IMPACT in Mathematics 2010

Improving Mathematical Practices through Algebraic Connections and Technology

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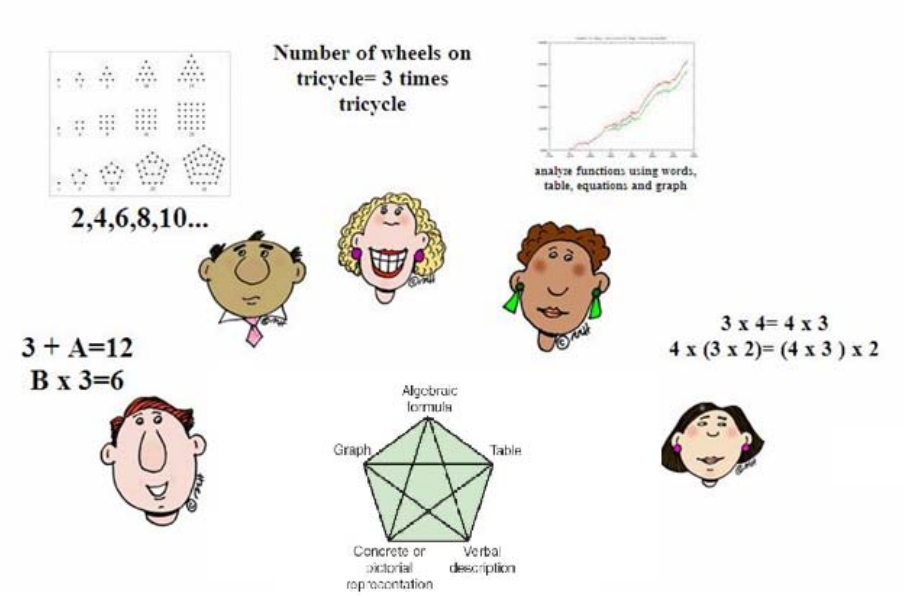
With the IMPACT TEAM GRADES 6-8 from 2009-2010

(Diane Dunn, Tammy Kraft, Mariam Shaker, Crystal Welsh, Nikki Wright)

