



Toothpick Squares

Lesson Study in 6th grade math

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Toothpick Squares

Finding patterns and
creating generalizations

Toothpick Squares

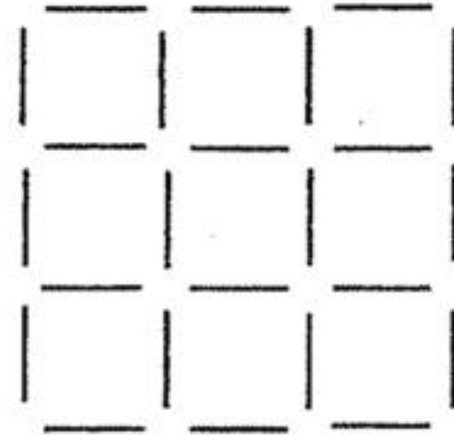
The Problem



#1



#2



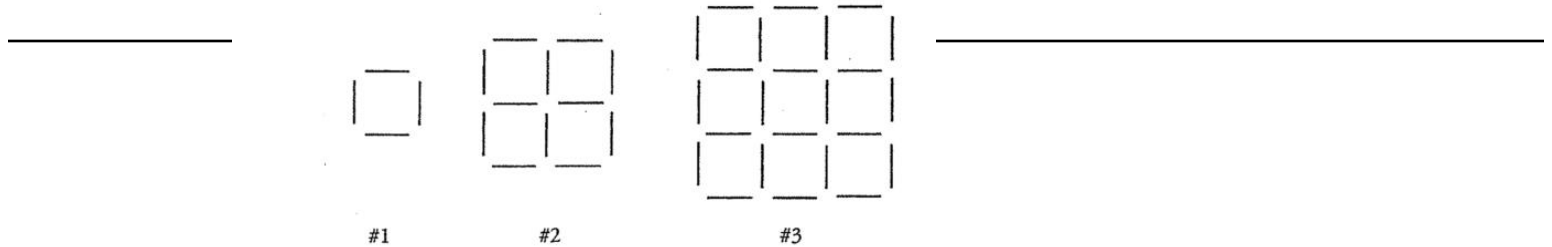
#3

1. Square #3 has 12 toothpicks in its perimeter.

- If you continue the sequence of squares, how many toothpicks will be in the perimeter of square #4?
- Write a rule that lets you predict how many toothpicks are in the perimeter of any large square in the sequence.

Toothpick Squares

The Problem

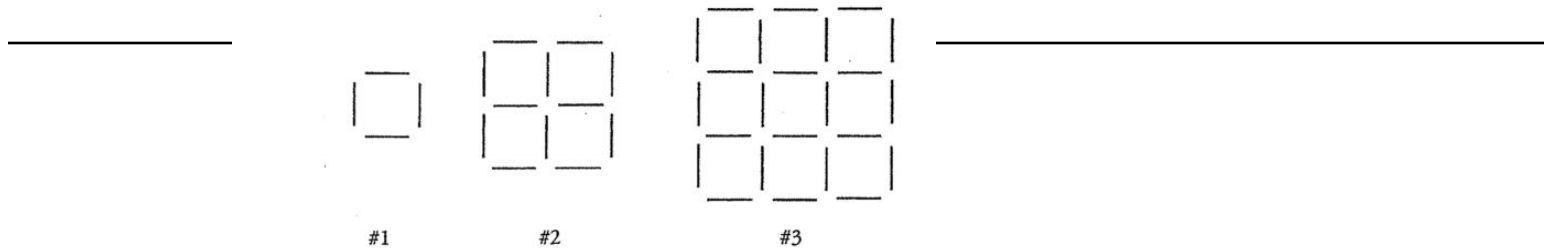


2. Square #3 is made up of 9 small squares.

- How many small squares would be needed for large squares #4, #5, #6, #7?
- Find a write a rule that lets you predict how many small squares would be needed in any square in the sequence (for instance, square #n).
- Is there a way to make a toothpick square that contains exactly 40 small squares? Explain your answer.
- Is there any way to make a toothpick square that contains exactly 144 small squares? Explain your answer.

Toothpick Squares

The Problem



3. The picture shows that there are 4 toothpicks in square #1 and 12 toothpicks in square #2.

- How many toothpicks are in square #3?
- How many toothpicks are in square #4?
- Predict the number of toothpicks needed for squares #5, #6, #10, & #15.
- Find and write a rule that lets you predict the number of toothpicks in square #n.

Mark starting the process.



