

THE PRODUCT GAME

Research Theme: Meaningful mathematics discourse in the classroom.

Broad Subject Matter Goals:

NCTM Standards and Expectations

Number & Operations 6-8

1. Use factors, multiples, prime factorization, and relatively prime numbers to solve problems.
2. Develop and analyze algorithms for computing with integers and develop fluency in their use.
3. Develop meaning for integers and represent and compare quantities with them.
4. Develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results.
5. Develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

Unit Learning Objectives: Students develop understanding of products, factors, multiples, and the relationships between them.

Learning Objectives:

Students will

- Review multiplication facts;
- Develop strategies for identifying factors and multiples;
- Understand that some products are the result of more than one factor pair;
- Develop understanding of factors and multiples and of the relationships between them.

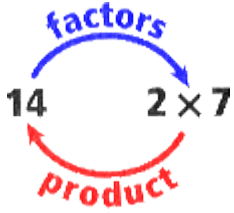
Materials:

- Paper clips (2 per pair of students)
- Colored chips (about 12 each of 2 colors per pair of students), or colored pens, markers, or pencils
- Product Game Rules Activity Sheet (One copy per pair of students)
- Game Board Overhead
- Copy of Game Boards Activity Sheet (One copy per pair of students)
- Illuminations The Product Game applet (alternative method for playing game) [Standalone version of the Product Game Applet]
- Making Your Own Product Game Activity Sheet (one copy per pair of students)

Instructional Plan

This lesson presents *The Produce Game*, an activity through which students see the relationship between products and factors. Though there are four lessons in this unit, the activities below only encompass the first two lessons of this unit.

Students play *The Product Game* in the first lesson. They start with factors and multiply to find the product. In the second lesson, they make their own game boards. The task of creating a new game is challenging to most students. They learn a lot by experimenting and by making mistakes about what factors and products to include in a game. The last lesson contains extensions and connections students can use to make connections and expand of what they have learned in the previous lessons.

PART 1			
<i>Steps</i>	<i>Main Learning Activities</i>	<i>Students' Anticipated Responses</i>	<i>Remarks on Teaching</i>
Introductory activities: Evoke prior knowledge	<ol style="list-style-type: none"> Before you introduce the game, make sure your students understand what a product is. Say: We are going to learn to play a new game called <i>The Product Game</i>. What does the word “product” mean? <ul style="list-style-type: none"> Use the <i>Fruyer</i> Model to discuss vocabulary used in this lesson (<u>product</u> and <u>factor</u>). The Model is attached to the end of this lesson plan. 	<p>Students often confuse the terms factor and multiple.</p> <p>Students may not understand how the terms product and multiple are related.</p>	<p>Use this diagram to help students see the relationship between factors and products:</p> 
Introductory activities: Understanding <i>The Product</i>	<ol style="list-style-type: none"> When you are satisfied students can give examples of products with understanding, introduce <i>The Product Game</i>. Project the <u>Game Board</u> overhead for 		

<p><i>Game</i></p>	<p>students to see. <i>The Product Game</i> board consists of a list of factors and a grid of products. Two players compete to get four squares in a row – up and down, across, or diagonally.</p> <ol style="list-style-type: none"> 4. Distribute a copy of the <u>Product Game Rules</u> activity sheet to each pair of students. 5. Explain that the list of numbers at the bottom of the board are factors and that the numbers in the grid are the products that can be made by multiplying any two factors. When you play the game, use two colors to mark the products; one to mark the class's products and the other to mark your own. <ul style="list-style-type: none"> • Player 1 puts a paper clip on a number in the factor list. No square on the product grid is marked with Player 1's color because only one factor has been marked; it takes two factors to make a product. • Player 2 puts the other paper clip on any number in the factor list (including the same number marked by Player 1) and then shades or covers the product of the two factors on the product grid. • Player 1 moves either one of the paper clips to another number and then shades or covers the new product. 	<p>Students may assume that each marker has to be on a different number.</p> <p>Students may assume that they must move only “their” marker.</p>	<p>Note that both markers can be on the same number resulting in a square number, and that they can move either paper clip to make their next move.</p> <p><u>Questions to consider:</u></p> <ol style="list-style-type: none"> 1. What are the factors of numbers and how do you find them? 2. What are the multiples of numbers and how do you find them?
--------------------	--	--	---

	<ul style="list-style-type: none"> Each player, in turn, moves a paper clip and marks a product. If a product is already marked, the player does not get a mark for that turn. The winner is the first player to mark four squares in a row -- up and down, across, or diagonally. 		
Exploration: Students play <i>The Product Game</i>	<p>6. Distribute the <u>Playing the Product Game</u> activity sheet for students to complete in pairs, OR students may use the “Illuminations” Applet to play the game. [<u>Standalone version of the Product Game Applet</u>]</p>	Students may use random moves.	<p>Use questioning as students play to ask what they are thinking about as they choose their next move.</p> <p>Have students play in pairs two or three times.</p>
Summary	<p>7. Have a class discussion about whether it is better to go first or second. Have students share any strategies they discovered while playing the game. Record student comments.</p> <p>8. Go over the follow-up questions with your class. This is especially important, since the word <i>multiple</i> is introduced for the first time.</p> <ul style="list-style-type: none"> What are the factors of numbers and how do you find them? What are multiples of numbers and how do you find them? What did you learn about prime and composite numbers while you were playing the Factor Game? Is the number 1 prime or composite? 		<p>Discuss whether it is better to go first or second. Focus on higher-level questioning (evaluation of strategies).</p> <p>Watch out for students who have not sorted out factor, product, multiple, and their relationship.</p> <p>** Make sure to end lesson with summary questions (bulleted) in the left column.</p> <p>Use the last question as a starting point for Part 2 of the lesson.</p>