http://mason.gmu.edu/~jsuh4/impact/index.htm



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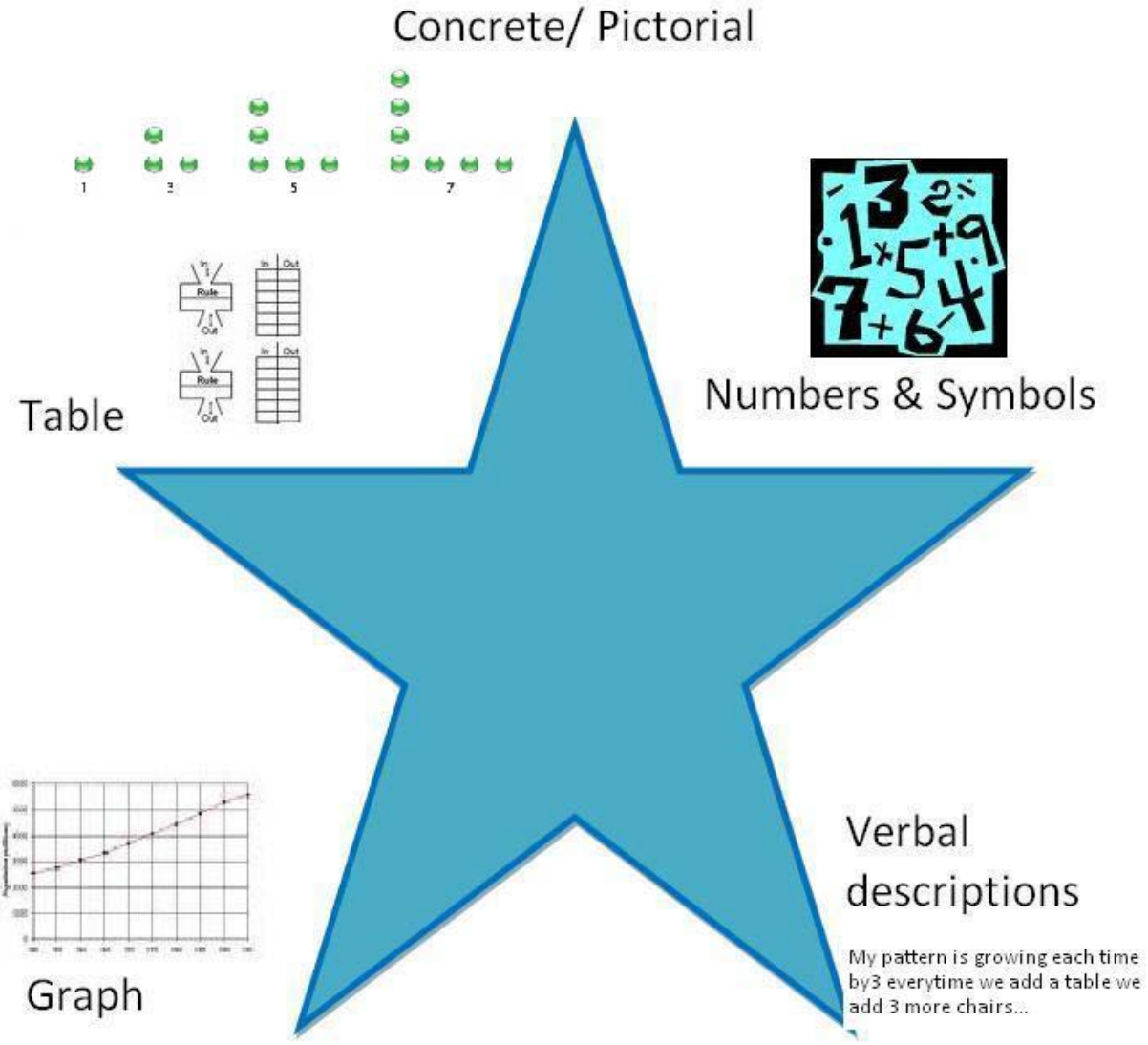
FIVE STAR REPRESENTATIONS



Algebraic Connection Talk

DOING AND UNDOING

How is this number in the sequence related to the one that came before?

What if I start at the end?

Which process reverses the one I'm using?

Can I decompose this number or expression into helpful components?

Building Rules to Represent Functions

Is there a rule or relationship here? Does my rule work for all cases?

How does the rule work and how is it helpful? Why does the rule work the way it does?

How are things changing?

Is there information here that lets me predict what's going to happen?

What steps am I doing over and over? Can I write down a mechanical rule that will do this job once and for all?

How can I describe the steps without using specific inputs?

When I do the same thing with different numbers, what still holds true? What changes?

Now that I have an equation, how do the numbers (parameters) in the equation related to the problem context?

Abstracting from Computation

How is this calculating situation like/unlike that one?

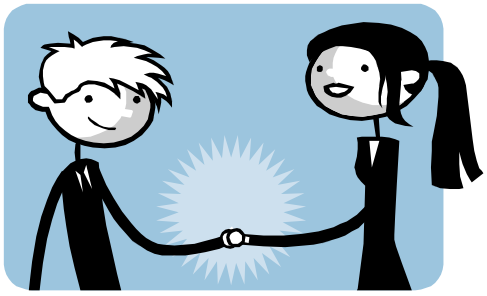
How can I predict what's going to happen without doing all the calculations?

What are my operation shortcut options for getting from here to there?

When I do the same thing with different numbers, what still holds true? What changes?

What are other ways to write that expression that will bring out hidden meaning?

How can I write the expression in terms of things I care about?How does this expression behave like that one?