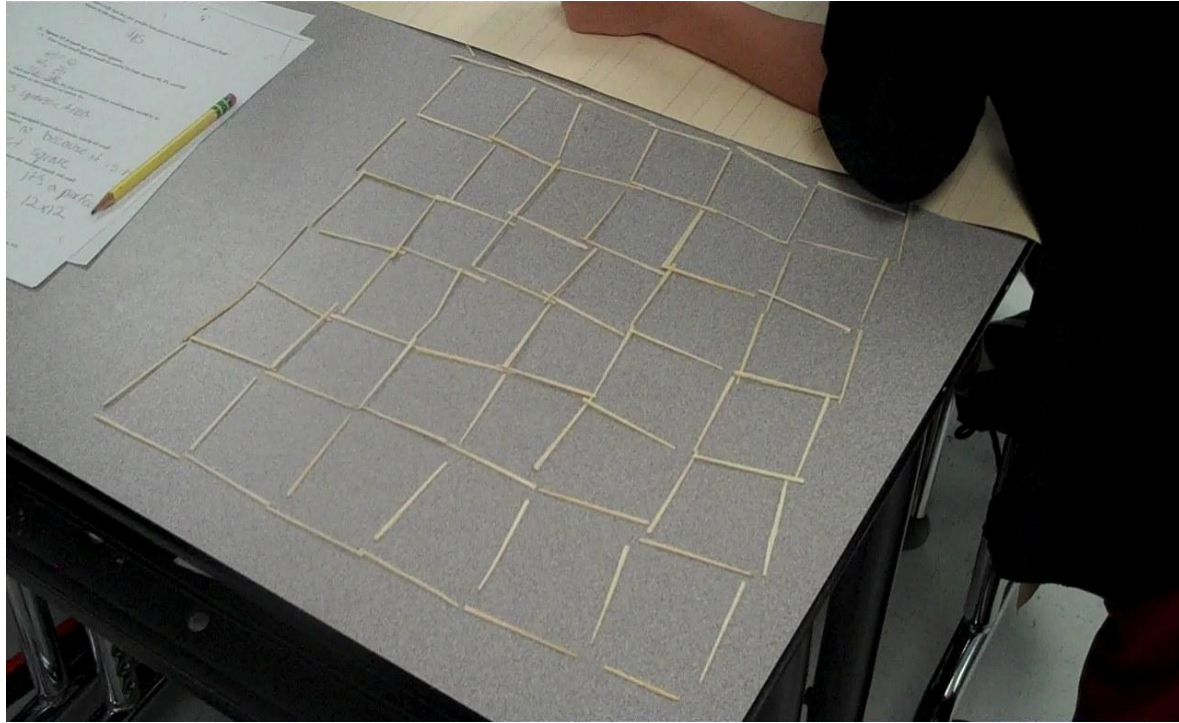
- We started with a review of basic geometric terms and formula.

Working it out



- The students used different methods to work on the problem.

Using manipulatives



- Many of the students used the toothpicks to set up the problem.

Hard at work



- Group discussing the problem, starting the table.

Math Talk



- Some groups had heated discussions about how to solve the problem.

Sharing



- Students had an opportunity to share their solutions with their classmates.

Engaged in discussion



- The students had lively and interesting discussions about the problem. Even as they were packing up to go to the next class the discussion continued.

Student Work

Sarah

ACTIVITY 2 TOOTHPICK SQUARES

High

Instructions
Work on the following math activity either individually or with one or two others.

The Problem

Examine the pictures above, which show three squares constructed out of toothpicks. Each large square is made up of some number of small squares, and each small square is one toothpick long on each side.

1. Square #3 has 12 toothpicks in its perimeter.
 * How many small squares would be needed to make square #4?
 16 toothpicks - 4 toothpicks x 4 sides

* Write a rule that lets you predict how many are in the perimeter of any large square in the sequence.

$n \times 4$ 4n-variable coefficient

2. Square #5 is made up of 9 small squares.
 How many small squares would be needed to make square #6, #7, and #8?
 * 4 = 16 * 6 = 36
 * 5 = 25 * 7 = 49

3. Find and write a rule that lets you predict how many small squares would be in any square in the sequence, any square #n.

n^2

4. Is there any way to make a toothpick square that contains exactly 40 small squares? Explain your answer.

Looked for factors, 40 = 4 x 10. No, the only factors of 40 are 1, 2, 4, 5, 8, 10, 20, 40. Is there any way to make a toothpick square that contains exactly 144 small squares? Explain your answer.

Yes, 12 x 12 (12^2) = 144.