	<p>Explain.</p> <ul style="list-style-type: none"> • Is it best to go first or second? Is there a best first move? • Is it possible to get every product with the factors given? • Are any products missing for these factors? • Suppose that the game is in progress and you want to cover the number 12 on the grid. Describe one way this can happen? Can you get 12 in more than one way? • Suppose the game is in progress and one of the paper clips is on the 5. What products can you make by moving the other paper clip? 		
--	---	--	--

PART 2			
<i>Steps</i>	<i>Main Learning Activities</i>	<i>Students' Anticipated Responses</i>	<i>Remarks on Teaching</i>
Introductory activities: Evoke prior knowledge	1. Ask students to use the words factor, divisor, product, multiple and divisible by in writing (or saying) sentences that can be determined from the math sentence $5 \times 6 = 30$. Then ask students to identify where the factors and products are on the game board.		

	<ol style="list-style-type: none"> 2. Ask students to explain the rules of the game. 3. Spend some time reviewing students' strategies for playing <i>The Product Game</i>. 		
<p>Introductory activities: Understanding how to derive products from factors</p>	<ol style="list-style-type: none"> 4. Redistribute <u>Product Game Rules Activity Sheet</u>. 5. Pose the following problem: "Look back at the board. Does it contain all the products you can make from the list of factors? What products would we need to add if we added 10 to the factor list?" 6. Participate in a discourse with the students about the products which would already be contained in the list and those products they would need to add. <ul style="list-style-type: none"> • (From Illuminations) - Multiplying 10 by the other factors on the list gives the products 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100. Help students to see that 10, 20, 30, and 40 are already on the board. (You might ask why.) Therefore, you would need to add only 50, 60, 70, 80, 90, and 100. You want students to see that every product, including the squares of the factors, must be on the game board to make a good game. Sometimes students have to experience frustration while making their game boards before they realize that every 	<p>Students may not notice that the game board contains <u>all</u> the products from the list of factors.</p>	<p>Examine the game board. Is it possible to get every number on the product grid by multiplying two of the numbers in the factor list? Explain.</p> <p>Can you find two numbers in the list of factors for the game whose product is not on the product grid?</p> <p>What products would we need to add if we added 10 to the factor list?</p> <p>Help students to see that 10, 20, 30, and 40 are already on the board. (You might ask why.) Therefore, you would need to add only 50, 60, 70, 80, 90, and 100.</p>

	product must be included.		
Exploration: Students create their own <i>Product Game</i>	<p>7. Distribute the <u>Making Your Own Product Game Activity Sheet</u> (one copy per pair of students)</p> <p>8. To create a new game, students might first decide what factors they want to use and then determine which products are possible.</p> <ul style="list-style-type: none"> • If, for example, students choose the factors 1, 2, 3, and 4, the products would be 1, 2, 3, 4, 6, 8, 9, 12, and 16. This would create a nice 3×3 game board. The rules could be modified so that three in a row would win. • Students need to use enough factors to make their game interesting. For example, the factors 1, 2, and 3 give the products 1, 2, 3, 4, 6, and 9. A 3×2 grid would accommodate these six products, but this would not make a very interesting game. Only two markers in a row would be required to win, so the game would end on the second turn of the first player! • Instead of choosing the factors first, students can select the size of the product grid they want, then work backward to find the factors needed to fill the board. 	<p>Students may choose too few factors to make the game interesting because the board is too small.</p> <p>Students may not know they can choose a board of any size, rectangular or square, and they don't have to fill every square (but it's not really a fair game if they don't).</p>	<p>Direct students to choose the grid size first, and then choose factors that help them fill the grid</p> <p>(You might want to help students organize their work in a table that lists the number of products created as each new factor is added to a list. (See <i>Illuminations Lesson Plan for example</i>)</p> <p>Check students list of products and help them get the products correct.</p> <p>What size game board will hold all of your products? Is this the smallest board you can use?</p> <p>When students have finished making their boards and trying them, ask them to work on the summary paragraph described in the Activity Sheet.</p>