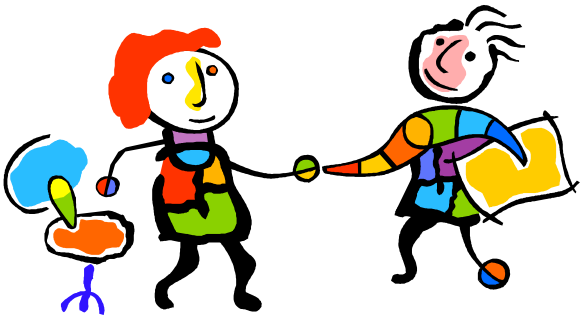


Handshake Problems

1. If everyone at your table shakes hands with everyone else, how many handshakes would there be?

2. If everyone in your class shakes hands with everyone else, how many handshakes would there be?

3. At a birthday party, every child shakes hands with every other child. If 190 different handshakes take place, how many children were at the party?



The locker problem

Here is the famous locker problem: Imagine you are in a hallway with 20 lockers, all shut.

> Suppose the first student goes along the row and opens every locker.
> The second student then goes along and shuts every other locker beginning with locker number 2.
> The third student *changes the state* of every third locker beginning with locker number 3. (If the locker is open the student shuts it, and if the locker is closed the student opens it.)
> The fourth student *changes the state* of every fourth locker beginning with number 4.

Imagine that this continues until the 20 students have followed the pattern with the 20 lockers. At the end, which lockers will be open and which will be closed? Which lockers have been switched the most often? Which lockers are still open after the twentieth student is finished?

Suppose there are 200 lockers. Which lockers are open after the 200^{th} student is finished? Which locker or lockers changed the most?

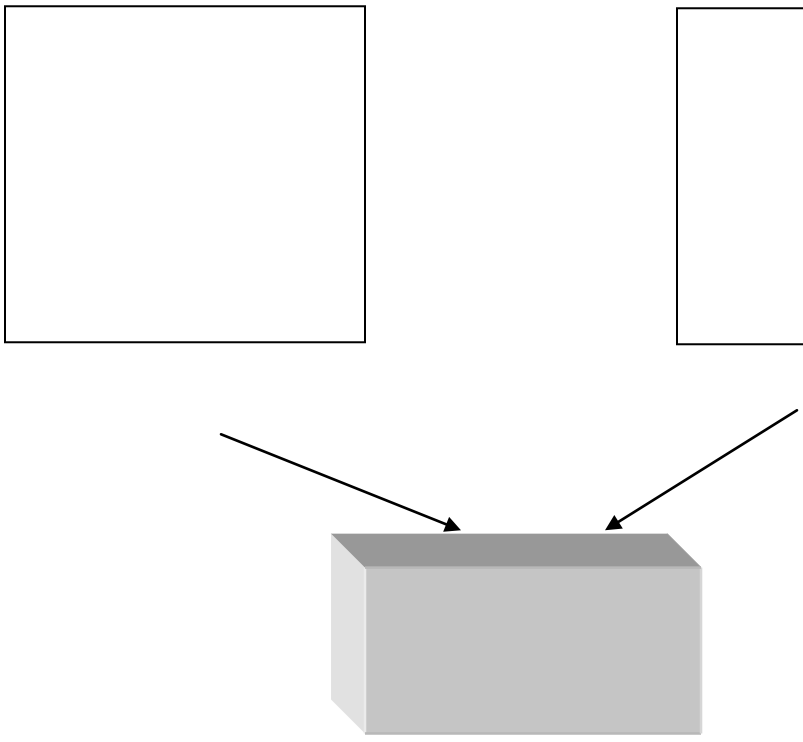
Ichiro's Problem

It has been one month since Ichiro's Mother has entered the hospital. He has decided to pray with his younger brother at a local temple every morning so that she will get better soon. There are 18 ten-yen coins in Ichiro's wallet and just 22 five-yen coins in the younger brother's wallet. They have decided to take one coin from each wallet everyday and put them in the offertory box and continue to pray until either wallet becomes empty.

One day, when they looked in to each other's wallets when they were done with their prayer, the younger brother's amount was greater than Ichiro's. When this happened, how many days had it been since they started their prayer?