Problem #1

24
40

$\frac{136}{24} = 5 \frac{20}{24}$
 $\frac{220}{40} = 5 \frac{20}{40}$

Can you predict the number of toothpicks needed for square #6, #10, and #17?
 * 6 = 84 * 10 = 400
 * 5 = 60

* 10 = 220

Find and write a rule that lets you predict the number of toothpicks in square #n

$(n+1) \times 2n$ partner's work
 $(n+1) \times (2+n)$
 $(n+1) \times 2n$

Problem #2

| Square # | # toothpicks | Square # | # small squares |
|----------|--------------|----------|-----------------|
| 1 | 4 | 1 | 1 |
| 2 | 8 | 2 | 4 |
| 3 | 12 | 3 | 9 |
| 4 | 16 | 4 | 16 |
| 5 | 20 | 5 | 25 |
| 6 | 24 | 6 | 36 |
| 7 | 28 | 7 | 49 |
| 8 | 32 | 8 | 64 |

Problem #3

24
40

$\frac{136}{24} = 5 \frac{20}{24}$
 $\frac{220}{40} = 5 \frac{20}{40}$

Can you predict the number of toothpicks needed for square #6, #10, and #17?
 * 6 = 84 * 10 = 400
 * 5 = 60

* 10 = 220

Find and write a rule that lets you predict the number of toothpicks in square #n

$(n+1) \times 2n$ partner's work
 $(n+1) \times (2+n)$
 $(n+1) \times 2n$

Caroline

ACTIVITY 2 TOOTHPICK SQUARES

Good

Instructions
Work on the following math activity either individually or with one or two others.

The Problem

Examine the pictures above, which show three squares constructed out of toothpicks. Each large square is made up of some number of small squares, and each small square is one toothpick long on each side.

1. Square #3 has 12 toothpicks in its perimeter.
 * How many small squares would be needed to make square #4?
 16 squares

* Write a rule that lets you predict how many are in the perimeter of any large square in the sequence.

You multiply the number by itself, then you get the answer.

Problem #1

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 * 6 = 84 * 10 = 400
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* 10 = 220

Find and write a rule that lets you predict the number of toothpicks in square #n

$(n+1) \times 2n$ partner's work
 $(n+1) \times (2+n)$
 $(n+1) \times 2n$

Problem #2

| Square # | # toothpicks | Square # | # small squares |
|----------|--------------|----------|-----------------|
| 1 | 4 | 1 | 1 |
| 2 | 8 | 2 | 4 |
| 3 | 12 | 3 | 9 |
| 4 | 16 | 4 | 16 |
| 5 | 20 | 5 | 25 |
| 6 | 24 | 6 | 36 |
| 7 | 28 | 7 | 49 |
| 8 | 32 | 8 | 64 |

Problem #3

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$\frac{136}{24} = 5 \frac{20}{24}$
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Can you predict the number of toothpicks needed for square #6, #10, and #17?
 * 6 = 84 * 10 = 400
 * 5 = 60

* 10 = 220

Find and write a rule that lets you predict the number of toothpicks in square #n

$(n+1) \times 2n$ partner's work
 $(n+1) \times (2+n)$
 $(n+1) \times 2n$

Student Work

Grey

LOWE

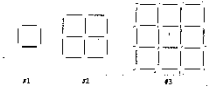
**ACTIVITY 2
TOOTHPICK SQUARES**

Divergent Thinker

Introduction
Work on the following math activity, either individually or with one or two others.

Very reluctant to show work explain

The Problem



Estimate the pictures above, which show three squares constructed out of toothpicks. Each large square is made up of some number of small squares, and each small square is one toothpick long on each side.

1. Square #3 has 12 toothpicks in its perimeter.
If the squares are combined, how many toothpicks could be in the perimeter of square #3?
16

2. What a rule that lets you predict how many are in the perimeter of any large square in the sequence?
4n

3. Square #3 is made up of 9 small squares.
How many small squares would be needed for large square #4 and #5?
12 15 18

Find and write a rule that lets you predict how many small squares would be in any square in the sequence, any square #n.
n^2 (n^2)

4. Is there any way to make a toothpick square that contains exactly 40 small squares? Explain your answer.
no, it's not a multiple of 3

5. Is there any way to make a toothpick square that contains exactly 144 small squares? Explain your answer.
yes, it's a multiple of 3^2

Not thinking about previous question

Challenge 1

| sq. # | # of toothpicks |
|-------|-----------------|
| 1 | 4 |
| 2 | 8 |
| 3 | 12 |
| 4 | 16 |
| 5 | 20 |
| n | 4n |

Challenge 2

| sq. # | # of toothpicks |
|-------|-----------------|
| 1 | 6 |
| 2 | 14 |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| n | 2 * (n+1) + n |

683

$(n+1) \times 2 \times n = A$

Challenge 3

| sq. # | # of toothpicks |
|-------|-----------------|
| 1 | 6 |
| 2 | 14 |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| n | 2 * (n+1) + n |

Notice how he goes straight to the "rule"

$n \quad 2 * (n+1) + n$

Challenge 1

| sq. # | # of toothpicks |
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| 1 | 4 |
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| 5 | 20 |
| n | 4n |

Challenge 2

| sq. # | # of toothpicks |
|-------|-----------------|
| 1 | 6 |
| 2 | 14 |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
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Challenge 3

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| 10 | |
| n | 2 * (n+1) + n |

683

$(n+1) \times 2 \times n = A$

no organization

$2 * (n+1) + n$

Notice how he goes straight to the "rule"

$n \quad 2 * (n+1) + n$