	<p>Interested students might be challenged to find the factors needed to create a 10×10 board (the factors 1 through 16 are needed, and there will be three blank spaces). You might want to help students organize in a table. One from NCTM's <i>Illuminations</i> site is included at the end of this lesson plan.</p> <p>9. Direct students back to the <u>Making Your Own Product Game Activity Sheet</u>. Circulate while students are making their games, and help keep them focused on the task.</p> <p>10. If a groups I having difficulty, check over their list of products and help them get the products correct. Questions you might pose include:</p> <ul style="list-style-type: none">• I notice that the product of 4 and 5 (or any others) is not on you list? Have you checked to make sure you have all of the products? <p>11. Also ask questions that will help students focus on the relationship between the list of products and the size of the game board. Some board will need to have blanks or free spaces. Questions you might pose include:</p> <ul style="list-style-type: none">• What size game board will hold all of your products? Is this the smallest board you can use?		
--	---	--	--

<p>Exploration: Students play theirs and others <i>Product Games</i></p>	<p>12. Students will play the created games in pairs.</p> <p>13. Ask students to make notes about anything interesting about the different game board they encounter. Share their comments with the class at the end of the activity.</p> <p>NOTE: Alternatively, students may use the Applet once again. The Product Game applet is below, with one change from the first lesson. Now there is a bar called "Customize." Click on the "Customize" bar, then change the list of factors and the number of squares needed for a winning "run" so it is the same as the game you designed. Click "OK" and the applet will make a game board from your factors. Play the game a few times with your partner. Decide together which game board is better (yours or the applet's) and why.</p>		<p>When students are playing each other's games, remind them that it is very important that they give good feedback.</p>
<p>Summary</p>	<p>14. Once students have finished making their boards and trying them, ask them to work on the summary paragraph described in the Activity Sheet.</p> <p>15. <i>You can summarize this activity with each group individually.</i> As you interact with a group, observe the problems they are having, and work to help them overcome these problems. Ask the group</p>		<p>Ask the group to explain the steps they went through to create the board. Ask what problems they had and how they solved these problems. Ask how they knew when they had all of the possible products and whether they needed to change the rules to play on their board.</p>

	<p>to explain the steps they went through to create the board. Ask what problems they had and how they solved these problems. Ask how they knew when they had all of the possible products and whether they needed to change the rules to play on their board.</p> <p>16. You also could summarize by having groups share their reports with the class. Use the reports to help students focus on characteristics of interesting game boards and the strategies that were used to create them.</p>		<p>Focus on characteristics of interesting game boards and the strategies that were used to create them.</p>
--	---	--	--

Definition (in your own words)

Facts/Characteristics

Product

Examples

Non-examples

Definition (in your own words)

A product is the number that results when two or more numbers are multiplied together.

The name given to the answer in a multiplication problem.

Facts/Characteristics

A product will be larger than the numbers multiplied together when all the numbers are greater than or equal to one.

A product will be a positive number when two negative numbers are multiplied together.

A product may be smaller than some of the numbers multiplied together when one or more of the numbers is a fraction.

Product

Examples

$$4 \times 5 = 20$$

$$4 \times \frac{1}{2} = 2$$

$$0.7 \times 0.8 = 0.56$$

$$-3 \times 5 = -15$$

$$-4 \times -7 = 28$$

$$-0.3 \times -1.2 = 0.36$$

$$2 \times 3 \times 5 = 30$$

$$\sqrt{2} \times \sqrt{2} = 2$$

$$\sqrt{2} \times 3 = 3\sqrt{2}$$

Non-examples

$$4 + 3 = 7 \quad (7 \text{ is a sum not a product})$$

$$18 - 12 = 6 \quad (6 \text{ is the difference})$$

$$20 \div 4 = 5 \quad (5 \text{ is a quotient})$$

(5 is also a factor of 20)

LESSON STUDY OBSERVATION GUIDELINES – LESSON #1 ON CONCEPTS OF PERCENT

(This page is for use by those who observe the selected lesson as it is taught. The focus of this observation is on the lesson activities and student learning; it is not an evaluation of the teacher.)

OBSERVATION OF RESEARCH LESSON

As you observe, keep the following questions in mind and make notes about what occurs. Please be strictly an observer and do not interact with the teaching or the learning; observers may ask students informational questions for purposes of clarification.

Note: ✓ indicates an observation question that is particularly relevant to our Research Theme and Subject-Matter Goals.

Academic

- ✓ What evidence did you see that students understood how the number of factors changed the size of the board?
- ✓ What were students' reactions when they made boards that were not perfectly rectangular?
- ✓ What type of understanding was demonstrated by each student? What misconceptions remain?

Motivation and Engagement:

- Did students appear to be engaged in the lesson? If so, how?
- Did students share strategies for making a game board or winning when playing a certain board configuration?

Social Behavior

- ✓ Did students discuss with their peers possible ways to choose factors for a game board?
- Were students' comments only given in response to the teacher's questions and comments or did students respond to and elaborate upon other students' comments?

Instructional Features:

- What instructional tasks appeared to be most effective in general? What tasks appeared to be most effective for students of different learning groups (visual, gifted, English language learners, special needs, etc.)? What types of understanding were demonstrated by these students?
- ✓ What questions did the teacher ask? What questions did the teacher ask about student strategies?
- ✓ How did the teacher respond to students' suggested strategies? What students were called on by the teacher?

Additional Observations: