the instructors to continue their professional learning through a *teacher-led professional development model called Lesson Study*.

Data Sources

- The data sources included videotapes of the class sessions and teacher mathematician's reflective journal in which they responded to assigned problems that included
- (1) teachers' algebra content knowledge and feeling about the problems
- (2) teachers' use of mathematics tools and technology and pedagogical strategies and
- (3) teachers' beliefs.

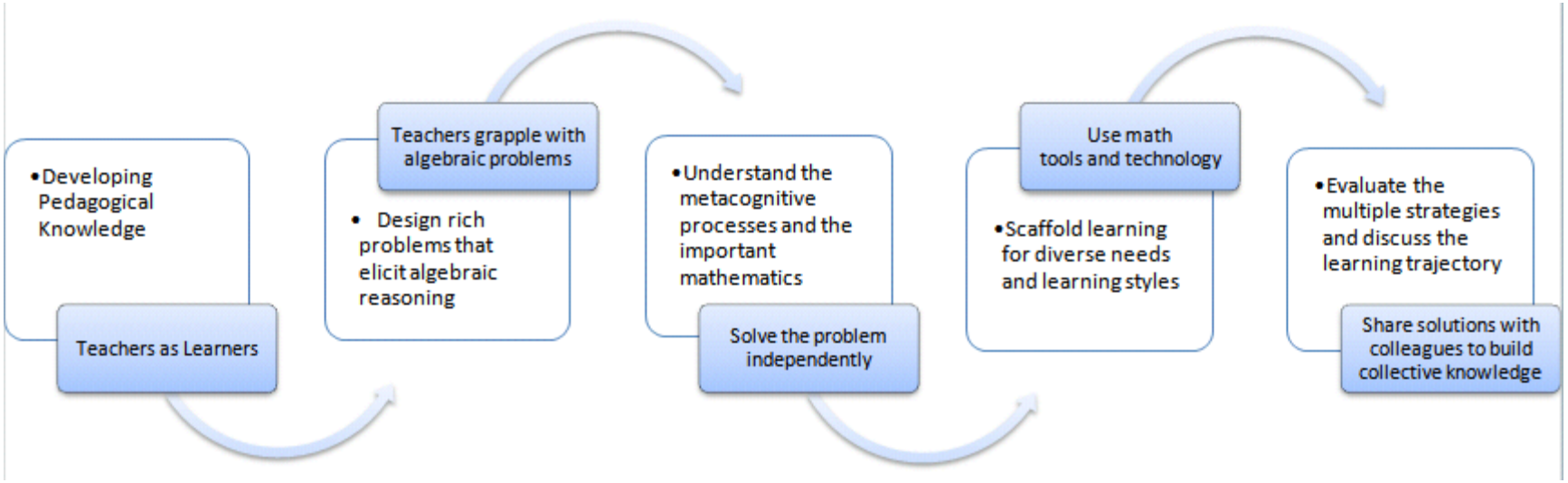


Teacher reflective entry shows "relearning" through problem solving

# toppings	# pizzas	0 topping	1 topping	2 topping	3 topping	4 topping	5 topping
2	4	1	2	1			
3	8	1	3	3	1		
4	16	1	4	6	4	1	
5	32	1	5	10	10	5	1
6	64	1	6	15	20	15	6
n	2^n	1	n	[n(n-1)]/2			