Ichiro's wallet

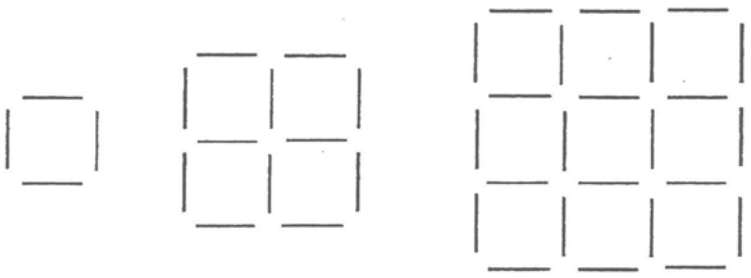
Brother's wallet



Tooth pick problem

Instructions
Work on the following math activity, either individually or with one or two others.

The Problem



#1 #2 #3

Examine the pictures above, which show three squares constructed out of toothpicks. Each large square is made up of some number of small squares, and each small square is one toothpick long on each side.

1. **Square #3 has 12 toothpicks in its perimeter.**
   - If the sequence of squares was continued, how many toothpicks would be in the perimeter of square #4?

- Write a rule that lets you predict how many are in the perimeter of any large square in the sequence.

2. **Square #3 is made up of 9 small squares.**
   - How many small squares would be needed for large squares #4, #5, and #6?

   - Find and write a rule that lets you predict how many small squares would be in any square in the sequence, say square #*n*.

   - Is there any way to make a toothpick square that contains exactly 40 small squares? Explain your answer.

   - Is there any way to make a toothpick square that contains exactly 144 small squares? Explain your answer.

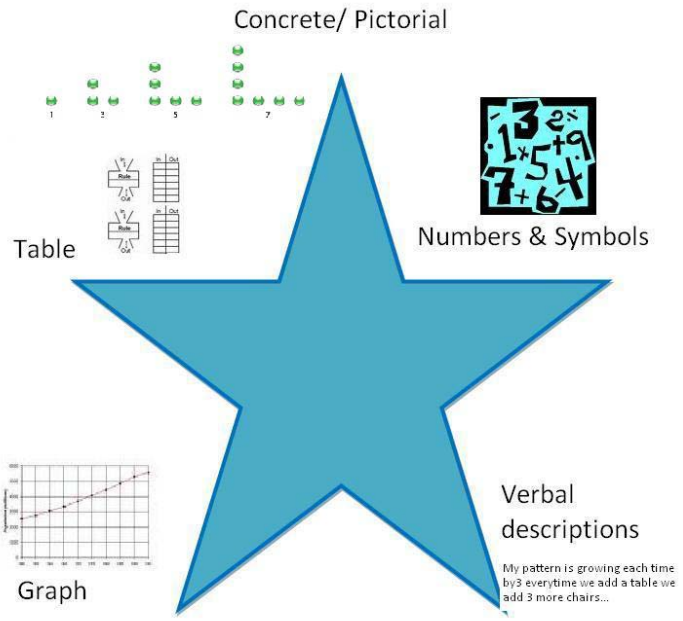
3. **The picture shows that there are 4 toothpicks in square #1 and 12 toothpicks in square #2.**
   - How many toothpicks are there in square #3?

   - In square #4?

   - Can you predict the number of toothpicks needed for squares #6, #10, and #15?

