# The Handshake Problem

Tamisha is in a Geometry class with 25 students. On the first day of class her teacher asks everyone to shake hands and introduce themselves to each other. Tamisha wants to know how many handshakes had just been exchanged. Brainstorm some ways that you could use to find an answer to Tamisha's question.

Let students discuss methods that they might use to find an answer. Four possible methods are presented below.

#### **METHOD 1**

One way is to have the students actually simulate the shaking of hands.

Shake hands with everyone in the room. How many handshakes occurred? How did you keep track of your handshakes to make sure that you shook everyone's hand exactly once? What was the total number of handshakes that occurred in the room?

[This can be cumbersome, confusing and time-consuming but is workable if the group is not too big. Sometimes students do not come up with the correct 300 handshakes.] You might to use a subset of the class if you actually wanted to model the problem.

### **METHOD 2**

Let's gather some data and make a table. Why not start with 2 students and work your way up? How many handshakes are needed for two students? Have students actually do this. How many handshakes are needed for three students to greet each other exactly once? How many handshakes for four students? Is there a pattern that is revealed? What is the total number of handshakes?

Number of Students	Number of Handshakes
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45
25	300

The total number of handshakes would be 300.

# **METHOD 3**

The first student (A) can shake hands with 24 other students. The second student (B) can shake hands with only 23 other students. He or she has already shaken hands with student A so that leaves 23 other students with whom he or she can shake hands. Using the same argument, student C can shake hands with 22 students (A and B have already been greeted and the student does not shake hands with himself). The total number of handshakes will be  $24 + 23 + 22 + 21 + \dots + 3 + 2 + 1 = 300$ .

# **METHOD 4**

Students could use a model for the handshakes. Call the 25 students A, B, C, D,.....W, X, Y. Each handshake will be represented by a pair of letters. For example, AB represents students A and B shaking hands, CF represents student C and F shaking hands. Do we need to represent BA? [No since AB and BA are the same two students shaking hands.]

If students are careful and do the handshake representation in order they should not miss any.

Counting by rows:  $24 + 23 + 22 + \dots + 3 + 2 + 1 = 300$ 

# **Class Discussion Questions:**

If there were 53 students in the room would it be practical to have each student shake everyone else's hand and count them up? Explain your reasoning.

# **Class Discussion Question**

Can you develop a formula for calculating the number of handshakes if there were n students in the room?

Let's return to the chart we created in the second method.

Number of Students	Total Number of Handshakes
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45
25	300

Is there a formula that relates the number of students in the first column to the total number of handshakes in the second column? Again let students discuss and brainstorm possible approaches.

If students need a prompt consider adding another column on the right and write the numbers in the last column as

$$1 = \frac{(2)(1)}{2} \qquad 3 = \frac{(3)(2)}{2} \qquad 6 = \frac{(4)(3)}{2} \qquad 10 = \frac{(5)(4)}{2} \qquad 15 = \frac{(6)(5)}{2}$$

# **More Class Discussion**

Do you see any pattern in the formula? How are the numbers in the formula related to the number of students who are shaking hands?

### **Answer:**

$$Total\ number\ of\ handshakes = \frac{(number\ of\ students)(number\ of\ students-1)}{2}$$

In more shorthand form:  $T = \frac{n(n-1)}{2}$  where T = total number of handshakes and n = number of people who are shaking hands.

# **Class Questions:**

a) How many handshakes are necessary if 43 people what to greet each other exactly once?

**Answer:** Total = 
$$\frac{(43)(42)}{2}$$
 = 903

b) There are ten soccer teams in a tournament. How many games must be played in order for each team to play every other team exactly once?

### **Answer:**

This is like the handshake question. Each team playing every other team is similar to each person shaking hands with every other person. Therefore the number of games that need to be played is  $G = \frac{(10)(9)}{2} = 45$ .

# **Extending The Problem**

How can we relate the handshake problem to finding the number of diagonals in a polygon? Here it might be helpful to have student work either with partners or small groups.

Have students draw a rectangle and then draw the diagonals.

[Answer: There should be 2.]

Does a triangle have any diagonals? [Answer: No]

Have students draw a five-sided figure and then draw the diagonals. [Answer = 5]

If there is time in the period have students work their way through six-, seven-, and eight-sided figures.

Keep a table on the board asking students to fill it in.

Number of Sides	Number of Diagonals
4	2
5	5
6	9
7	14
8	20
9	27

# Can we develop a formula for finding the number of diagonals for an n-sided figure?

Let's look at the problem in the context of handshakes. When we were investigating people it was clear that person A shakes hands with everyone except himself, which was represented by n-1. Thus the formula was *Total number of handshakes* =  $\frac{(n)(n-1)}{2}$ .

Draw a rectangle and label it ABCD. Think of the four points as persons A, B, C, and D. With respect to the handshake scenario a diagonal is a handshake with everyone except yourself and the two people adjacent to you. Thus person A cannot shake hands with himself, or persons B and C. This would imply that you have n people minus the three non-handshakes, i.e., n - 3.

Person B can only shake hands with C, since A and D are adjacent, and B won't shake hands with herself. Again we have n-3 rather than n-1.

# **Class Question**

Can you suggest a conjecture for finding the total number of diagonals of an n-sided figure?

#### **Answer:**

If Total number of handshakes = 
$$\frac{(n)(n-1)}{2}$$
 then

Total number of diagonals = 
$$\frac{(n)(n-3)}{2}$$

### **Class Ouestion:**

How can we verify that this formula is correct?

#### **Answer:**

Refer back to the table on the board and have students verify by computation that the number of diagonals that they got from their constructions were correct.