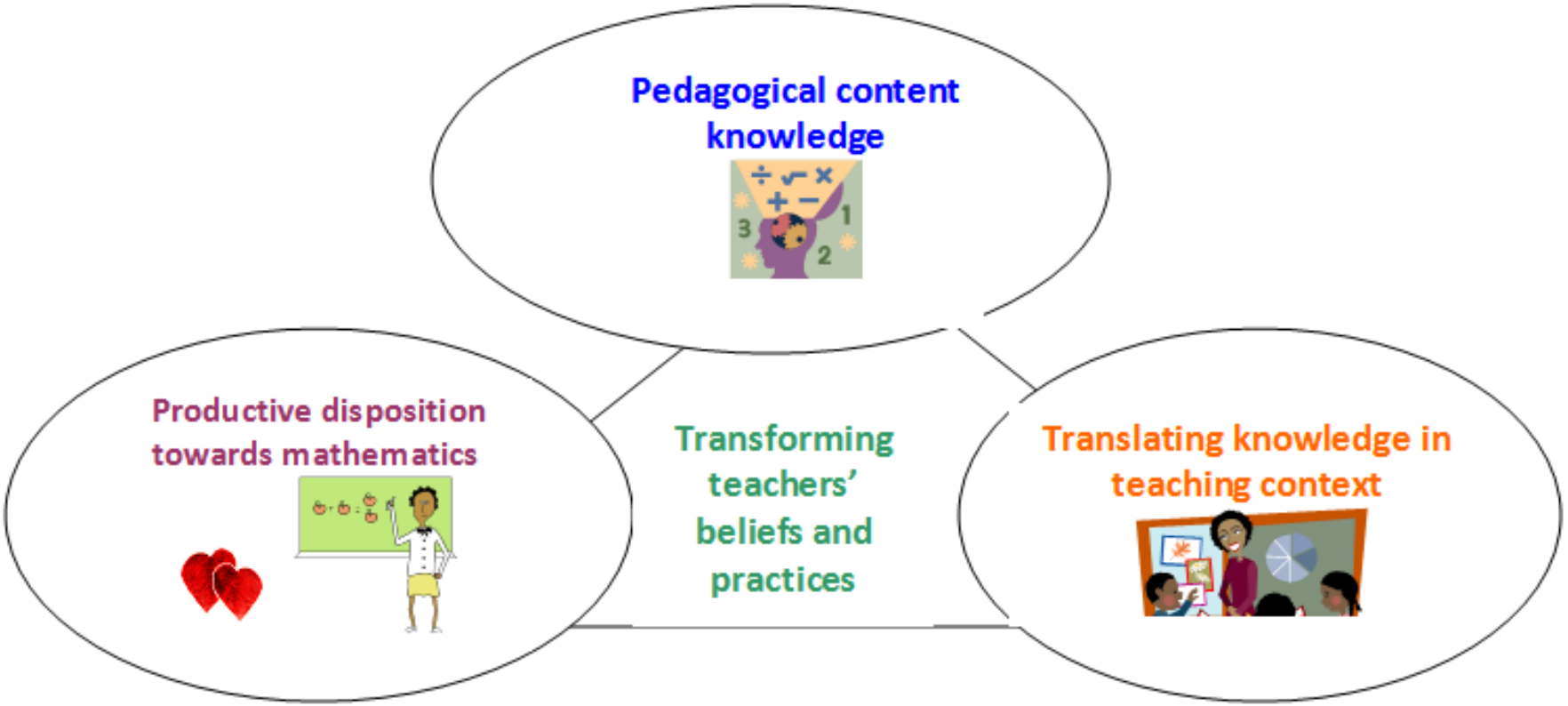
- *"Taking the chart that I made to compare toppings to pizzas, I decided to add some columns showing the number of combinations for each amount of toppings... I also noticed the symmetrical feeling the pattern in the table had, where the first and last numbers are the same, and the second and second to the last numbers were the same. I shared my discovery with my group, and Lucy noticed a triangular number pattern in something she was doing as well. When Jamie looked at the table, she quickly stated that it was Pascal's triangle. WOW… I totally missed that! The really big WOW was still to come for this problem. When we were working on binomials, Dr. S said something that really blew my mind. After already multiplying out $(x + y)^2$, he said to look at the coefficients when you multiply out $(x + y)^3$. So, I did it all out and got $x^3 + 3x^2y + 3xy^2 + y^3$, the coefficients were 1, 3, 3, 1. While I have held a belief that math makes sense and is full of patterns, this reaffirmed that belief and added on another level."*

Two levels of learning while engaged in algebraic problem solving



The first level was the learning that took place for teachers as learners as indicated by the blue boxes in the diagram below (see figure 3). The teachers learn to grapple and solve algebraic problems, use math tools such as graphs, tables, formulas, pictures and technology and finally evaluate multiple solutions with colleagues. On the second level, teachers gained pedagogical knowledge while engaged in this process as indicated in the white boxes. Each process that teachers participated in as learners, allowed them to consider the pedagogical implications to making algebraic connections such as, understanding the importance of designing rich problems that elicit algebraic reasoning and understanding the metacognitive processes and mathematical concepts important within these problems.