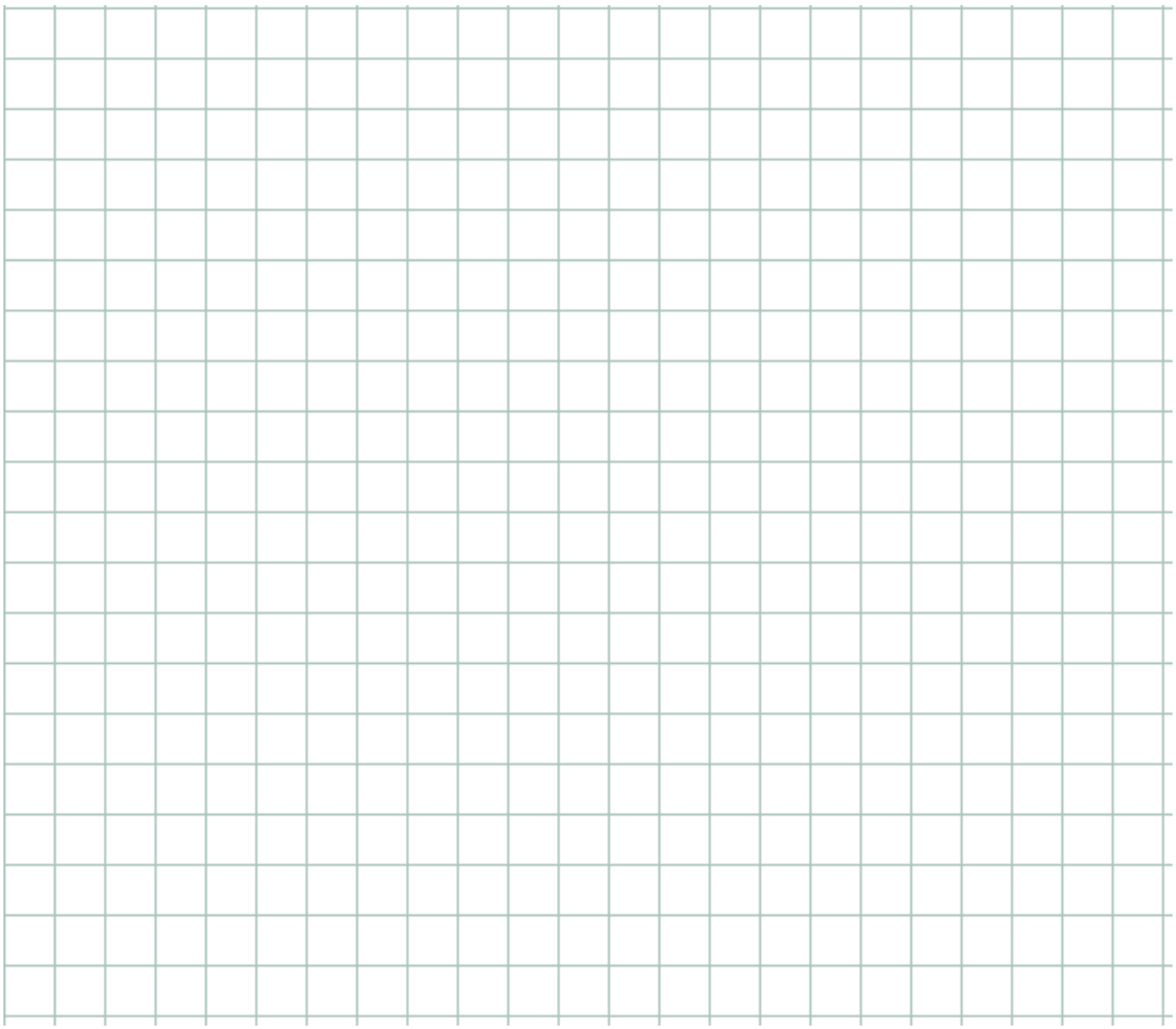
   - Find and write a rule that lets you predict the number of toothpicks in square #*n*?

Five Star-Algebraic Connections

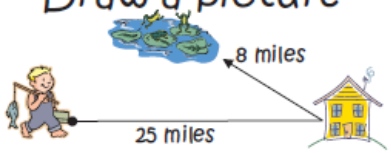
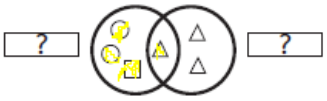
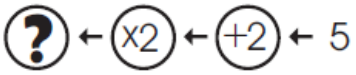
Math Investigation:	Choose a representational strategy.
	

Numbers, Pictures, Table, Graph & Words.

Graph



General Rule:

Problem Solving Strategies...	**Look for a pattern.** □○△□○△	**Draw a picture** 
Act it out. 	**Guess and Check.** 	**Write a number sentence.** $5 \times 4 = 20$
Use logical reasoning. 	**Work backwards.** 	**Make an organized list.**  - vanilla with chocolate - vanilla with rainbow - vanilla with raisins
Make a table. \# of people: 1 2 3 4 5 \# of eyes: 2 4 6 8 10	**Use a simpler problem.**  If I know 3×7=21 then I know 6×7 is 21 doubled = 42 	**_____'s Strategy.**