Critical areas of development necessary for transforming teachers' beliefs and practices



You have decided to use your allowance to buy an mp3 purchase plan. Your friend Alex is a member of i-sound and pays $1 for each download. Another one of your friends, Taylor, belongs to Rhaps and pays $13 a month for an unlimited number of downloads. A third friend, Chris, belongs to e-musical and pays a $4 monthly membership fee and $0.40 a month per download. Each friend is trying to convince you to join their membership plan. Under what circumstances would you choose each of these plans and why?

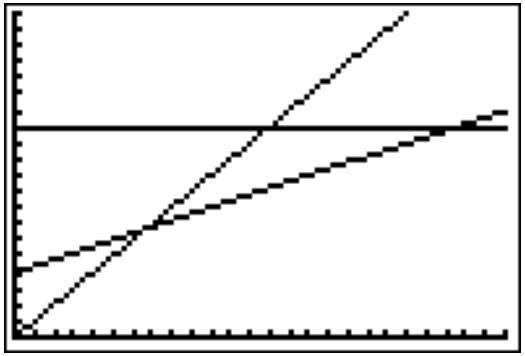
Multiple Representations

Verbal expressions were created together as a class.

Table

Number of Downloads	i-sound	Rhaps	e-musical
0	0	13	4
1	1	13	4.40
2	2	13	4.80
3	3	13	5.20
4	4	13	5.60
5	5	13	6
6	6	13	6.40
7	7	13	6.80
8	8	13	7.20
9	9	13	7.60
10	10	13	8
11	11	13	8.40
12	12	13	8.80
13	13	13	9.20
14	14	13	9.60
15	15	13	10
16	16	13	10.40
17	17	13	10.80
18	18	13	11.20
19	19	13	11.60
20	20	13	12
21	21	13	12.40
22	22	13	12.80
23	23	13	13.20
24	24	13	13.60
25	25	13	14

- **Graph**



- **Algebraic Formulas**

y = x (i-Sound)
y = 13 (Rhaps)
y = 4 + .4x (e-musical)

Teachers' voices…about "relearning"

"I used to be uncomfortable with kids struggling with problems that I think I may have "guided" their thinking which also "robbed" the critical thinking process. Through this experience, I realized that the struggling part is part of the problem solving process that mathematicians need to go through to make sense of the mathematics."

"I really need to change how I teach. I need to do more of these problems so that they can make those algebra connections."

"I will use more problems like these to teach the students to think and to make them more independent and confident learners. I use a lot of direct instruction which does help the Special Ed students but more "thinking" problems will be a great balance for my class."

"As a student in the class I am learning the power and benefit of struggling through a concept as a student; instead of simply receiving an equation or an answer. When I teach this year I want my students to feel this disequilibrium and then have satisfaction through understanding more through time as their mathematics learning continues."

"Unlike the algebra courses that I have taken before, this course is fulfilling my expectation of challenging me to "think algebraically."

"It really has been good to put me back into thinking like my students. I feel the frustration that I feel, and the great questioning and example setting gives me great insight as to how I can teach in my own classroom."

Teachers' voices…about Lesson Study

"The fact that I taught the lesson twice that day provided for us a nice opportunity to debrief after the first time, and modify the lesson for the second time. My analogy was that of a football coaching staff making modifications to the game plan at halftime. Once again, this was a very balanced and constructive process, with everyone's input considered and valued." (teacher who lead the Lesson Study with 8th graders)

"The debriefing / enhancing discussion was done in a very supportive environment. The term "enhancing" a lesson immediately helped me feel like any suggestions were put towards the lesson, and not as a critique of me. I hope that my students feel as safe as I felt taking this class!" (teacher who lead a lesson in a 6th grade class)

"This lesson study format allowed me to challenge even the lowest of my students. All children can learn through this method and having colleagues to bounce ideas off of made it so much more valuable." (a 4th grade teacher observer and participant in Lesson Study)

"Lesson study is a very powerful tool to gain insight into student learning and understanding of a specific topic. I learned more about my students during this one hour lesson than I have any other day in math this year. By engaging in the cooperative lesson study cycle I felt stronger as a teacher and more knowledgeable about where my students are and where I need to push them. Lesson study is not something that can be done alone and requires a unique blend of people who are willing to take risks and work collaboratively." (a 5th grade teacher who lead a lesson)